Power flow along transmission lines

At any distance $d$, we can use an equivalent impedance $Z(d)$ to represent the stuff to the right of $d$.

Question: What is the impedance “felt” by the incident wave at $d$?
Power flow along transmission lines

At any distance $d$, we can use an equivalent impedance $Z(d)$ to represent the stuff to the right of $d$.

**Question:** What is the impedance “felt” by the incident wave at $d$?

Let’s now have a quantitative look at the energy flowing along the transmission line.

Let’s first do this with the “real quantities” instead of phasors.

$$v(d,t) = \text{Re} \left( V(d) e^{j\omega t} \right)$$

We forgo the convenience provided by phasors for the moment, in order not to forget the real world quantities.

The incident wave

$$v_{inc}(d,t) = \text{Re} \left( \frac{V_0^+ e^{j\beta d} e^{j\omega t}}{a \text{ complex number}} \right)$$

$$= \text{Re} \left( |V_0^+| e^{j(\omega t + \beta d + \phi^+)} \right)$$

$$= |V_0^+| \cos(\omega t + \beta d + \phi^+)$$

Make sure you can do the conversion
The incident wave

\[ v_{\text{inc}}(d, t) = |V_0^+| \cos(\omega t + \beta d + \phi^+) \]

Notice that we are talking about lossless transmission lines thus \( Z_0 \) is real. Also notice that \( P_{\text{inc}} \) is always positive.

The average flow:

\[ P_{\text{inc}, \text{av}} = \frac{1}{T} \int_0^T P_{\text{inc}}(d, t) \, dt = ? \]
The incident wave

\[ v_{inc}(d,t) = |V_0^+| \cos(\omega t + \beta d + \phi^+) \]

Notice that we are talking about lossless transmission lines thus \( Z_0 \) is real. Also notice that \( P_{inc} \) is always positive.

The average flow:

\[ P_{inc,av} = \frac{1}{T} \int_0^T P_{inc}(d,t) \, dt = \frac{|V_0^+|^2}{2Z_0} \]
Similarly, the reflected wave

\[ v_{ref} (d, t) = \text{Re} \left( \Gamma V_0^+ e^{-j \beta d} e^{j \omega t} \right) \]

\[ = \text{Re} \left[ |\Gamma| |V_0^+| e^{j(\omega t - \beta d + \phi^+ + \phi_r)} \right] \]

\[ = |\Gamma| |V_0^+| \cos(\omega t - \beta d + \phi^+ + \phi_r) \]

reflected

\[ P_{ref} = -\frac{v_{ref}^2}{2 \varepsilon_0} = -|\Gamma|^2 \frac{|V_0^+|^2}{2 \varepsilon_0} \cos^2(\omega t - \beta d - \phi^+ + \phi_r) \]

\[ P_{ref, av} = -|\Gamma|^2 \frac{|V_0^+|^2}{2 \varepsilon_0} \]

The average total or net energy flow is

\[ P_{av} = P_{inc, av} + P_{ref, av} = \frac{|V_0^+|^2}{2 \varepsilon_0} (1 - |\Gamma|^2) \]
You may do it the phasor way and just get the average values.

\[ P_{av} = \frac{1}{2} \Re \left( \widetilde{V} \widetilde{I}^* \right) \]  \hspace{1cm} (1)

Apply the same to the incident, the reflected, and the total or net. You will get:

\[ P_{inc, av} = \frac{\left| V_0 \right|^2}{2Z_0} \]

\[ P_{ref, av} = -\left| \Gamma \right|^2 \frac{\left| V_0 \right|^2}{2Z_0} \]

\[ P_{av} = \frac{\left| V_0 \right|^2}{2Z_0} \left( 1 - \left| \Gamma \right|^2 \right) \]  \hspace{1cm} (2)

Now, look at them. Are they just intuitively obvious?

Do it on your own:

For the total or net power flow, derive Eq (2) from (1) on your own.
Think about a special case: 

$$|\Gamma| = 0 \implies P_{av} = \frac{|V_o + l|^2}{2Z_0}$$

$$P_{av} = P_{inc,av} = \frac{|V_o + l|^2}{2Z_0}$$

$$P_{ref,av} = -|\Gamma|^2 \frac{|V_o + l|^2}{2Z_0} = 0$$

Consider a class of special cases: 

$$|\Gamma| = 1$$

Review Textbook Section 2-9.
Finish HW3.