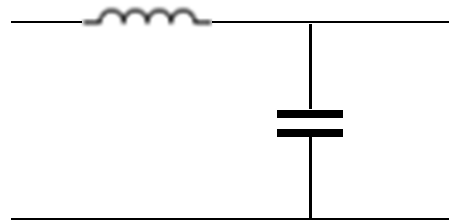
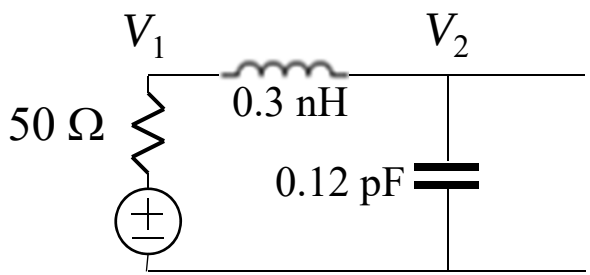
Project

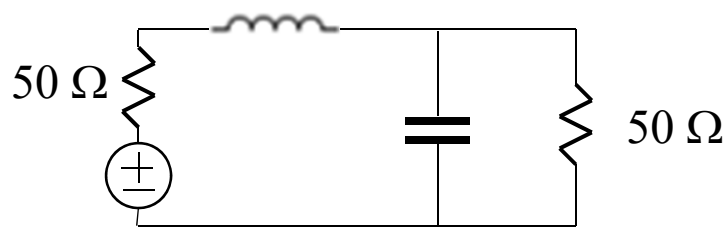
A circuit simulation project to transition you from lumped component-based circuit theory

In Part 1 and Part 2, you built an LC network:



And, you did transient simulations of the following circuits with the generator signal being voltage steps with different rise times:



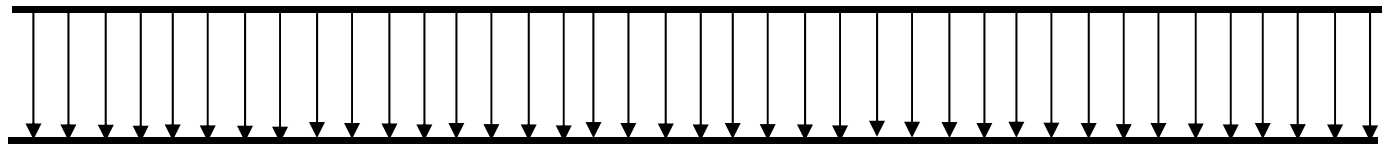


Part 3: Now, create a new network that is a cascade of 10 instances of the above LC network. You may create a symbol for this new network for convenience. Using the same inductance and capacitance values to do the same simulations you have done for the above single LC network. (Same generators with same internal impedance. Simulate for both open circuit and 50-ohm loads, the two rise times for each case, as done for the single LC.)

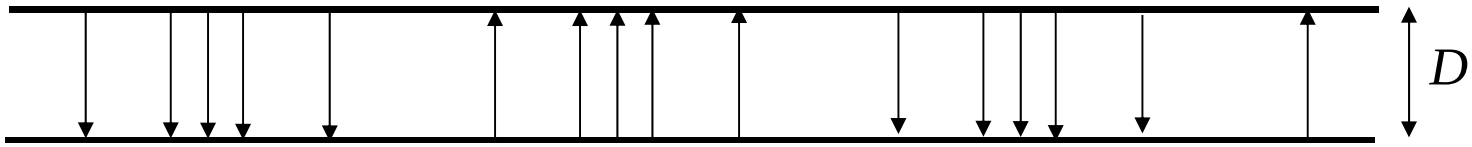
Ongoing project. Stay tuned for next steps.

Consider a pair of wires **ideal** wires)

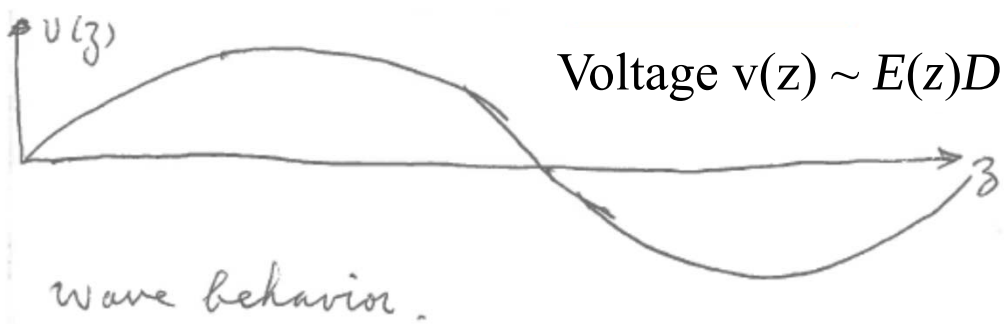
Length << λ



Length >> λ, say, infinitely long



Voltage along a cable can vary!



Voltage v(z) ~ $E(z)D$

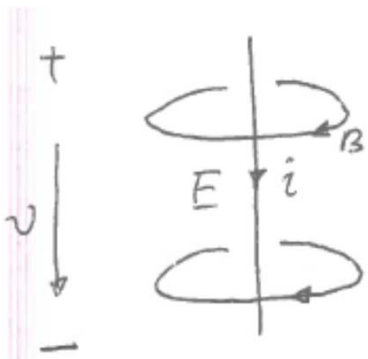
We can actually get to this wave behavior by using circuit theory, w/o going into details of the EM fields!

