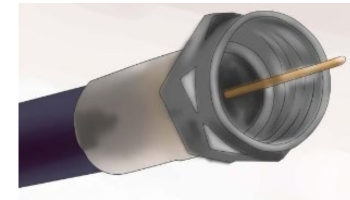
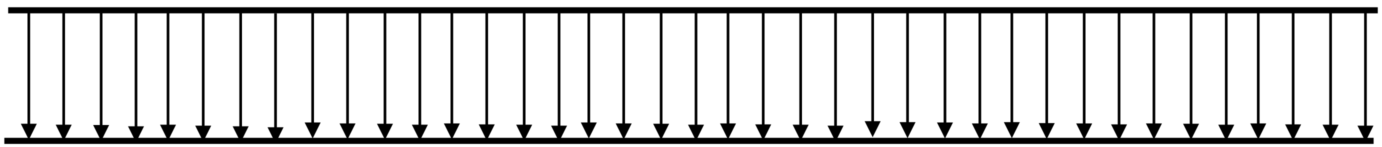
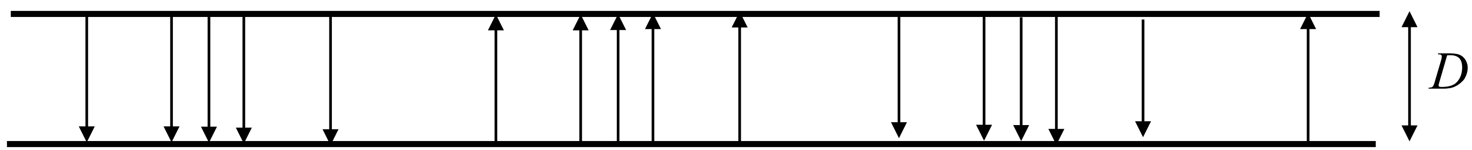
Consider a pair of wires **ideal** wires)



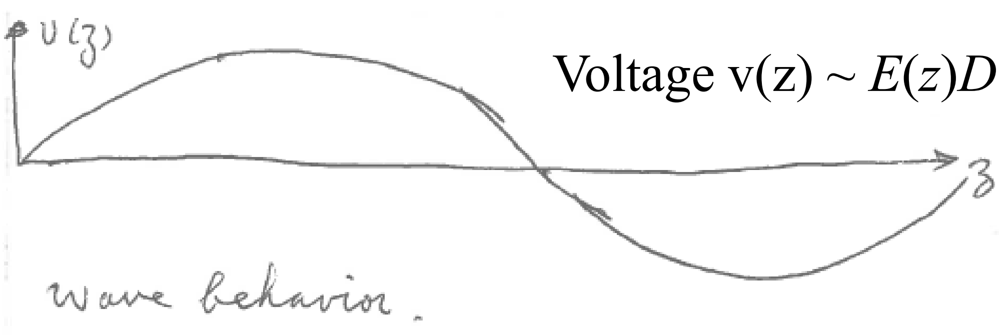
Length << λ



Length >> λ, say, infinitely long



Voltage along a cable can vary!



Voltage v(z) ~ $E(z)D$

We can actually get to this wave behavior by using circuit theory, w/o going into details of the EM fields!

There is capacitance between any two pieces of conductors.
A pair of plates, wires, etc., or the core and shields of a co-ax cable.

$$C = C'\Delta z$$

C': capacitance per length

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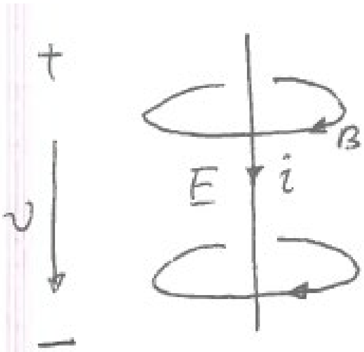
A piece of wire is actually an inductor



$B \propto i$

When i changes w/ t, so does B $\quad \frac{di}{dt} \rightarrow \frac{dB}{dt}$

$$v \propto E \propto \frac{\partial B}{\partial t} \Rightarrow v \propto \frac{di}{dt}$$

$$v = L\frac{di}{dt}$$

Voltage v along the wire

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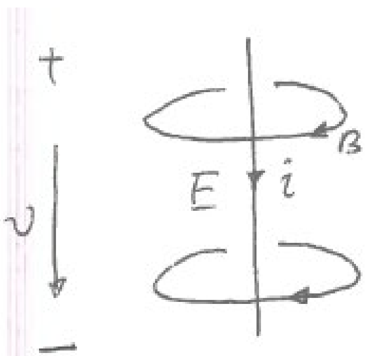

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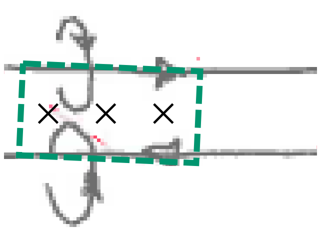
Voltage v along the wire

Similarly, a pair of coupled wires



Voltage v around the loop!

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A pair of plates, wires, etc., or the core and shields of a co-ax cable.

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C': capacitance per length

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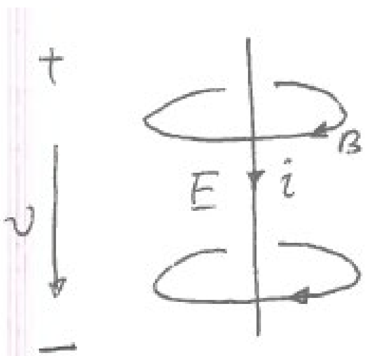

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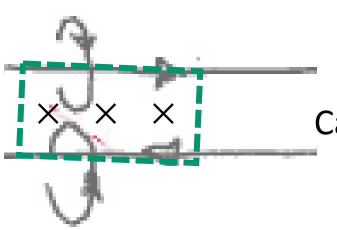
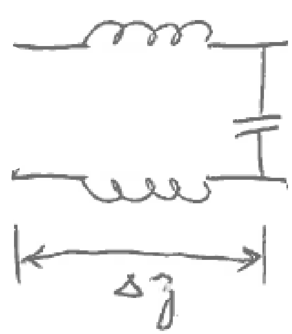
Voltage v along the wire

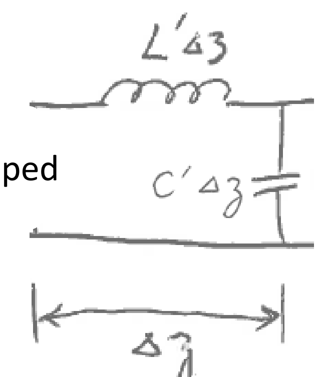
Similarly, a pair of coupled wires



Capacitance also considered



Inductance lumped to one side

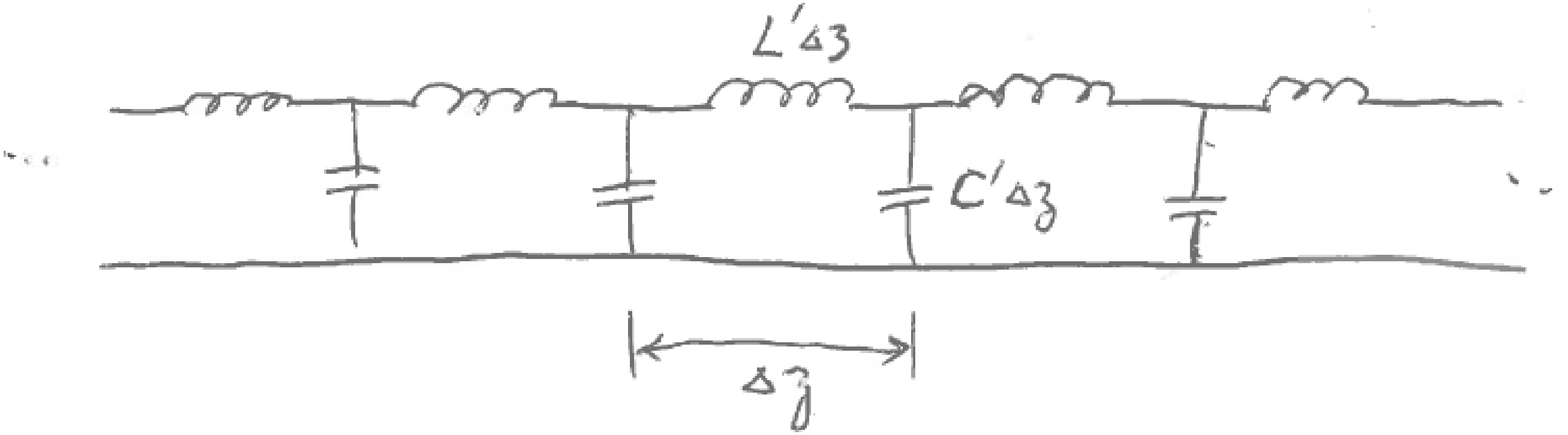


L' is inductance per length

$L = L'\Delta z$

Voltage v around the loop!

To make things simple, we first consider a pair of ***ideal*** wires.
No resistance, no shunt (leakage).



Pay close attention. We take a different approach than does the book.

