Power flow along transmission lines

At any distance $d$, we can use an equivalent impedance $Z(d)$ to represent the stuff to the right of $d$.

Question: What is the impedance “felt” by the incident wave at $d$?

Let’s now have a quantitative look at the energy flowing along the transmission line.

Let’s first do this with the “real quantities” instead of phasors.

$$v(d, t) = \Re \left( \tilde{V}(d) e^{j\omega t} \right)$$

We forgo the convenience for the moment, in order not to forget the real world quantities.

The incident wave

$$v_{inc}(d, t) = \Re \left( \frac{V_0}{\sqrt{\varepsilon}} e^{j\beta d + \phi^*} e^{j\omega t} \right)$$

$$= \Re \left( |V_0| e^{j(\omega t + \beta d + \phi^*)} \right)$$

$$= |V_0^+| \cos(\omega t + \beta d + \phi^*)$$

Make sure you can do the conversion
The incident wave

\[ v_{\text{inc}}(d, t) = |V_0^+| \cos(\omega t + \beta d + \phi^+) \]

Notice that we are talking about lossless transmission lines thus \( Z_0 \) is real. Also notice that \( P_{\text{inc}} \) is always positive.

The average flow:

\[ P_{\text{inc, av}} = \frac{1}{T} \int_0^T P_{\text{inc}}(d, t) \, dt = \frac{|V_0^+|^2}{2Z_0} \]
Similarly, the reflected wave

\[
\nu_{\text{ref}}(d,t) = \text{Re} \left( \Gamma V_0^+ e^{-j \beta d} e^{j \omega t} \right) \\
= \text{Re} \left[ |\Gamma| |V_0^+| e^{j (\omega t - \beta d + \phi^+ + \theta_r)} \right] \\
= |\Gamma| |V_0^+| \cos (\omega t - \beta d + \phi^+ + \theta_r)
\]

reflected

\[
P_{\text{ref}} = -\frac{\nu_{\text{ref}}^2}{Z_0} = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0} \cos^2 (\omega t - \beta d - \phi^+ + \theta_r)
\]

\[
P_{\text{ref, av}} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}
\]

The average total or net energy flow is

\[
P_{\text{av}} = P_{\text{inc, av}} + P_{\text{ref, av}} = \frac{|V_0^+|^2}{2Z_0} \left( 1 - |\Gamma|^2 \right)
\]
You may do it the phasor way and just get the average values.

\[ P_{av} = \frac{1}{2} \Re(e^{j\theta}) \]

Apply the same to the incident, the reflected, and the total or net. You will get:

\[ P_{inc, av} = \frac{|V_0|^2}{2Z_0} \]

\[ P_{ref, av} = -|\Gamma|^2 \frac{|V_0|^2}{2Z_0} \]

\[ P_{av} = \frac{|V_0|^2}{2Z_0} (1-|\Gamma|^2) \]

Now, look at them. Are they just intuitively obvious?

Think about a special case:

\[ |\Gamma| = 0 \quad \Rightarrow \quad P_{av} = \frac{|V_0|^2}{2Z_0} \]

\[ P_{av} = P_{inc, av} = \frac{|V_0|^2}{2Z_0} \]

\[ P_{ref, av} = -|\Gamma|^2 \frac{|V_0|^2}{2Z_0} = 0 \]