There is capacitance between any two pieces of conductors.
A pair of plates, wires, etc., or the core and shields of a co-ax cable.

$$\boxed{C = C' \Delta z}$$

C': capacitance per length

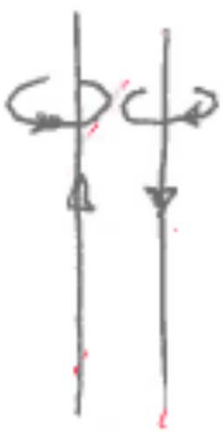
A piece of wire is actually an inductor



$B \propto i$

When i changes w/ t
so does B $\quad \frac{di}{dt} \rightarrow \frac{dB}{dt}$

$$v \propto E \propto \frac{\partial B}{\partial t} \Rightarrow v \propto \frac{di}{dt} \qquad v = L\frac{di}{dt}$$

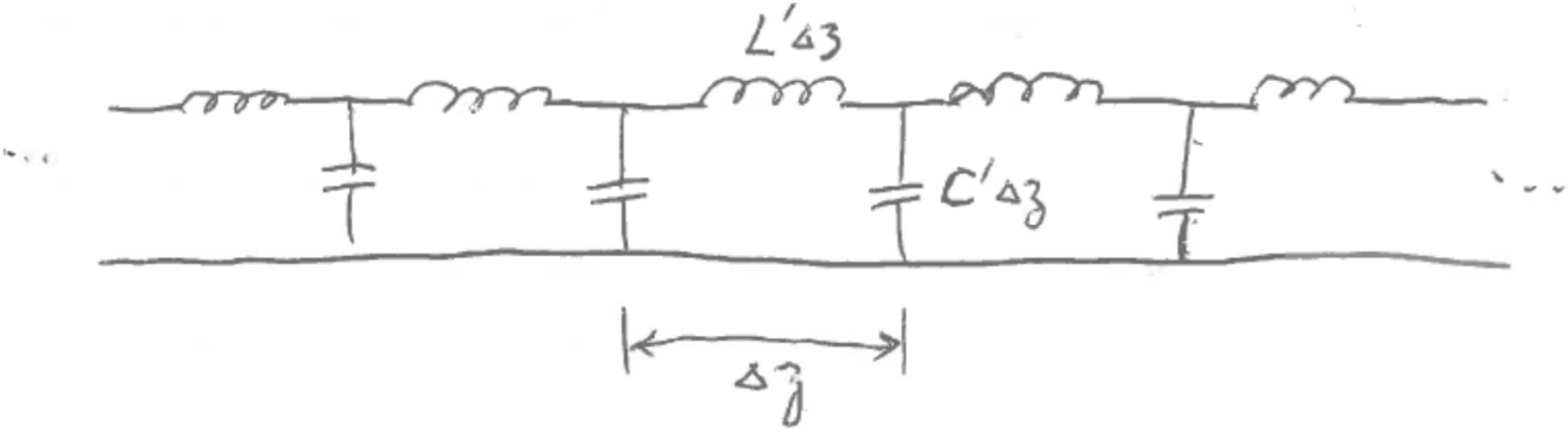


A pair of wires, coupled, but similar.

$$\boxed{L = L' \Delta z}$$

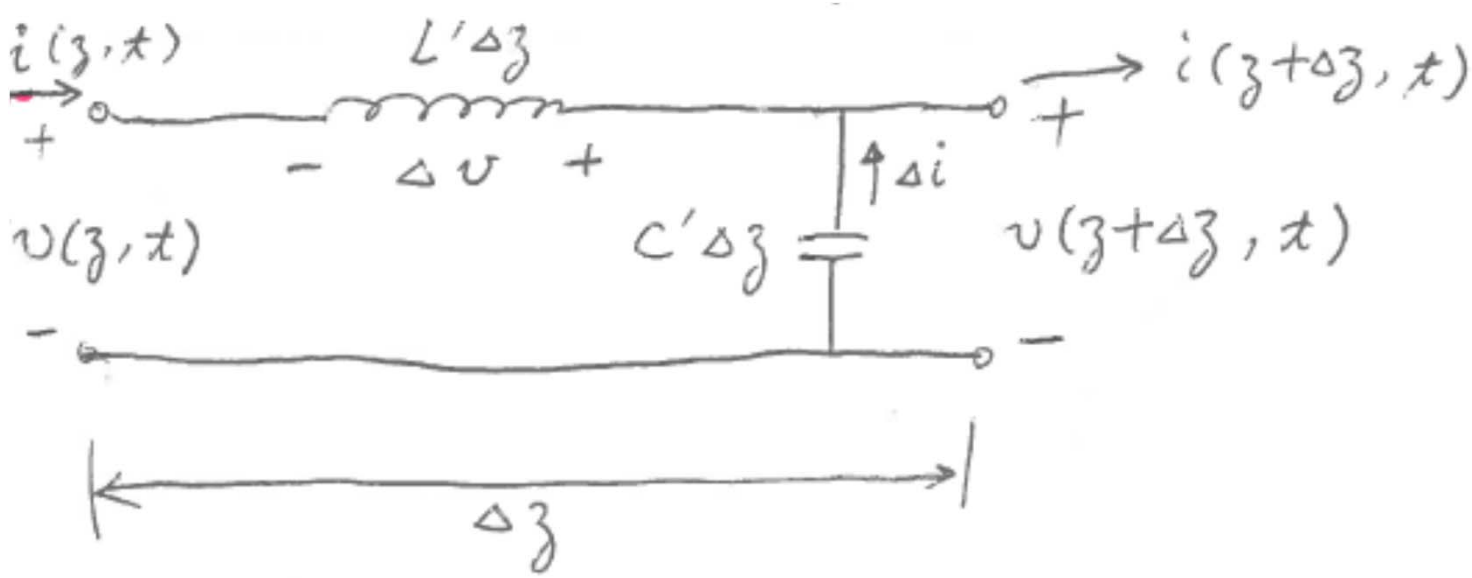
L' is the inductance per length

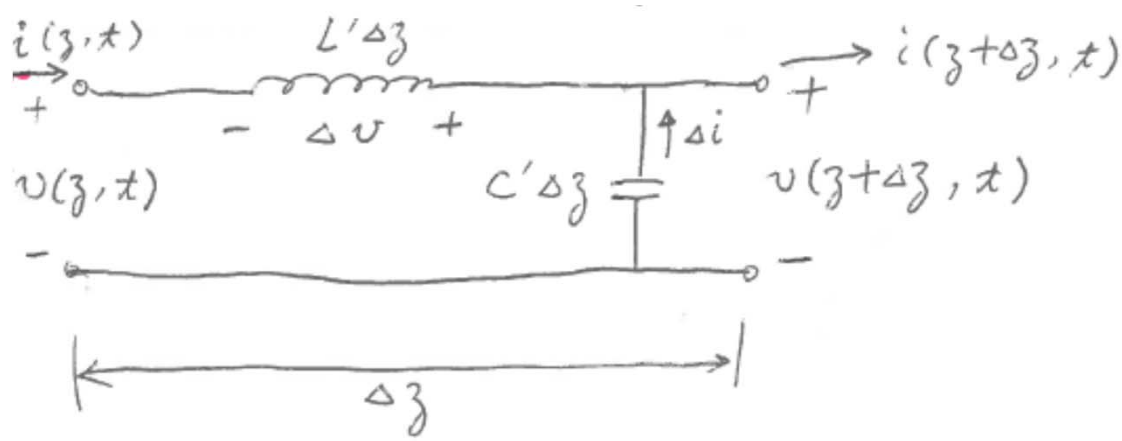
To make things simple, we first consider a pair of ***ideal*** wires.
No resistance, no shunt (leakage).