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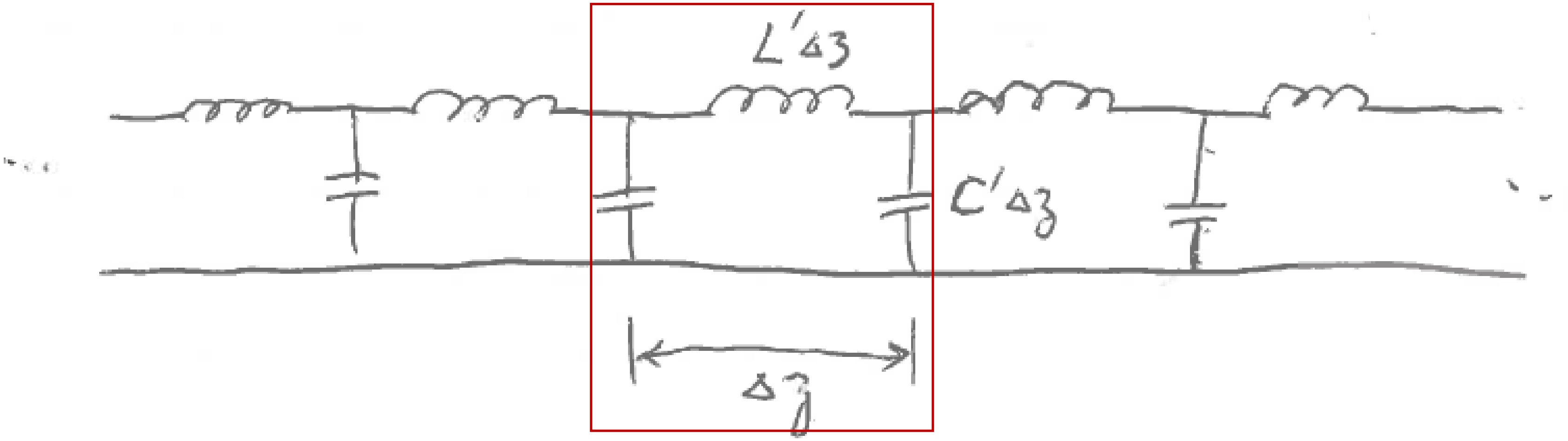
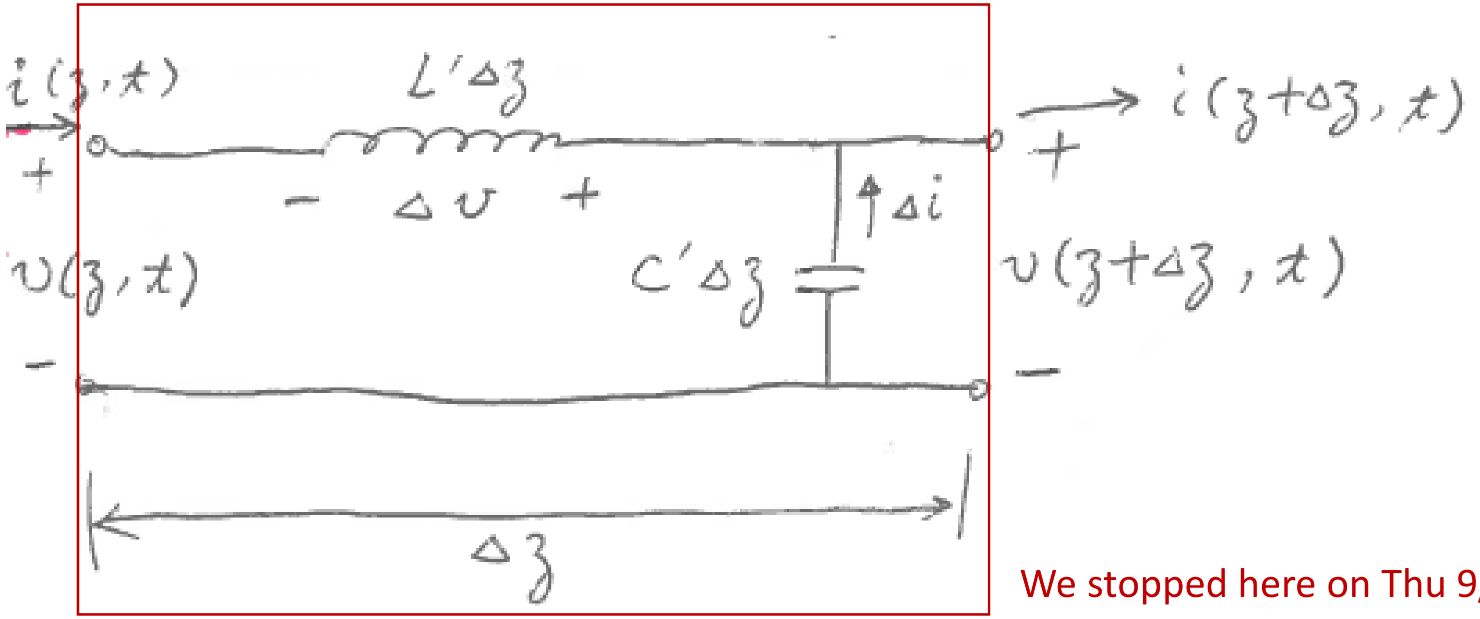
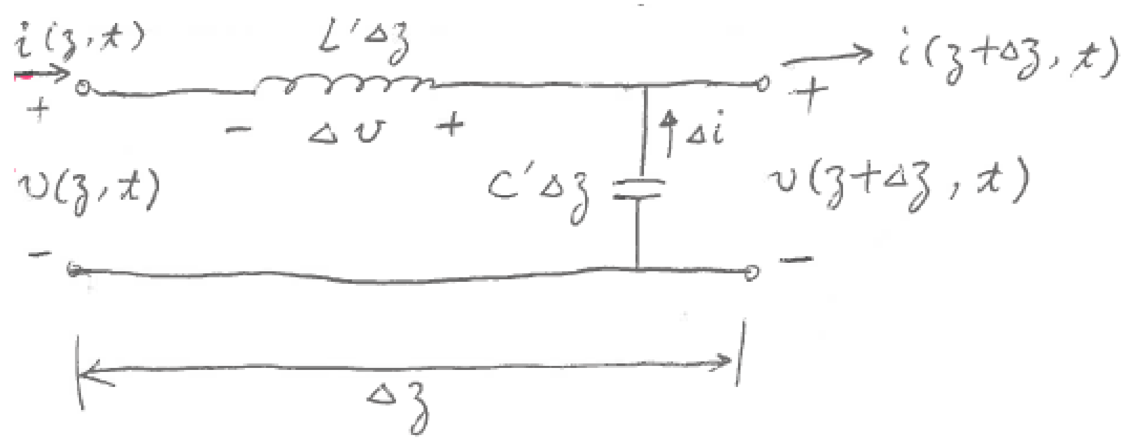


Pay close attention. We take a different approach than does the book.

Now, zoom in on one segment:



We stopped here on Thu 9/1/2022.



Inductor

$$\Delta v = v(z+\Delta z, t) - v(z,t) = -L'\Delta z \frac{\partial i}{\partial t}$$

$$\frac{\partial v}{\partial z} = \lim_{\Delta z \to 0} \frac{\Delta v}{\Delta z} = -L' \frac{\partial i}{\partial t}$$

Capacitor

$$\Delta i = i(z+\Delta z, t) - i(z,t) = -C'\Delta z \frac{\partial v}{\partial t}$$

$$\frac{\partial i}{\partial z} = \lim_{\Delta z \to 0} \frac{\Delta i}{\Delta z} = -C' \frac{\partial v}{\partial t}$$

Take derivatives with regard to *z*, *t*

$$\frac{\partial^2 v}{\partial z^2} = -L' \frac{\partial^2 i}{\partial t \partial z}$$

$$\frac{\partial^2 v}{\partial z \partial t} = -L' \frac{\partial^2 i}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial z^2} = -C' \frac{\partial^2 v}{\partial t \partial z}$$

$$\frac{\partial^2 i}{\partial z \partial t} = -C' \frac{\partial^2 v}{\partial t^2}$$

Inductor

$$\frac{\partial^2 v}{\partial z^2} = -L' \frac{\partial^2 i}{\partial t \partial z}$$

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Capacitor

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$$\frac{\partial^2 i}{\partial z \partial t} = -C' \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 v}{\partial z^2} = L'C' \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial z^2} = L'C' \frac{\partial^2 i}{\partial t^2}$$

Partial differential equations

Do these 2 equations look familiar to you? What are they?

Let $v_p = \frac{1}{\sqrt{L'C'}}$, we have $\frac{\partial^2 v}{\partial z^2} = \frac{1}{v_p^2} \frac{\partial^2 v}{\partial t^2}$

$v = f(v_p t - z)$ is the general solution to this equation.

Do it on your own: verify this.

$$\frac{\partial^2 v}{\partial z^2} = L'C' \frac{\partial^2 v}{\partial t^2}$$

Let $v_p = \frac{1}{\sqrt{L'C'}}$, we have $$\frac{\partial^2 v}{\partial z^2} = \frac{1}{v_p^2} \frac{\partial^2 v}{\partial t^2}$$

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$v = f(v_p t - z)$ is the general solution to this equation.

This is the wave equation in 1D!
More strictly, the lossless, dispersionless, linear wave equation.
Assume: no resistance, no leakage; v_p independent of frequency; v_p independent of voltage v

The equation for i is in the same form – formally the same.
Therefore, formally same solution.

$$\frac{\partial^2 i}{\partial z^2} = L'C'\frac{\partial^2 i}{\partial t^2}$$

These two together are called the telegrapher's equations.

Formally same wave equations for voltage & current

$$\begin{cases} \dfrac{\partial^2 v}{\partial z^2} = L'C' \dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial^2 i}{\partial z^2} = L'C' \dfrac{\partial^2 i}{\partial t^2} \end{cases}$$

Solution to both take general form $f(v_p t - z)$.

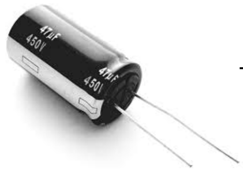
Is this amazing?

We arrived at the wave equation from circuit theory, regardless of frequency.

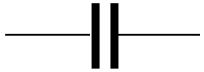
Why does this approach work?

Circuit theory is a simple part of EM (**black boxes: lumped elements**)

component



element



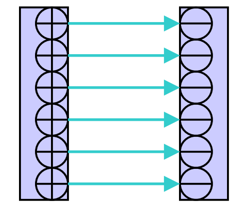
$$i \propto dv/dt$$

Inside the black boxes:

$Q \propto v \Rightarrow Q \equiv Cv$

C is a proportional constant.

$$C\frac{dv}{dt} = \frac{dQ}{dt} = i$$



Current is charge flow per time.

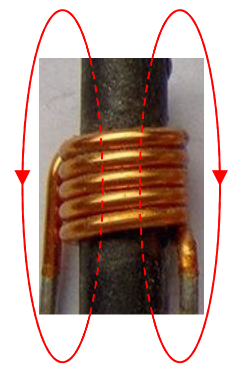
component

element

$$v = L\frac{di}{dt}$$

$B \propto i$

$di/dt \propto dB/dt \propto E \propto v$



Changing **B** field induces **E**.

A wire (or a pair of wires) is also an inductor!

These "lumped" element models are valid only when dimensions << wavelength

A circuit element is a model of a physical phenomenon, not necessarily a circuit component.

Formally same wave equations for voltage & current

$$\begin{cases} \dfrac{\partial^2 v}{\partial z^2} = L'C' \dfrac{\partial^2 v}{\partial t^2} \\[2ex] \dfrac{\partial^2 i}{\partial z^2} = L'C' \dfrac{\partial^2 i}{\partial t^2} \end{cases}$$

Solution to both take general form $f(v_p t - z)$.

Is this amazing?
We arrived at the wave equation from circuit theory, regardless of frequency.
Why does this approach work?

One more agreement, for 2-wire cables:

$$L' = \frac{\mu}{\pi} \ln\left[\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right]$$

$$C' = \frac{\pi \varepsilon}{\ln\left[\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right]}$$

$$\therefore \quad v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

See Table 2-1, pp. 52 in 8/E (pp. 45 in 7/E, pp.53 in 6/E)

Consistent with EM theory (to be discussed later)!
Check offline for other transmission lines.

(There will be homework problems for you to learn about other types of transmission lines, as well as non-ideal co-ax cables)

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{v_p^2} \frac{\partial^2 v}{\partial t^2}$$

(the equation for i is in the same form – formally the same.)

$v = f(v_p t - z)$ is the general solution to this equation.

What are the single frequency, simple harmonic solutions?

Single frequency, simple harmonic solutions:

$$v(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi_{v0}^+)$$

$$\frac{\omega}{\beta} = v_p$$

$$i(z,t) = |I_0^+| \cos(\omega t - \beta z + \phi_{i0}^+)$$

(formally the same equation, thus formally the same solution.)

Here, V_0^+ and I_0^+ are "complex amplitudes" that we will talk about later. For the waves, they are not the phasors of the waves. We will talk about the distinction.

$|V_0^+|$ and $|I_0^+|$ are the real amplitudes, or simply amplitudes.

We have not yet shown the voltage and current waves are in phase. But they are. You can take this as a conclusion for now. The proof is on next page.

Here we show that the voltage and current waves are in phase with each other:

$$v\frac{\partial v}{\partial z} = \beta\,|V_0^+|\sin(\omega t - \beta z + \phi_{v0}^+)$$

$$i(z,t) = |I_0^+|\cos(\omega t - \beta z + \phi_{i0}^+)$$

$$\frac{\partial v}{\partial z} = \beta\,|V_0^+|\,\sin(\omega t - \beta z + \phi_{v0}^+)$$

$$\frac{\partial i}{\partial t} = -\omega\,|I_0^+|\,\sin(\omega t - \beta z + \phi_{i0}^+)$$

Recall that $\frac{\partial v}{\partial z} = \lim_{\Delta z \to 0} \frac{\Delta v}{\Delta z} = -L'\frac{\partial i}{\partial t}$

For this to hold for any arbitrary z, we must have $\phi_{v0}^+ = \phi_{i0}^+ \equiv \phi_0^+$

So, in phase!

Do not just read through.
Use scratch paper!

We also get a by-product:

$$v(z,t) = |V_0^+|\cos(\omega t - v_p\ \phi_{v0}^+)$$

Unit: $\frac{\text{m}}{\text{s}}\frac{\text{H}}{\text{m}} = \text{H/s} = \Omega$

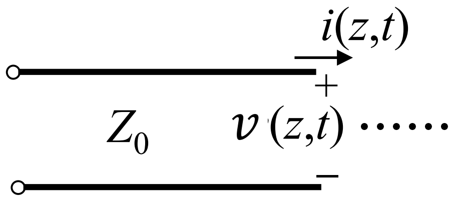
These conclusions are **important**!

Anywhere, any time $v(z,t)/i(z,t)$ = constant

Define $\dfrac{|V_0^+|}{|I_0^+|} = \dfrac{\omega L'}{\beta} = v_p L' \equiv Z_0$

($Z_0 = v(z,t)/i(z,t)$ is real, i.e., purely resistive, for lossless lines)

Consider:



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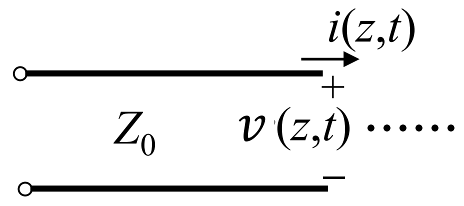
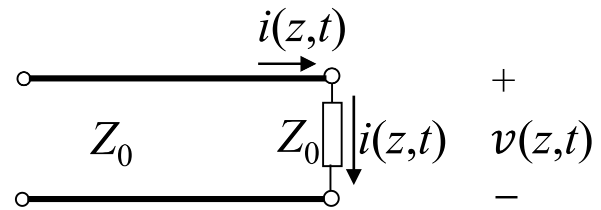


Infinitely long

vs.



There is no way to tell the difference just by measuring v and i:

Energy propagating away vs. energy dissipated

Analogy: laser beam going to infinity or hitting a totally black wall

Define $\dfrac{|V_0^+|}{|I_0^+|} = \dfrac{\omega L'}{\beta} = v_p L' \equiv Z_0$

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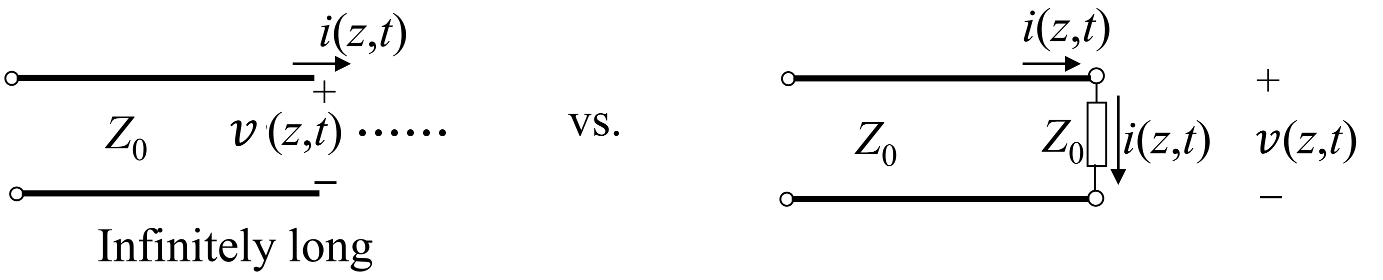

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THE most important new concept in the first half of this course

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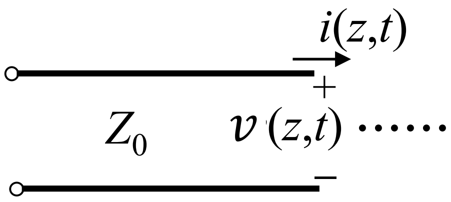
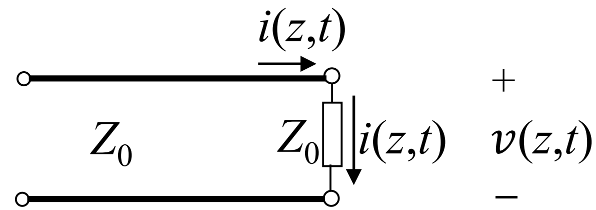


Infinitely long

vs.



There is no way to tell the difference just by measuring v and i:

Energy propagating away vs. energy dissipated

Analogy: laser beam going to infinity or hitting a totally black wall

THE most important new concept in the first half of this course

You may also use $\dfrac{\partial i}{\partial z} = \lim\limits_{\Delta z \to 0} \dfrac{\Delta i}{\Delta z} = -C' \dfrac{\partial v}{\partial t}$

Doing the derivatives in a similar way as in last page, you will also see the voltage and current waves are in phase.

You will have a similar "by-product" about the v/i ratio.

It may look different, but you should be able to show they are equal.

Do it on your own. Hint: use $v_p = \dfrac{1}{\sqrt{L'C'}}$ Use scratch paper!

Now you can work on HW2: P1, P2

The solution copied:

$$v^+(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi_0^+)$$

$$i^+(z,t) = |I_0^+| \cos(\omega t - \beta z + \phi_0^+)$$

In what direction do these waves propagate?

Waves propagating the other way are also solutions to the same equations:

$$v^-(z,t) = |V_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

$$i^-(z,t) = |I_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

The superscript + or – signifies propagation direction: $+z$ or $-z$.

Of course, any linear combinations of waves in opposite directions are also solutions:

$$v(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi_0^+) + |V_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

$$i(z,t) = |I_0^+| \cos(\omega t - \beta z + \phi_0^+) + |I_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

They may represent combinations of incident and reflected waves.

Recall that we have a mathematical tool to

1. Avoid the pain of dealing trigonometric functions, and
2. Turn partial differential equations to ordinary differential equations by putting aside the known time variation

Express waves with phasors

$$v(z,t) = |V_0^+|\cos(\omega t - \beta z + \phi_0^+) + |V_0^-|\cos(\omega t + \beta z + \phi_0^-)$$

phasor

$$\widetilde{V} = \widetilde{V}^+ + \widetilde{V}^- = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Where do the phases go?

Express waves with phasors

$$v(z,t) = |V_0^+|\cos(\omega t - \beta z + \phi_0^+) + |V_0^-|\cos(\omega t + \beta z + \phi_0^-)$$

phasor

$$\widetilde{V} = \widetilde{V}^+ + \widetilde{V}^- = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Where do the phases go?

Recall the details how we convert a time-domain function to a phasor:

By adding an "imaginary partner"

$$v(z,t) \rightarrow |V_0^+| e^{j(\omega t - \beta z + \phi_0^+)} + |V_0^-| e^{j(\omega t + \beta z + \phi_0^-)} = [(|V_0^+| e^{j\phi_0^+})e^{-j\beta z} + (|V_0^-| e^{j\phi_0^-})e^{j\beta z}]e^{j\omega t}$$

This is not the phasor yet.

Throw away the known time variation $e^{j\omega t}$

and define the complex amplitudes $V_0^+ = |V_0^+| e^{j\phi_0^+}$ and $V_0^- = |V_0^-| e^{j\phi_0^-}$

We get the phasor:

$$\widetilde{V} = \widetilde{V}^+ + \widetilde{V}^- = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Positive going

Negative going

Notice sign and direction

Express waves with phasors

The current wave is similar

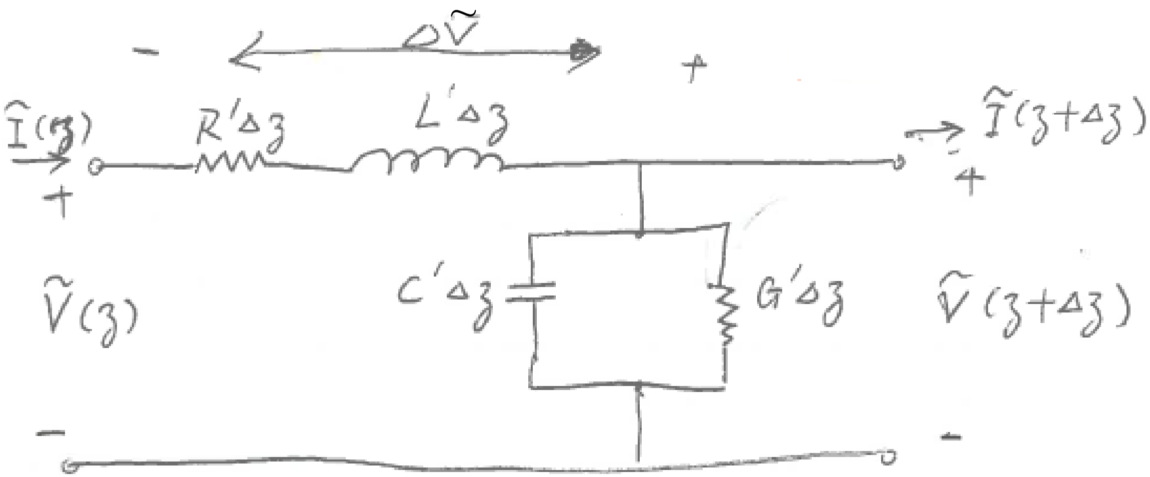
$$i(z,t) = |I_0^+|\cos(\omega t - \beta z + \phi_0^+) + |I_0^-|\cos(\omega t + \beta z + \phi_0^-)$$

phasor

$$\tilde{I} = \tilde{I}^+ + \tilde{I}^- = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$$

This tool makes our life much easier when we deal with a more complicated situation.