Pay close attention. We take a different approach than does the book.

Now, zoom in on one segment:





Inductor

$$\Delta v = v(z+\Delta z) - v(z,t)$$
$$= -L'\Delta z \frac{\partial i}{\partial t}$$

$$\boxed{\frac{\partial v}{\partial z} = \lim_{\Delta z \to 0} \frac{\Delta v}{\Delta z} = -L' \frac{\partial i}{\partial t}}$$

Capacitor

$$= i(z+\Delta z, t) - i(z,t)$$

$$\Delta i = -C'\Delta z \frac{\partial v}{\partial t}$$

$$\frac{\partial i}{\partial z} = \lim_{\Delta z \to 0} \frac{\Delta i}{\Delta z} = -C' \frac{\partial v}{\partial t}$$

Take derivatives with regard to *z*, *t*

$$\frac{\partial^2 v}{\partial z^2} = -L' \frac{\partial^2 i}{\partial t \partial z}$$
$$\frac{\partial^2 v}{\partial z \partial t} = -L' \frac{\partial^2 i}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial z^2} = -C' \frac{\partial^2 v}{\partial t \partial z}$$
$$\frac{\partial^2 i}{\partial z \partial t} = -C' \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 v}{\partial z^2} = -L' \frac{\partial^2 i}{\partial t \partial z}$$

$$\frac{\partial^2 v}{\partial z \partial t} = -L' \frac{\partial^2 i}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial z^2} = -C' \frac{\partial^2 v}{\partial t \partial z}$$

$$\frac{\partial^2 i}{\partial z \partial t} = -C' \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 v}{\partial z^2} = L'C' \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial z^2} = L'C' \frac{\partial^2 i}{\partial t^2}$$

Partial differential equations

Do these 2 equations look familiar to you?
What are they?

Let $v_p = \frac{1}{\sqrt{L'C'}}$, we have $\frac{\partial^2 v}{\partial z^2} = \frac{1}{v_p^2} \frac{\partial^2 v}{\partial t^2}$

$v = f(v_p t - z)$ is the general solution to this equation.

Do it on your own: verify this.

Let $v_p = \frac{1}{\sqrt{L'C'}}$, we have $\frac{\partial^2 v}{\partial z^2} = \frac{1}{v_p^2}\frac{\partial^2 v}{\partial t^2}$

$v = f(v_p t - z)$ is the general solution to this equation.

So, this is the wave equation!
More strictly, the lossless, dispersionless, linear wave equation.
Assume: no resistance, no leakage; v_p independent of frequency;
v_p independent of v

Is this amazing?
We arrived at the wave equation from circuit theory, regardless of frequency.
Why does this approach work?

One more agreement:

$$\boxed{L' = \frac{\mu}{\pi}\ln\left[\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right]}$$

$$\boxed{C' = \frac{\pi\varepsilon}{\ln\left[\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right]}}$$

$$\therefore\ v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

See Table 2-1, pp. 45 in 7/E, pp.53 in 6/E

Consistent with EM theory! Check this offline for other transmission lines.

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{v_p^2}\frac{\partial^2 v}{\partial t^2}$$

(the equation for i is in the same form – formally the same.)

$$\frac{\partial^2 i}{\partial z^2} = L'C'\frac{\partial^2 i}{\partial t^2}$$

$$v_p = \frac{1}{\sqrt{L'C'}}$$

The wave equation.
More strictly, the lossless, dispersionless, linear wave equation.
Assume: no resistance, no leakage; v_p independent of frequency;
v_p independent of v or i

$v = f(v_p t - z)$ is the general solution to this equation.

What are the single frequency, harmonic solutions?

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{v_p^2}\frac{\partial^2 v}{\partial t^2}$$

(the equation for i is in the same form – formally the same.)

$v = f(v_p t - z)$ is the general solution to this equation.

Single frequency, harmonic solutions:

$$v(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi_0^+)$$

$$i(z,t) = |I_0^+| \cos(\omega t - \beta z + \phi_0^+)$$

$$\frac{\omega}{\beta} = v_p$$

Here, V_0^+ and I_0^+ are "complex amplitudes" that we will talk about later.
For the waves, they are not the phasors of the waves.
We will talk about the distinction.

$|V_0^+|$ and $|I_0^+|$ are the real amplitudes, or simply amplitudes.

We have not shown the voltage and current waves are in phase.
But they are. You can take this as a conclusion for now.
Or, if you are interested, read the proof next page.

Here we show that the voltage and current waves are in phase with each other:

$$v(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi_{v0}^+) \qquad i(z,t) = |I_0^+| \cos(\omega t - \beta z + \phi_{i0}^+)$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\frac{\partial v}{\partial z} = \beta |V_0^+| \sin(\omega t - \beta z + \phi_{v0}^+) \qquad \frac{\partial i}{\partial t} = -\omega |I_0^+| \sin(\omega t - \beta z + \phi_{i0}^+)$$

Recall that $\boxed{\frac{\partial v}{\partial z} = \lim_{\Delta z \to 0} \frac{\Delta v}{\Delta z} = -L' \frac{\partial i}{\partial t}}$

For this to hold for any arbitrary z, we must have $\boxed{\phi_{v0}^+ = \phi_{i0}^+ \equiv \phi_0^+}$

So, in phase!