Here's the more complicated situation:



No wires are ideal.
Any wire has some resistance.

$$R = R'\Delta z$$

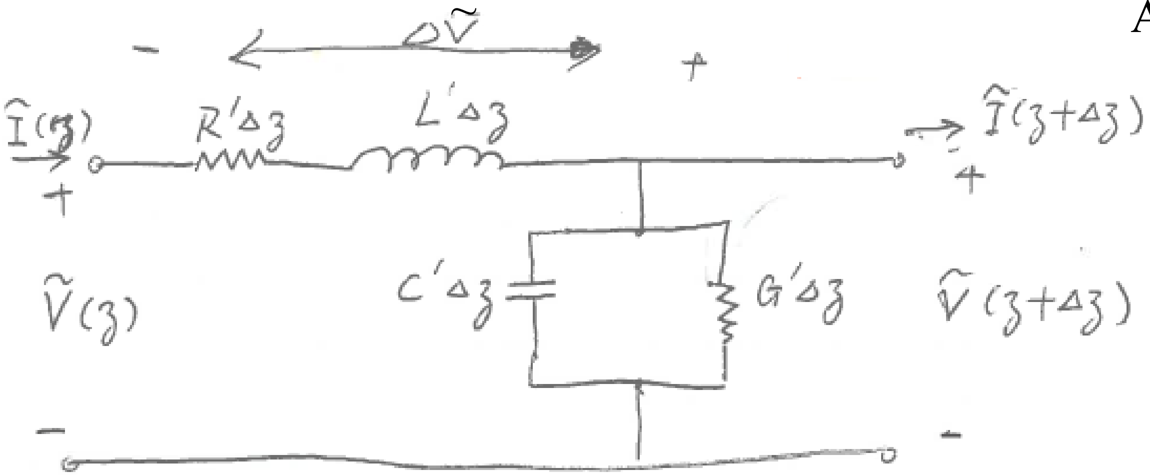
Resistance per lenght

There is always some shunt conductance between two wires

$$G = G'\Delta z$$

Shunt conductance per length

Notice that *R'* and *G'* describes two different things. $R' \neq 1/G'$



Analyze the circuit in the phasor way

$$\frac{d\tilde{V}}{dz} = \lim_{\Delta z \to 0} \frac{\Delta \tilde{V}}{\Delta z} = -(R' + j\omega L')\tilde{I}$$

$$\frac{d\tilde{I}}{dz} = \lim_{\Delta z \to 0} \frac{\Delta \tilde{I}}{\Delta z} = -(G' + j\omega C')\tilde{V}$$

Take derivatives on one equation and insert it into the other, you get

$$\frac{d^2 \tilde{V}}{dz^2} - (R' + j\omega L')(G' + j\omega C')\tilde{V}(z) = 0$$

and a formally same equation for the current.

This is an ordinary differential equation. Because we used phasors.

We have arrived at this equation just by circuit analysis using phasors.
You could also first do the circuit analysis in the time domain, arriving at partial differential equations, and then convert quantities to phasors and arrive at the same ordinary differential equations, as done in the book (Sections 2-3 & 2-4).

The partial differential equations for the general, more complicated situation are, well, too complicated. We don't even bother to tackle them.
Let's look at the simpler ordinary differential equation:

$$\frac{d^2\tilde{V}}{dz^2} - (R' + j\omega L')(G' + j\omega C')\tilde{V}(z) = 0$$

Before solving this equation, let's first have a digression back to the ideal case

$$R' = 0 \qquad G' = 0$$

$$\frac{d^2\tilde{V}}{dz^2} + \omega^2 L'C'\tilde{V} = 0 \quad \Longrightarrow \quad \tilde{V}(z) = V_0^{\pm} e^{\mp j\omega\sqrt{L'C'}z}$$

Recall that $\frac{\omega}{\beta} = v_p = \frac{1}{\sqrt{L'C'}} \Rightarrow \beta = \omega\sqrt{L'C'}$

we have $\tilde{V} = V_0^{\pm} e^{\mp j\beta z}$

Notice that these are actually two solutions.
What are the difference between the two solutions?

Waves propagating in two opposite directions: $\tilde{V} = \tilde{V}^{+} + \tilde{V}^{-} = V_0^{+} e^{-j\beta z} + V_0^{-} e^{j\beta z}$

No surprise. The solutions we got earlier for the ideal case.

Now, back to the more complicated, general case

$$\frac{d^2 \tilde{V}}{dz^2} - (R' + j\omega L')(G' + j\omega C')\tilde{V}(z) = 0$$

Let $\gamma^2 = (R' + j\omega L')(G' + j\omega C')$

we can write $\dfrac{d^2 \tilde{V}}{dz^2} - \gamma^2 \tilde{V}(z) = 0$

Compare this to the ideal case $\dfrac{d^2 \tilde{V}}{dz^2} + \omega^2 L'C' \tilde{V} = 0$

$\beta = \omega\sqrt{L'C'}$ $\qquad$ β^2 $\qquad$ $\tilde{V} = V_0^{\pm} e^{\mp j\beta z}$

These two equations are "formally" the same, except $-\gamma^2$ is complex.

$\beta^2 \rightarrow -\gamma^2$ thus $-j\beta \rightarrow -\gamma$ and $j\beta \rightarrow \gamma$

So, the solutions are $\tilde{V} = V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z}$

What kind of waves are they?

$$\tilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

What kind of waves are they?

Let $\gamma = \alpha + j\beta$ (we are doing nothing. Any complex number can be written as this), we have the first solution

$$\tilde{V}^+ = V_0^+ e^{-\gamma z} = V_0^+ e^{-(\alpha + j\beta) z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

What kind of wave is this?

Similarly, $\tilde{I}^+ = I_0^+ e^{-\gamma z} = I_0^+ e^{-(\alpha + j\beta) z} = I_0^+ e^{-\alpha z} e^{-j\beta z}$

Why do the waves attenuate when there is resistance or shunt leakage? (Why is there no attenuation when the wire is made of a perfect conductor and the medium between them is a perfect insulator)?

Recall that, in circuit theory, reactive versus resistive..., ...

How to express the decaying wave propagating in the $-z$ direction?

Again, there can be waves going the other way.

$$\widetilde{V} = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z}$$

sign

$$\widetilde{I} = I_0^+ e^{-\alpha z} e^{-j\beta z} + I_0^- e^{\alpha z} e^{j\beta z}$$

sign

We were here at the end of class on Tue 9/6/2022.

Again, there can be waves going the other way.

$$\tilde{V} = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z}$$

$$\tilde{I} = I_0^+ e^{-\alpha z} e^{-j\beta z} + I_0^- e^{\alpha z} e^{j\beta z}$$

sign

sign

Again, we discuss **the most important concept** of the first half of the semester:

$$\tilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

Take derivatives ⟹

$$\frac{d\tilde{V}}{dz} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z}$$

Again, there can be waves going the other way.

$$\tilde{V} = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \qquad \tilde{I} = I_0^+ e^{-\alpha z} e^{-j\beta z} + I_0^- e^{\alpha z} e^{j\beta z}$$

sign sign

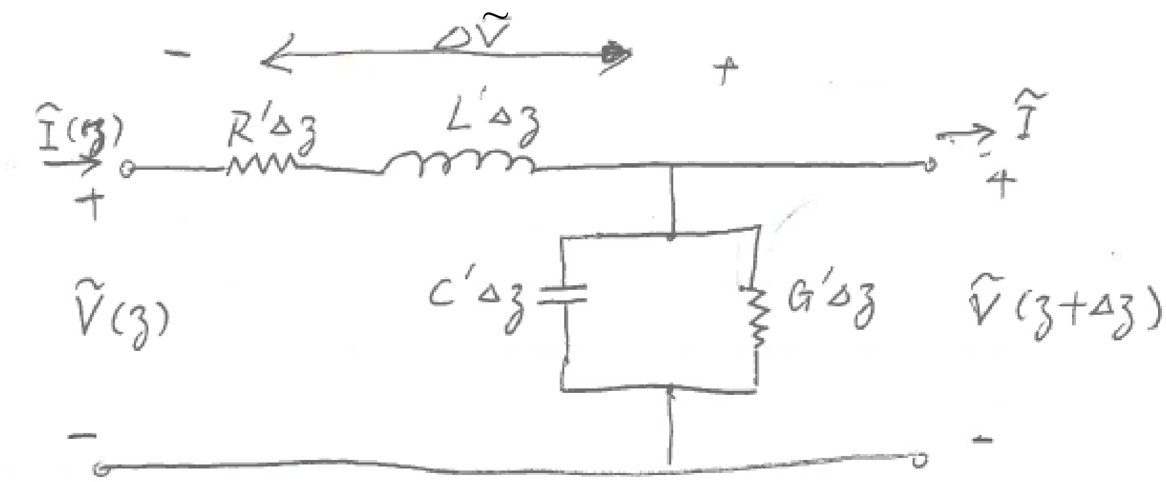
Again, we discuss **the most important concept** of the first half of the semester:

Take derivatives

$$\tilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \Longrightarrow \quad \frac{d\tilde{V}}{dz} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z}$$

From circuit analysis

$$\frac{d\tilde{V}}{dz} = \lim_{\Delta z \to 0} \frac{\Delta \tilde{V}}{\Delta z} = -(R' + j\omega L')\tilde{I} = -(R' + j\omega L') I_0^+ e^{-\alpha z} e^{-j\beta z} - (R' + j\omega L') I_0^- e^{\alpha z} e^{j\beta z}$$



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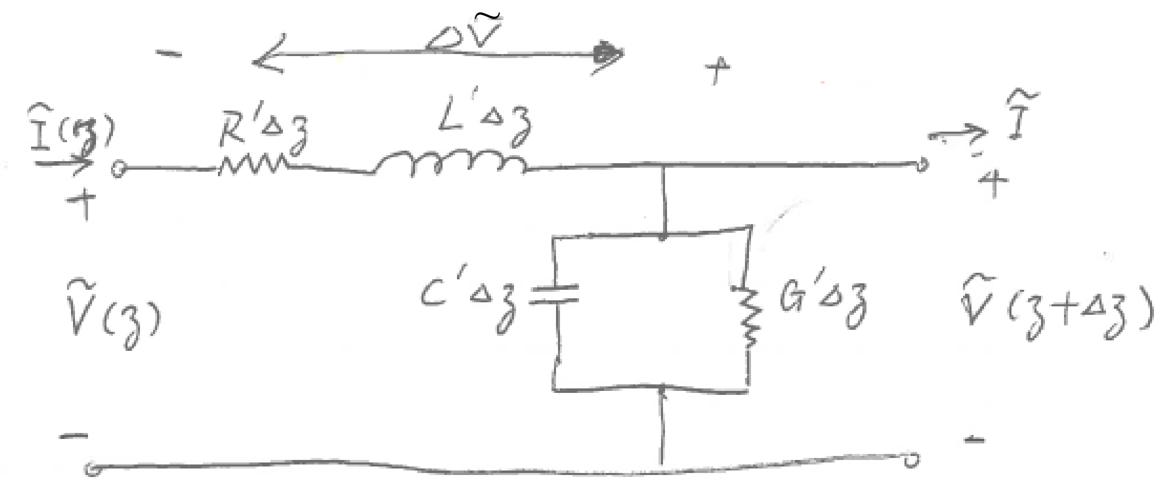

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$$\tilde{V} = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \qquad \tilde{I} = I_0^+ e^{-\alpha z} e^{-j\beta z} + I_0^- e^{\alpha z} e^{j\beta z}$$

sign sign

Again, we discuss **the most important concept** of the first half of the semester:

$$\tilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \xrightarrow{\text{Take derivatives}} \quad \frac{d\tilde{V}}{dz} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z}$$

From circuit analysis

$$\frac{d\tilde{V}}{dz} = \lim_{\Delta z \to 0} \frac{\Delta \tilde{V}}{\Delta z} = -(R' + j\omega L')\tilde{I} = -(R' + j\omega L') I_0^+ e^{-\alpha z} e^{-j\beta z} - (R' + j\omega L') I_0^- e^{\alpha z} e^{j\beta z}$$

For the identity to hold for all z, we must have the following:

For the $e^{-\gamma z}$ term,

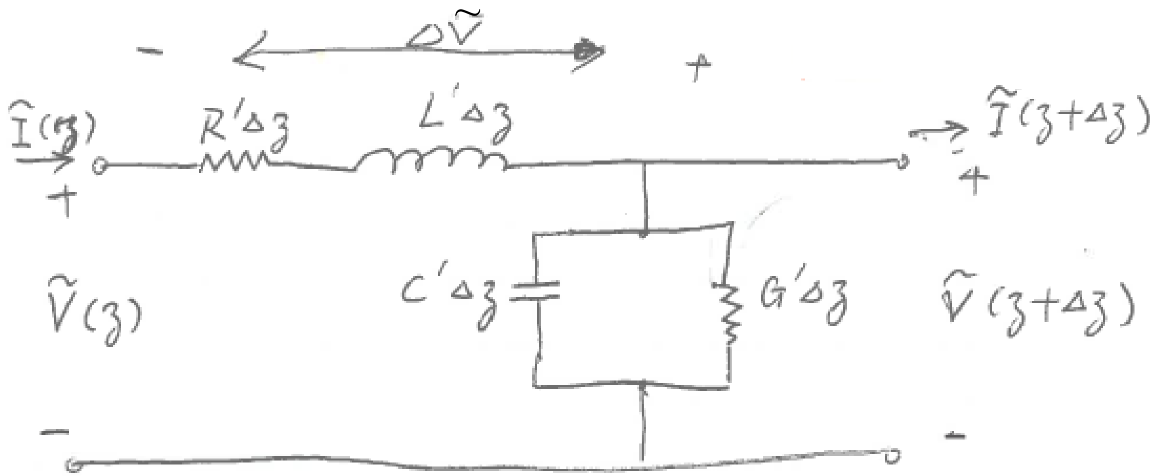
$$\gamma V_0^+ = (R' + j\omega L') I_0^+$$

$$\Rightarrow \quad \frac{V_0^+}{I_0^+} = \frac{R' + j\omega L'}{\gamma} = \frac{R' + j\omega L'}{\sqrt{(R' + j\omega L')(G' + j\omega C')}} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

For the $e^{\gamma z}$ term, $\gamma V_0^- = -(R' + j\omega L')I_0^-$

$$\Rightarrow \quad \frac{V_0^-}{I_0^-} = -\frac{R' + j\omega L'}{\gamma} = -\frac{R' + j\omega L'}{\sqrt{(R' + j\omega L')(G' + j\omega C')}} = -\sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Notice negative signs. Just because of sign convention (see circuit diagram)



Define $Z_0 \equiv \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}}$

the characteristic impedance

Complex and explicitly dependent on frequency in the general (lossy) case.

For the wave traveling towards $+z$,

$$\tilde{V}^+ = V_0^+ e^{-\gamma z}$$

$$\tilde{I}^+ = I_0^+ e^{-\gamma z}$$

At any z, $\dfrac{\tilde{V}^+(z)}{\tilde{I}^+(z)} = \dfrac{V_0^+}{I_0^+} = Z_0$

For the wave traveling towards $-z$,

$$\tilde{V}^- = V^- e^{\gamma z}$$

$$\tilde{I}^- = I_0^- e^{\gamma z}$$

At any z, $\dfrac{\tilde{V}^-(z)}{\tilde{I}^-(z)} = \dfrac{V_0^-}{I_0^-} = -Z_0$

Again, notice this negative sign.

These relations are **separately** held by the two waves in opposite directions.

In general, $Z_0 \equiv \sqrt{\dfrac{R'+j\omega L'}{G'+j\omega C'}}$ is complex and explicitly dependent on frequency.

Are V_0^+ and I_0^+ in phase in the general case?

For the lossless transmission line, $R' = 0$ and $G' = 0$,

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

Real. No explicit frequency dependence.

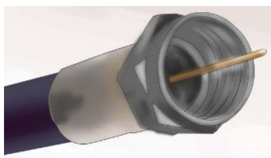
Take-home messages

- Voltage v and current i follow the same differential equation
- Therefore solutions in same form
- Therefore there is a constant ratio between their amplitudes and there is a constant shift between their phases for harmonic waves going in **one direction**
- In the phasor form, the complex amplitude ratio is $\pm Z_0$
- Being a voltage/current ratio, Z_0 has the dimension of impedance
- In general (lossy case), Z_0 is complex and explicitly depends on ω
- In the lossless case, Z_0 is real w/o explicit frequency dependence
- Z_0 being real means the voltage and the current are in phase. It also means the equivalent circuit for a (semi-)infinitely long transmission line is simply a resistor with a resistance value Z_0.

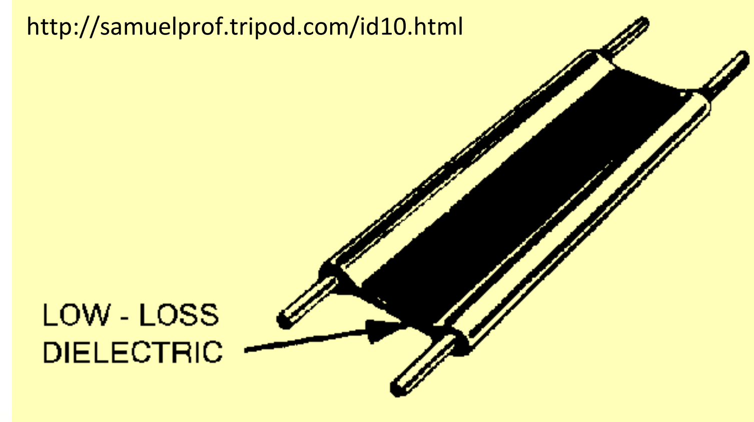
At this point, review textbook up to Section 2-4.
Finish P1 through P6 of HW2.

The cables

Coaxial cable

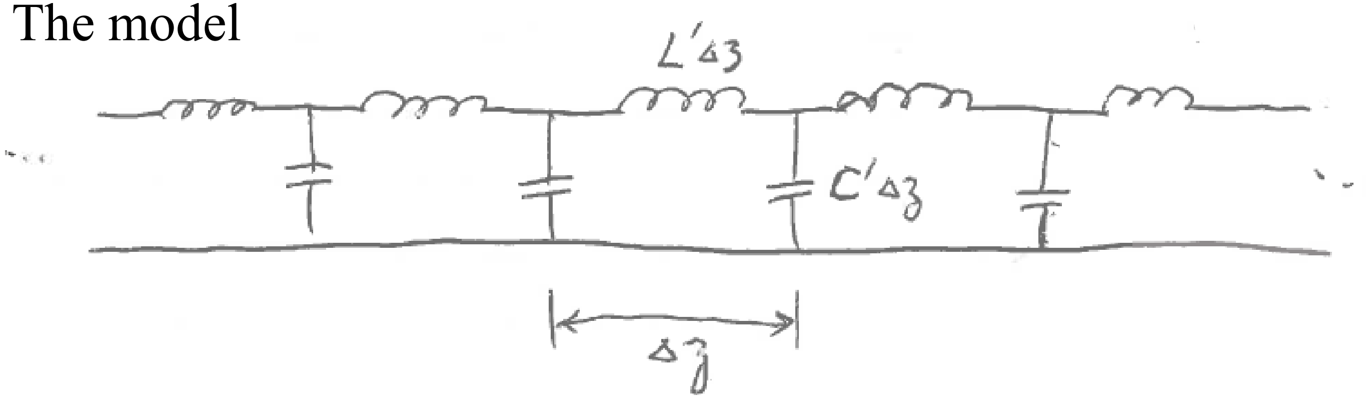


Ribbon twin lead: (used to be) used to connect a television receiving antenna to a home television set.