We also get a by-product:

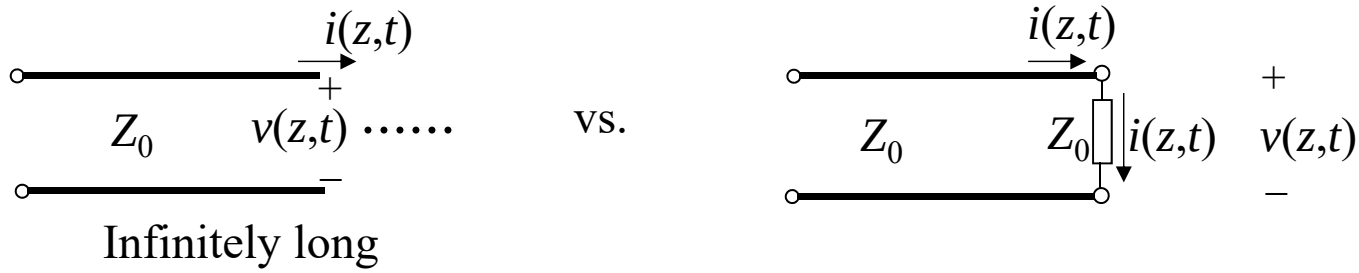
$$\boxed{\begin{aligned} &\frac{|V_0^+|}{|I_0^+|} = \frac{\omega L'}{\beta} = v_p L' \\ &\text{Unit:} \quad \frac{\text{m}}{\text{s}} \frac{\text{H}}{\text{m}} = \text{H/s} = \Omega \end{aligned}}$$

These conclusions are important!

Anywhere, any time $\boxed{v(z,t)/i(z,t) = \text{constant}}$

Define $\dfrac{|V_0^+|}{|I_0^+|} = \dfrac{\omega L'}{\beta} = v_p L' \equiv Z_0$ (Z_0 is real, i.e., purely resistive)

Consider:



There is no way to tell the difference just by measuring *v* and *i*.

Energy propagating away vs. energy dissipated

Analogy: laser beam going to infinity or hitting a totally black wall

You may also use $\dfrac{\partial i}{\partial z} = \lim_{\Delta z \to 0} \dfrac{\Delta i}{\Delta z} = -C' \dfrac{\partial v}{\partial t}$

Doing the derivatives in a similar way as in last page, you will also see the voltage and current waves are in phase.
You will have a similar "by-product" about the *v/i* ratio.
It may look different, but you should be able to show they are equal.

Do it on your own. Hint: use $v_p = \dfrac{1}{\sqrt{L'C'}}$

Rewrite:

$$v^{+}(z,t) = |V_0^{+}|\cos(\omega t - \beta z + \phi_0^{+})$$

$$i^{+}(z,t) = |I_0^{+}|\cos(\omega t - \beta z + \phi_0^{+})$$

In what direction do these waves propagate?

Waves propagating the other way are also solutions to the same equations:

$$v^-(z,t) = |V_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

$$i^-(z,t) = |I_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

Of course, any linear combinations of waves in opposite directions are also solutions:

$$v(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi_0^+) + |V_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

$$i(z,t) = |I_0^+| \cos(\omega t - \beta z + \phi_0^+) + |I_0^-| \cos(\omega t + \beta z + \phi_0^-)$$

They may represent the combinations of incident and reflected waves.

Recall that we have a mathematical tool to

1. Avoid the pain of dealing trigonometric functions, and
2. Turn partial differential equations to ordinary differential equations by putting aside the known time variation

Express waves with phasors

phasor

$$v(z,t) = |V_0^+|\cos(\omega t - \beta z + \phi_0^+) + |V_0^-|\cos(\omega t + \beta z + \phi_0^-)$$

$$\tilde{V} = \tilde{V}^+ + \tilde{V}^- = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Where do the phases go?

Recall the details how we convert a time-domain function to a phasor:

By adding an "imaginary partner"

$$v(z,t) \rightarrow |V_0^+| e^{j(\omega t - \beta z + \phi_0^+)} + |V_0^-| e^{j(\omega t + \beta z + \phi_0^-)} = [(|V_0^+| e^{j\phi_0^+}) e^{-j\beta z} + (|V_0^-| e^{j\phi_0^-}) e^{j\beta z}] e^{j\omega t}$$

This is not the phasor yet.

Throw away the known time variation $e^{j\omega t}$

and define the complex amplitudes $V_0^+ = |V_0^+| e^{j\phi_0^+}$ and $V_0^- = |V_0^-| e^{j\phi_0^-}$

We get the phasor:

$$\tilde{V} = \tilde{V}^+ + \tilde{V}^- = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Positive going Negative going

Notice sign and direction

Express waves with phasors

The current wave is similar

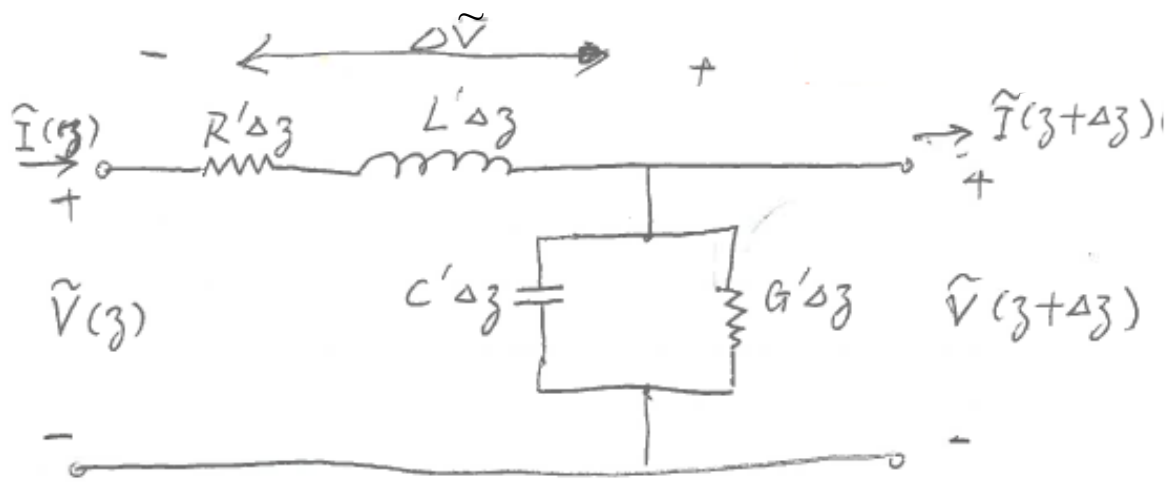
$$i(z,t) = |I_0^+|\cos(\omega t - \beta z + \phi_0^+) + |I_0^-|\cos(\omega t + \beta z + \phi_0^-)$$

phasor

$$\tilde{I} = \tilde{I}^+ + \tilde{I}^- = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$$

This tool makes our life much easier when we deal with a more complicated situation.