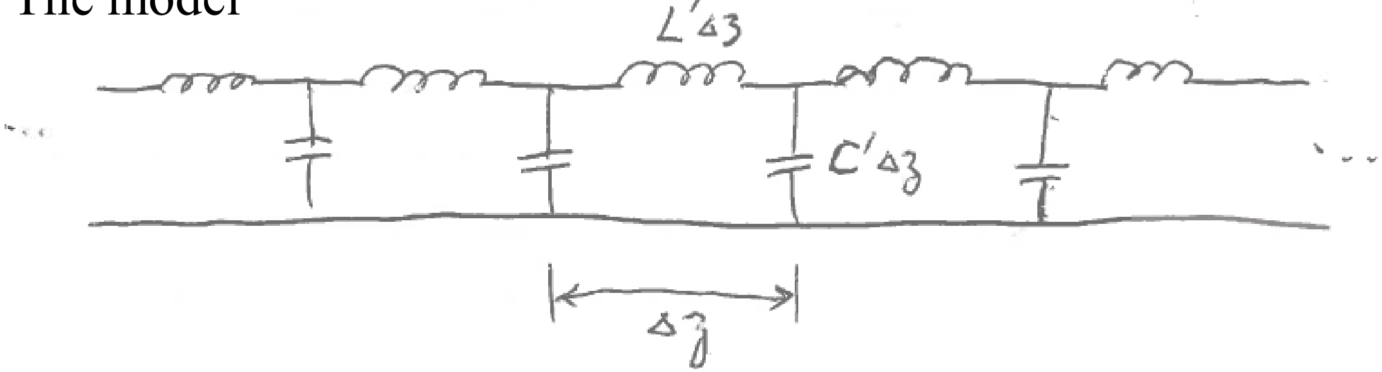
https://www.moonraker.eu/antennas/antenna-hardware/ribbon-feeder/75-m-75-ohm-twin-feeder

The model



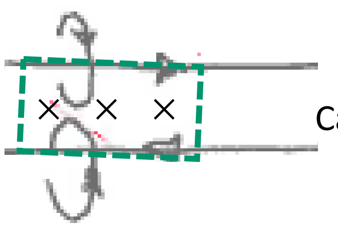
The inductors (and resistors in lossy lines) are on only one side.
Which side is which wire???

The model



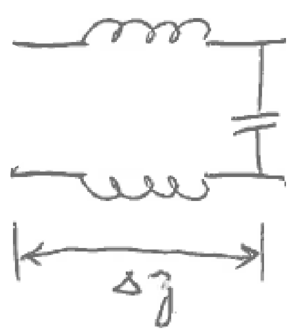
The inductors (and resistors in lossy lines) are on only one side.
Which side is which wire???

A pair of coupled wires

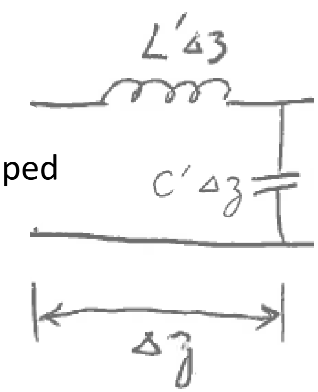


Voltage v around the loop!

Capacitance also considered



Inductance lumped to one side

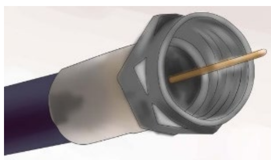


$L = L'\Delta z$

L' is inductance per length

The cables

Coaxial cable



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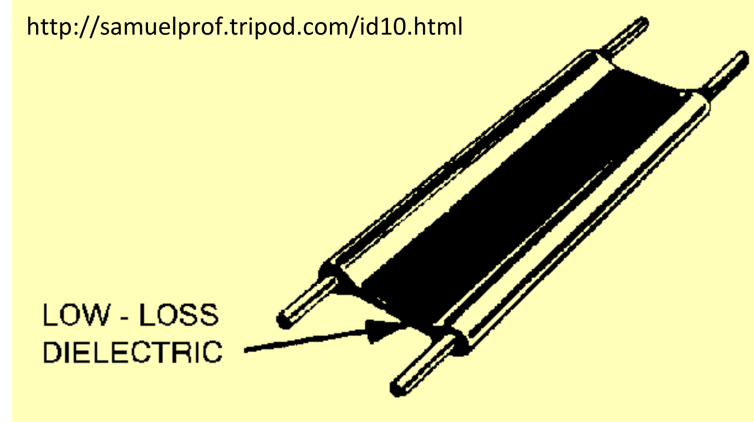
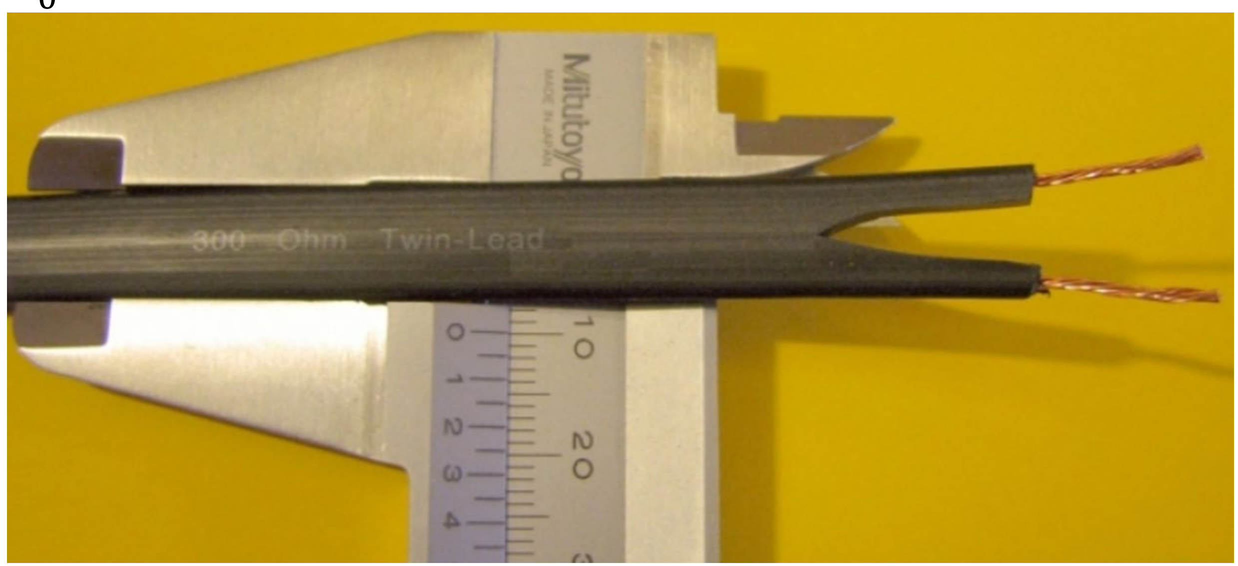


$Z_0 = 75\ \Omega$

https://www.moonraker.eu/antennas/antenna-hardware/ribbon-feeder/75-m-75-ohm-twin-feeder

$Z_0 = 300\ \Omega$



https://www.dx-wire.de/lng/en/wire-cable/300-ohm-twinlead/300-ohm-twin-lead.html

Question:

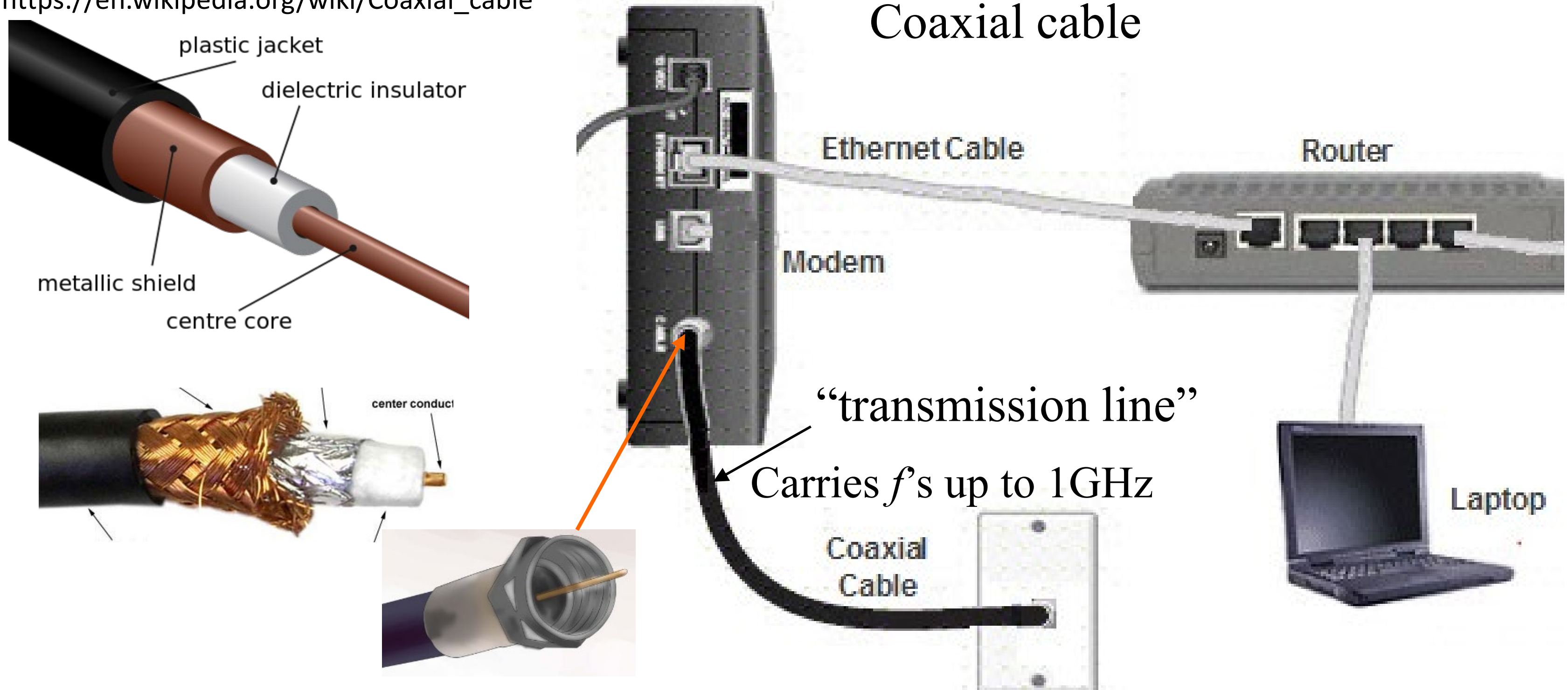
How to vary the Z_0 of the transmission line of a given construction?

Hint: Consider the simple lossless case

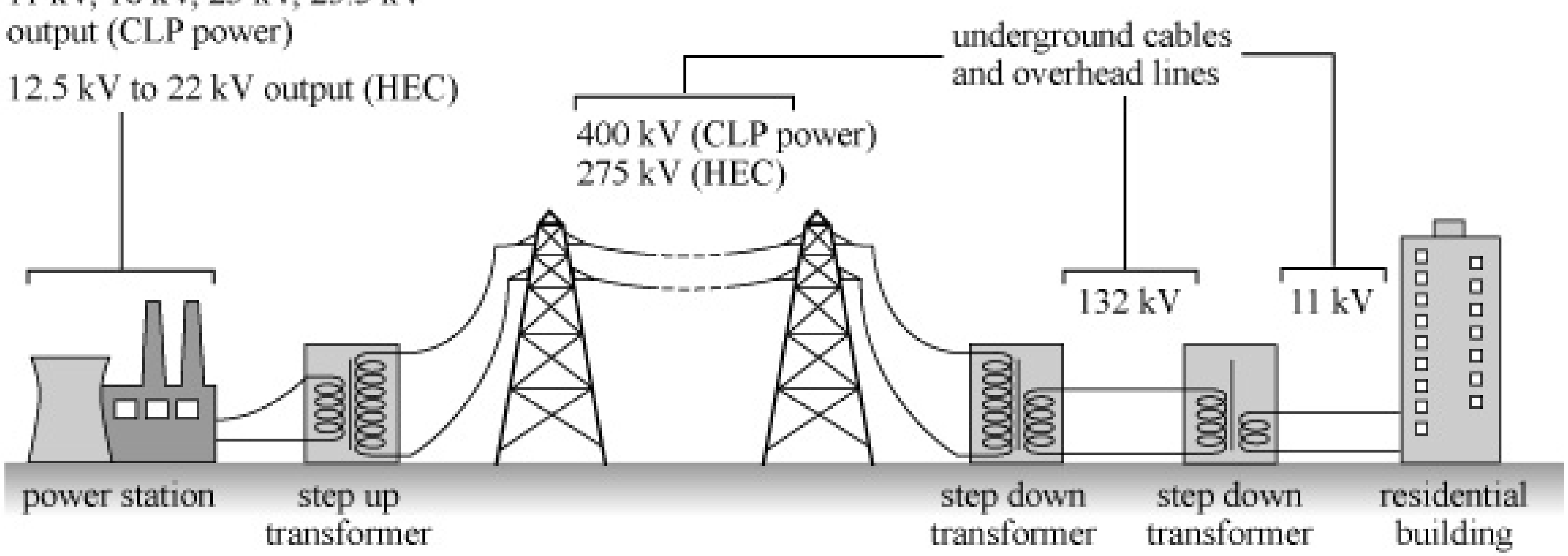
$$Z_0 = \sqrt{\frac{L'}{C'}}$$

and use Table 2-1 in Textbook.

https://en.wikipedia.org/wiki/Coaxial_cable

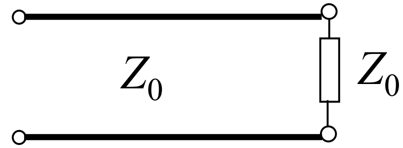


Power transmission line



Infinitely long

vs.

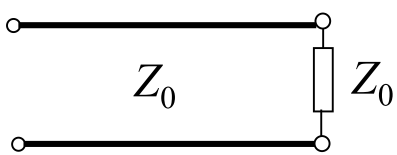


There is no way to tell the difference just by measuring v and i.

Energy propagating away vs. energy dissipated

Analogy: laser beam going to infinity or hitting a totally black wall

Impedance match



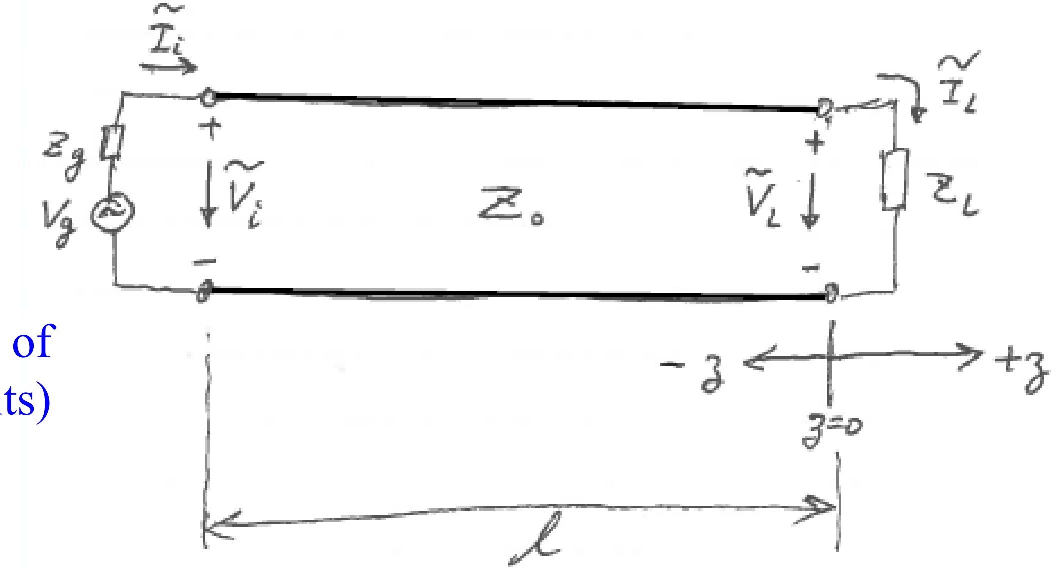
By the way, transmission line (thick line) versus "wire wires" (thin lines)

The same as the infinitely long line!

We want impedance match!

(Reasons?)

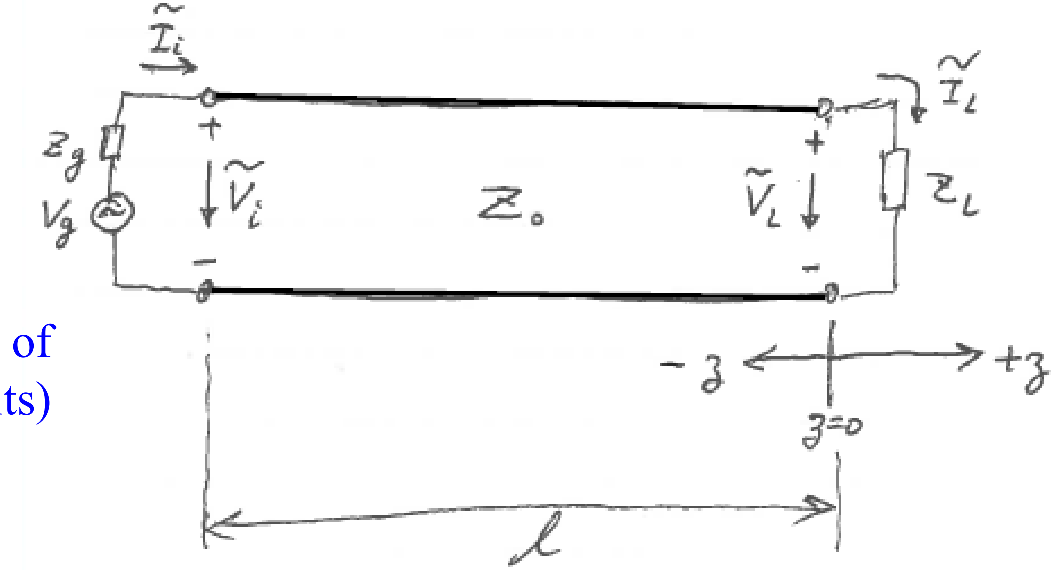
Now, let's look at a transmission line with a source and a load.



If $Z_L = Z_0$, impedance matched.
All energy delivered to load. Good!
(we can view this from the vantage point of equivalent circuits)

Now, we just focused on the load. Will talk about the line and generator later.

Now, let's look at a transmission line with a source and a load.



If $Z_L = Z_0$, impedance matched.
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What if $Z_L \neq Z_0$?

The load says $\dfrac{\tilde{V}(0)}{\tilde{I}(0)} = \dfrac{\tilde{V}_L}{\tilde{I}_L} = Z_L$

If there were only the incident wave, $\dfrac{\tilde{V}^+(0)}{\tilde{I}^+(0)} = \dfrac{V_0^+}{I_0^+} = Z_0$

Now, we just focused on the load. Will talk about the line and generator later.

Something has to happen to resolve this "conflict." That something is reflection.

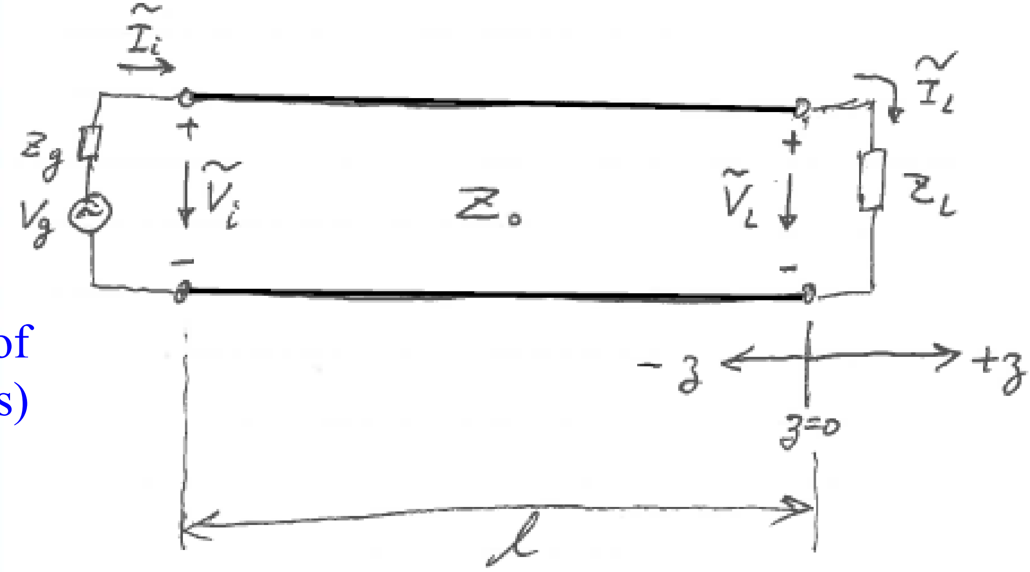
$$\tilde{V}_L = \tilde{V}(z=0) = V_0^+ + V_0^-$$

$$\tilde{I}_L = \tilde{I}(z=0) = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

Sign due to convention

Both waves separately follow $V_0^{\pm} = \pm I_0^{\pm} Z_0$

Now, let's look at a transmission line with a source and a load.