Here's the more complicated situation:



No wires are ideal.
Any wire has some resistance.

$$R = R'\Delta z$$

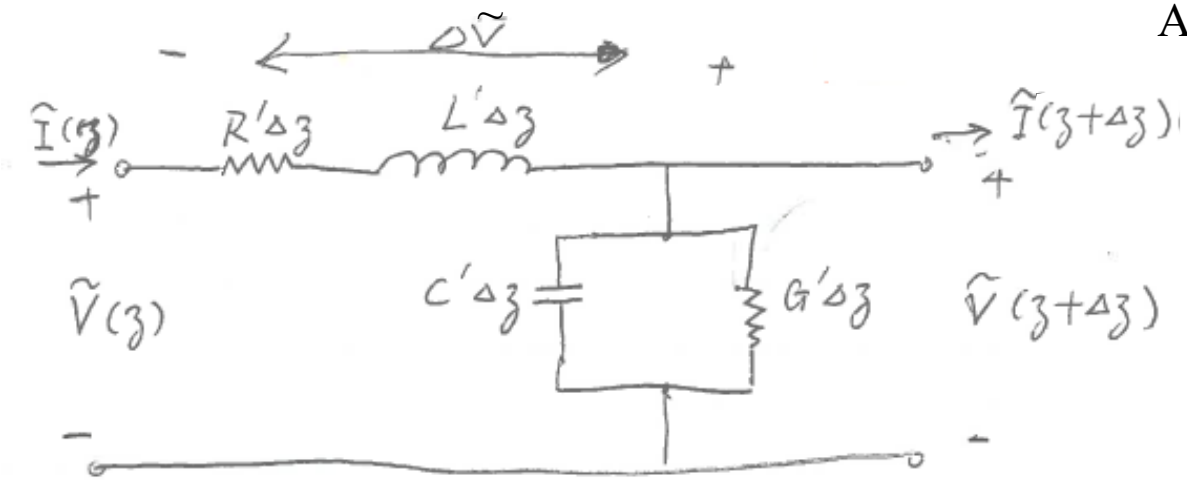
Resistance per lenght

There is always some shunt conductance between two wires

$$G = G'\Delta z$$

Shunt conductance per length

Notice that *R'* and *G'* desribes two different things. $R' \neq 1/G'$



Analyze the circuit in the phasor way

$$\frac{d\tilde{V}}{dz} = \lim_{\Delta z \to 0} \frac{\Delta \tilde{V}}{\Delta z} = -(R' + j\omega L')\tilde{I}$$

$$\frac{d\tilde{I}}{dz} = \lim_{\Delta z \to 0} \frac{\Delta \tilde{I}}{\Delta z} = -(G' + j\omega C')\tilde{V}$$

Take derivatives on one equation and insert it into the other, you get

$$\frac{d^2\tilde{V}}{dz^2} - (R' + j\omega L')(G' + j\omega C')\tilde{V}(z) = 0$$

and a formally same equation for the current.

This is an ordinary differential equation. Because we used phasors.

We have arrived at this equation just by circuit analysis using phasors.
You could also first to the circuit analysis in the time domain, arriving at partial differential equations, and then convert quantities to phasors and arrive at the same ordinary differential equations, as done in the book (Sections 2-3 & 2-4).

The partial differential equations for the general, more complicated situation are, well, too complicated. We don't even bother to tackle them.
Let's look at the simpler ordinary differential equation:

$$\frac{d^2\tilde{V}}{dz^2} - (R' + j\omega L')(G' + j\omega C')\tilde{V}(z) = 0$$

Before solving this equation, let's first have a digression back to the ideal case

$$R' = 0 \qquad G' = 0$$

$$\frac{d^2\tilde{V}}{dz^2} + \omega^2 L'C'\tilde{V} = 0 \Longrightarrow \tilde{V}(z) = V_0^{\pm} e^{\mp j\omega\sqrt{L'C'}\,z}$$

Recall that $\frac{\omega}{\beta} = v_p = \frac{1}{\sqrt{L'C'}} \Longrightarrow \beta = \omega\sqrt{L'C'}$

we have $\tilde{V} = V_0^{\pm} e^{\mp j\beta z}$

Notice that these are actually two solutions.
What are the difference between the two solutions?

Waves propagating in two opposite directions: $\widetilde{V} = \widetilde{V}^{+} + \widetilde{V}^{-} = V_0^{+} e^{-j\beta z} + V_0^{-} e^{j\beta z}$

No surprise. The solutions we got earlier for the ideal case.

Now, back to the more complicated, general case

$$\frac{d^2 \tilde{V}}{dz^2} - (R' + j\omega L')(G' + j\omega C')\,\tilde{V}(z) = 0$$

Let $\gamma^2 = (R' + j\omega L')(G' + j\omega C')$

we can write $\dfrac{d^2 \tilde{V}}{dz^2} - \gamma^2 \tilde{V}(z) = 0$

Compare this to the ideal case $\dfrac{d^2 \tilde{V}}{dz^2} + \omega^2 L' C' \tilde{V} = 0$

These two equations are "formally" the same, except $-\gamma^2$ is complex.

So, the solutions are $\widetilde{V} = V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z}$

What kind of waves are they?

$$\widetilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

What kind of waves are they?

Let $\gamma = \alpha + j\beta$ (we are doing nothing. Any complex number can be written as this), we have the first solution

$$\widetilde{V}^+ = V_0^+ e^{-\gamma z} = V_0^+ e^{-(\alpha + j\beta)z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

What is this?

Similarly, $\widetilde{I}^+ = I_0^+ e^{-\gamma z} = I_0^+ e^{-(\alpha + j\beta)z} = I_0^+ e^{-\alpha z} e^{-j\beta z}$

Why do the waves attenuate when there is resistance or shunt leakage? (Why is there no attenuation when the wire is made of a perfect conductor and the medium between them is a perfect insulator)?