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Sign due to convention

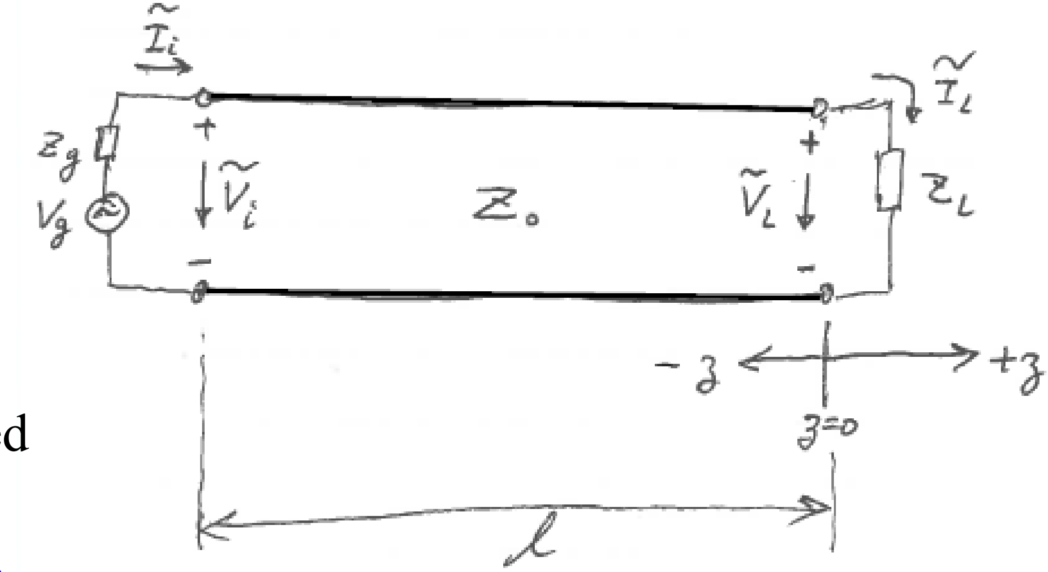
Both waves separately follow $V_0^{\pm} = \pm I_0^{\pm} Z_0$

By definition, $Z_L = \dfrac{\tilde{V}_L}{\tilde{I}_L} = \left(\dfrac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0$

Solve it and we have $V_0^- = \dfrac{Z_L - Z_0}{Z_L + Z_0} V_0^+$

If $Z_L \neq Z_0$, there has to be a reflected wave.

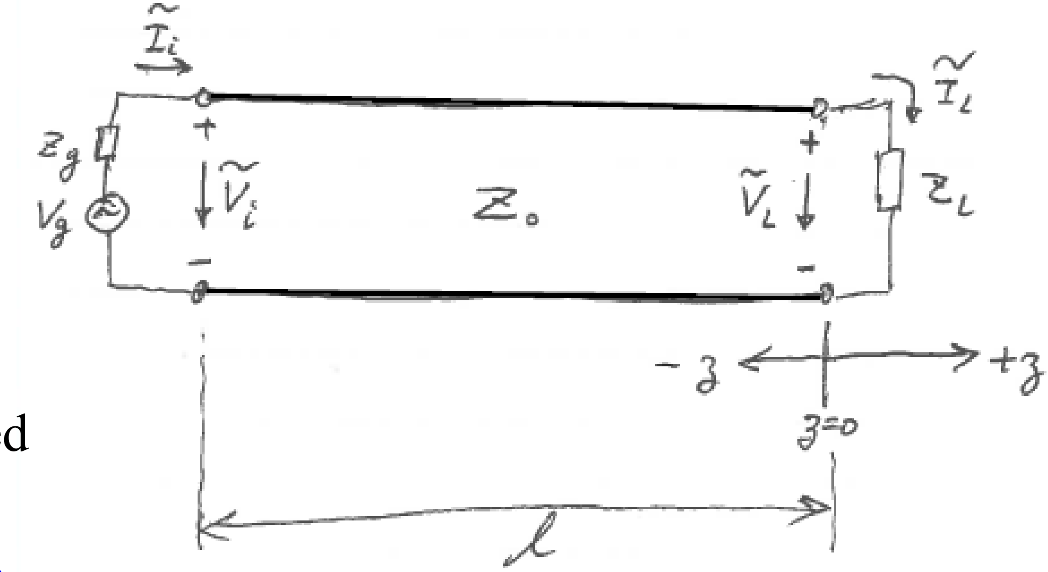
$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$



The load does not get all energy carried by the incident wave.

Where does the rest of the energy go?

If $Z_L \neq Z_0$, there has to be a reflected wave.



$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$

The load does not get all energy carried by the incident wave.

Where does the rest of the energy go?

Consider analogy: laser beam hitting wall not totally black/dark.

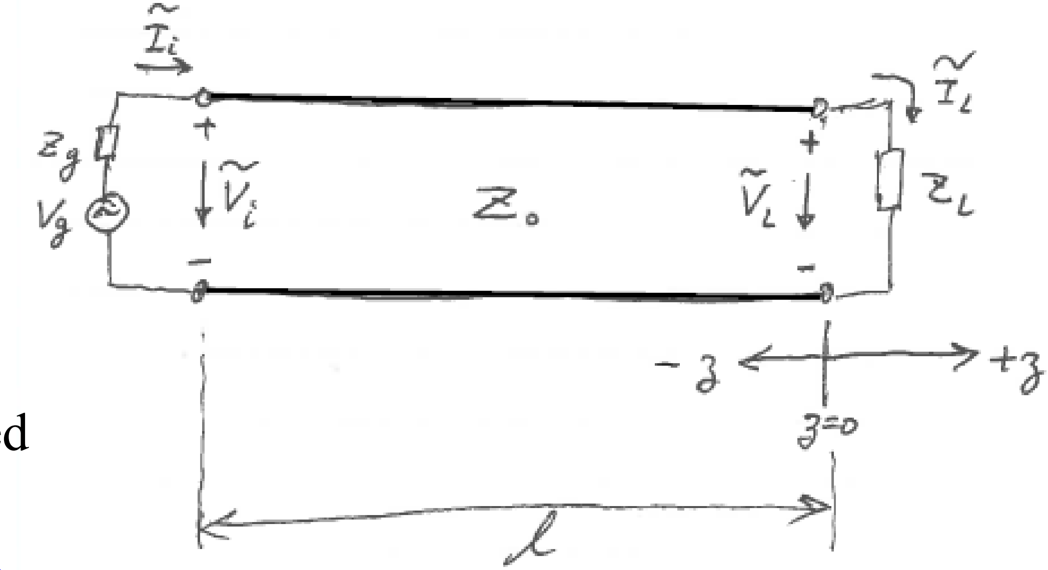
Define the voltage reflection coefficient $$\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

$$\Downarrow$$

$$\frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma}$$

If $Z_L \neq Z_0$, there has to be a reflected wave.

$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$



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$$\Downarrow$$

$$\frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma}$$

- One-to-one mapping between Γ and Z_L/Z_0
- The ratio Z_L/Z_0 more important than Z_L itself

Therefore we define the normalized load impedance $z_L = \dfrac{Z_L}{Z_0}$

Thus,

$$\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{z_L - 1}{z_L + 1}$$

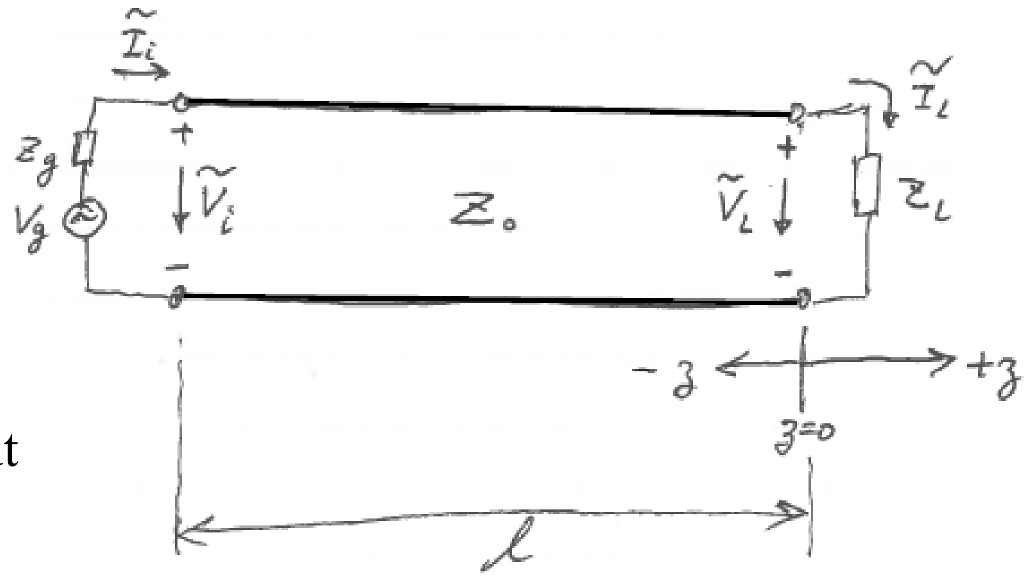
$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

Notice the one-to-one mapping. This is very important!

For the current

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma$$

Where does the negative sign come from?



$Z_0 = \sqrt{\dfrac{L'}{C'}}$ is real for a lossless line, but

$Z_0 \equiv \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}}$ is complex in general.

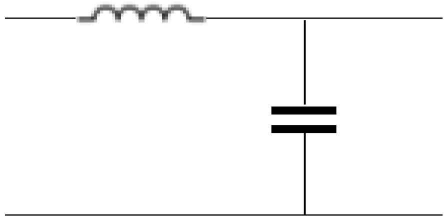
Z_L is complex in general. Thus, Γ is complex in general.

Read the textbook: Section 2-6 overview, 2-6.1

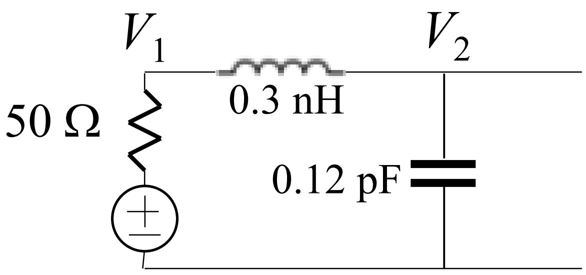
We finished this lecture on Thu 9/8/2022

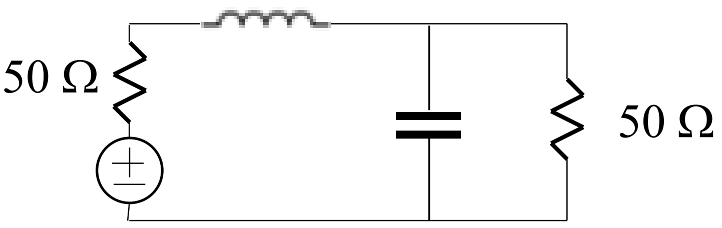
Project

Circuit simulations to transition you from lumped element-based circuit theory



Part 1





Generator: 1 V step, rise time = 0.1 ns. Internal impedance 50 Ω.

Plot the two voltages V_1 and V_2 for the above two cases.

Hint: You may make mistakes. Do a sanity check by a "back of an envelope" analysis.
At the very least, find out the steady state.
Does the simulation give you more or less what you expect?

Ongoing project. Stay tuned for next steps.

ADS Tutorial

Tue 9/13, in class. Bring your lap top.

Follow instructions on Canvas to get access to ADS **before** the class.

If you need more info or miss the class, see tutorial document on course website.
In the document, these is a link to a YouTube video.

Contact TAs for help if needed: GTA Tsotne Kvelashvili tkvelash@utk.edu and UGTA Graham Travis tgraham9@vols.utk.edu