Recall that, in circuit theory, reactive versus resistive..., ...

How to express the decaying wave propagating in the –z direction?

Some Clarifications

- Class notes and other information on course website
  - All class notes, homework, homework answers, quiz answers (after the quiz, of course). Looking into Canvas.
  - Read the notes, in conjunction with the textbook sections indicated in the schedule (also on website), **before** you do homework
  - The notes contains math derivations that I do not go over in detail and tell you to go over offline
  - The notes also contain contents that I call “the extra mile,” for you to ponder upon
- Homework
  - Do it on your own **before** checking the answer sheets
  - Homework problems are for your exercise. Effort level needed to achieve same learning outcome differs by individual. Therefore homework is not graded.
- Quizzes
  - You will do well on quizzes if you do the above. Example: Quizzes 1 & 2.
  - Graded (generously). Not a “gotcha” thing. Done with purposes.
  - Will give answers right after in-class quizzes. Primary purpose is to make sure you are prepared for the content to be covered in class. Example: Quiz 2 prepares you for last, this, and next lectures.
- Project
  - On going. Report to be submitted when done (after Test 1).
  - Take notes, save results. You will better understand the results later; you can go back to any step and do/observe more when your understanding deepens.
  - TA office hour for CAD tool support?

Again, there can be waves going the other way.

$$\tilde{V} = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \qquad \tilde{I} = I_0^+ e^{-\alpha z} e^{-j\beta z} + I_0^- e^{\alpha z} e^{j\beta z}$$

Now, we discuss the most important concept of the first half of the semester:

$$\tilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \xrightarrow{\text{Take derivatives}} \frac{d\tilde{V}}{dz} = \boxed{-\gamma V_0^+ e^{-\gamma z}} \boxed{+\gamma V_0^- e^{\gamma z}}$$

From circuit analysis

$$\frac{d\tilde{V}}{dz} = \lim_{\Delta z \to 0} \frac{\Delta \tilde{V}}{\Delta z} = -(R' + j\omega L')\tilde{I} = \boxed{-(R' + j\omega L') I_0^+ e^{-\alpha z} e^{-j\beta z}} \boxed{-(R' + j\omega L') I_0^- e^{\alpha z} e^{j\beta z}}$$

For the identity to hold for all z, we must have the following:

For the $e^{-\gamma z}$ term:

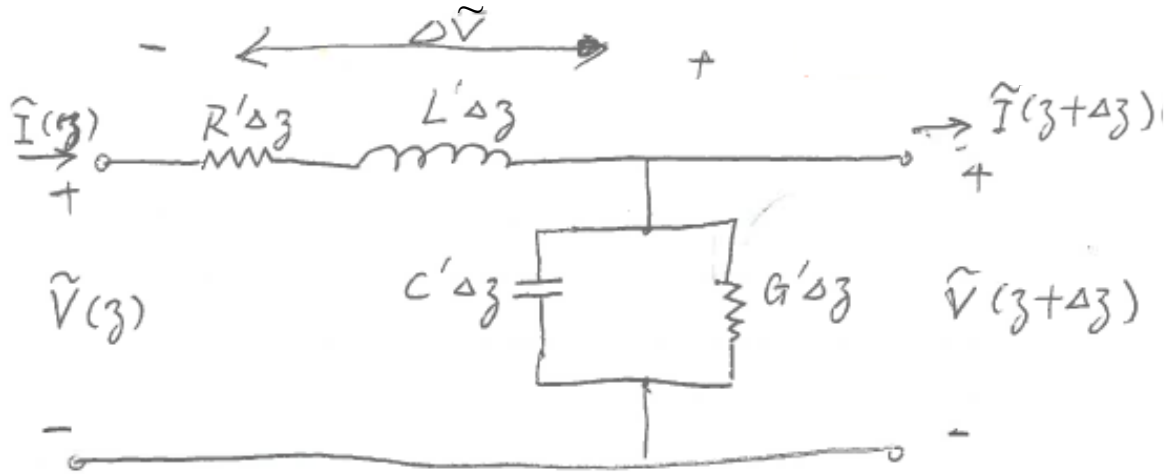
$$\gamma V_0^+ = (R' + j\omega L') I_0^+$$

$$\Rightarrow \quad \frac{V_0^+}{I_0^+} = \frac{R' + j\omega L'}{\gamma} = \frac{R' + j\omega L'}{\sqrt{(R' + j\omega L')(G' + j\omega C')}} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

For the $e^{\gamma z}$ term: $\gamma V_0^- = -(R' + j\omega L')I_0^-$

$$\Rightarrow \quad \frac{V_0^-}{I_0^-} = -\frac{R' + j\omega L'}{\gamma} = -\frac{R' + j\omega L'}{\sqrt{(R' + j\omega L')(G' + j\omega C')}} = -\sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Notice negative signs. Just because of sign convention (see circuit diagram)



Define $Z_0 \equiv \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}}$

the characteristic impedance

Complex and explicitly dependent on frequency in the general (lossy) case.

For the wave traveling towards +z,

$$\tilde{V}^{+} = V_0^{+} e^{-\gamma z}$$

$$\tilde{I}^{+} = I_0^{+} e^{-\gamma z}$$

At any z, $\dfrac{\tilde{V}^{+}(z)}{\tilde{I}^{+}(z)} = \dfrac{V_0^{+}}{I_0^{+}} = Z_0$

For the wave traveling towards –z,

$$\tilde{V}^{-} = V^{-} e^{\gamma z}$$

$$\tilde{I}^{-} = I_0^{-} e^{\gamma z}$$

At any z, $\dfrac{\tilde{V}^{-}(z)}{\tilde{I}^{-}(z)} = \dfrac{V_0^{-}}{I_0^{-}} = -Z_0$

Again, notice this negative sign.

These relations are **separately** held by the two waves in opposite directions.

In general, $Z_0 \equiv \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}}$ is complex and explicitly dependent on frequency.

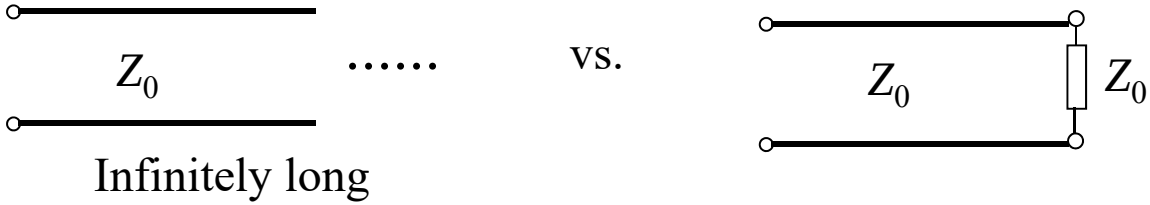
For the lossless transmission line, $R' = 0$ and $G' = 0$,

$$\boxed{Z_0 = \sqrt{\frac{L'}{C'}}}$$

Real. No explicit frequency dependence.

Take-home messages

- Voltage v and current i follow the same differential equation
- Therefore same solution
- Therefore there is a constant ratio between their amplitudes and there is a constant shift between their phases for harmonic waves going in **one direction**
- In the phasor form, the complex amplitude ratio is $\pm Z_0$
- Being a voltage/current ratio, Z_0 has the dimension of impedance
- In general (lossy case), Z_0 is complex and explicitly depends on ω
- In the lossless case, Z_0 is real w/o explicit frequency dependence
- Z_0 being real means the voltage and the current are in phase. It also means the equivalent circuit for a (semi-)infinitely long transmission line is simply a resistor with a resistance value Z_0 – see next page.

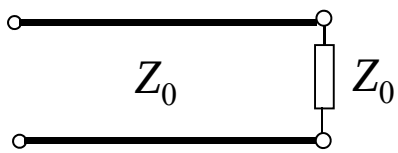


There is no way to tell the difference just by measuring v and i.

Energy propagating away vs. energy dissipated

Analogy: laser beam going to infinity or hitting a totally black wall

Impedance match



By the way, transmission line (thick line) versus "wire wires" (thin lines)

The same as the infinitely long line!

We want impedance match!

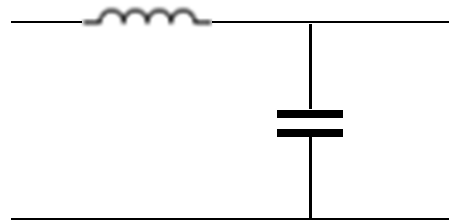
(Reasons?)

At this point, read textbook up to Section 2-4.

Project

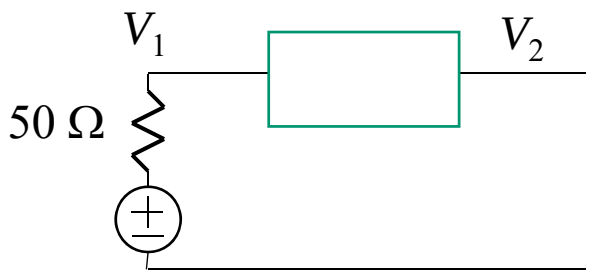
A circuit simulation project to transition you from lumped component-based circuit theory

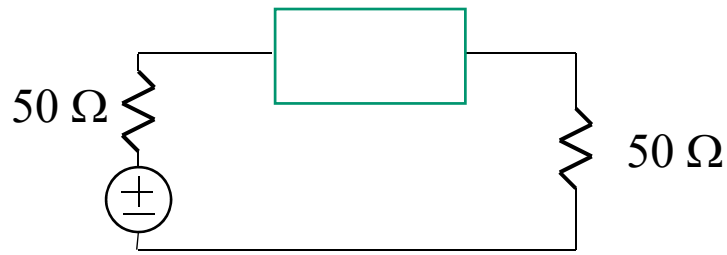
In Part 1 and Part 2, you built an LC network:



In Part3, you built a cascade of 10 instances of this LC network.

And, you did transient simulations of the following circuits (with the 1-unit and 10-unit networks) with the generator signal being voltage steps with different rise times:

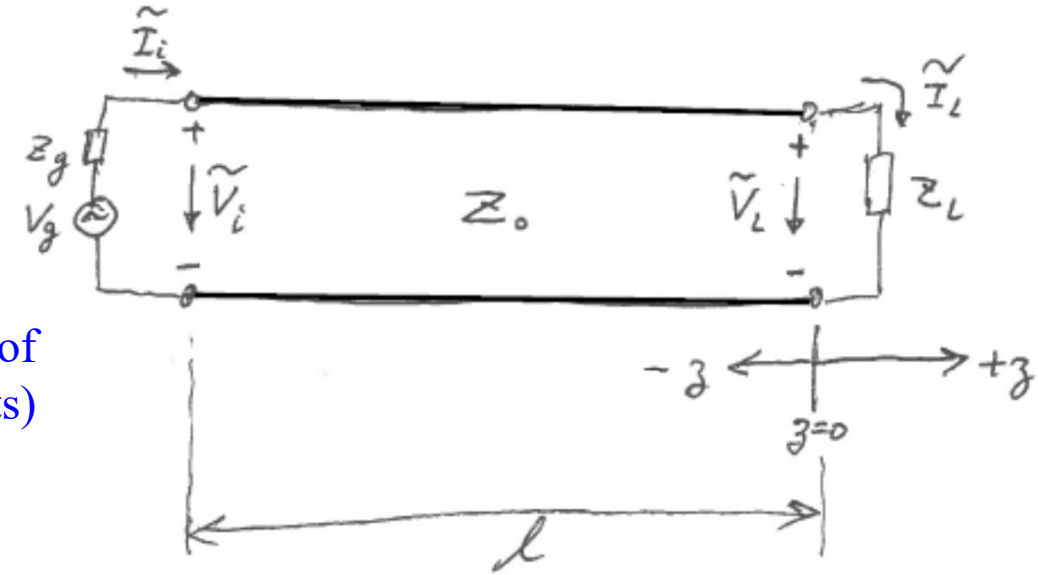




Part 4: Now, create a new network that is a cascade of 10 instances of the 10-unit network, so that this new network contains 100 units. You may create a symbol for this new network for convenience. Using the same inductance and capacitance values to do the same simulations you have done for the above 1- and 10-unit networks. (Same generators with same internal impedance. Simulate for both open circuit and 50-ohm loads, the two rise times for each case, as done for the single LC.)

Ongoing project. Stay tuned for next steps.

Now, let's look at a transmission line with a source and a load.



If $Z_L = Z_0$, impedance matched.
All energy delivered to load. Good!
(we can view this from the vantage point of equivalent circuits)

What if $Z_L \neq Z_0$?

The load says $\dfrac{\tilde{V}(0)}{\tilde{I}(0)} = \dfrac{\tilde{V}_L}{\tilde{I}_L} = Z_L$

If there is only the incident wave, $\dfrac{\tilde{V}^+(0)}{\tilde{I}^+(0)} = \dfrac{V_0^+}{I_0^+} = Z_0$

Now, we just focused on the load. Will talk about the line and generator later.

Something has to happen to resolve this "conflict." That something is reflection.

$$\tilde{V}_L = \tilde{V}(z=0) = V_0^+ + V_0^-$$

$$\tilde{I}_L = \tilde{I}(z=0) = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

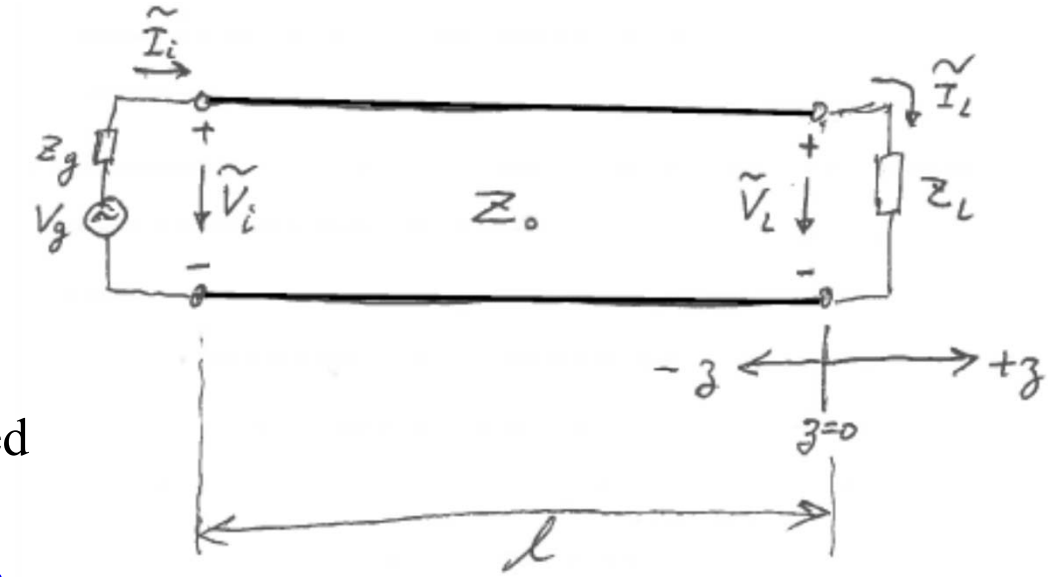
Sign due to convention

By definition, $Z_L = \dfrac{\tilde{V}_L}{\tilde{I}_L} = \left(\dfrac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0$

Solve it and we have $V_0^- = \dfrac{Z_L - Z_0}{Z_L + Z_0} V_0^+$

If $Z_L \neq Z_0$, there has to be a reflected wave.

$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$



The load does not get all energy carried by the incident wave.

Where does the rest of the energy go?

Consider analogy: laser beam hitting wall not totally black/dark.

Define the voltage reflection coefficient $\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$

$$\Downarrow$$

$$\frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma}$$

- One-to-one mapping between Γ and Z_L/Z_0
- The ratio Z_L/Z_0 more important than Z_L itself

Therefore we define the normalized load impedance $z_L = \dfrac{Z_L}{Z_0}$

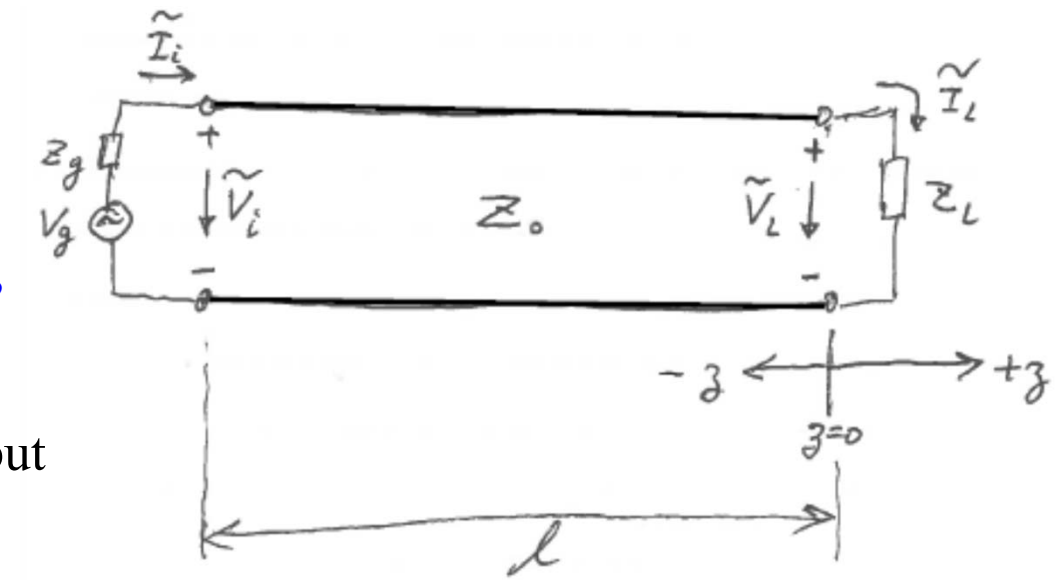
Thus,

$$\boxed{\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{z_L - 1}{z_L + 1}} \qquad \boxed{z_L = \frac{1+\Gamma}{1-\Gamma}}$$

Notice the one-to-one mapping. This is very important!

For the current

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma$$



Where does the negative sign come from?

$Z_0 = \sqrt{\dfrac{L'}{C'}}$ is real for a lossless line, but

$Z_0 \equiv \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}}$ is complex in general.

Z_L is complex in general. Thus, Γ is complex in general.

Read the textbook: Section 2-6 overview, 2-6.1