Impedance Matching

Generally, $Z_L = R_L + jX_L, X_L \neq 0$. You need to “turn two knobs” to achieve match.

Example

This time, we do not want to cut the line to insert a matching network. Instead, we tap something at some location to achieve matching. Therefore, we work with admittance.

First, locate this load on the chart.
Then, find $y_L$ using the chart.
\[ z_L = 0.5 - j \]
\[ y_L = 0.4 + 0.8j \]
\[ z_L = 0.5 - j \quad y_L = 0.4 + 0.8j \]

Move away from the load, i.e., along the constant-SWR circle clockwise.

We will hit this circle:

\[ g = 1 \]

When we use the chart for \( z \leftrightarrow \Gamma \) mapping, we called this circle the \( r = 1 \) circle.

The Smith chart is whichever you consider it to be (a \( z \)-chart or a \( y \)-chart).
\[ z_L = 0.5 - j \quad \text{and} \quad y_L = 0.4 + 0.8j \]

Move away from the load, i.e., along the constant-SWR circle clockwise.

We will hit this circle:
\[ g = 1 \]

On this circle,
\[ y = 1 + jb, \text{ i.e., } \quad Y = Y_0 + jB \]

The real part (conductance) is already matched to \( Y_0 \).

If we can somehow cancel the \( jB \) part, we are done.

Now, read one of the two sets of ticks of the outer scale.

How far do we need to move to reach this point?
\[ z_L = 0.5 - j \quad y_L = 0.4 + 0.8j \]

On this circle, \( g = 1 \)
\( y = 1 + jb \), i.e.,
\( Y = Y_0 + jB \)

The real part (conductance) is already matched to \( Y_0 \).
If we can somehow cancel the \( jB \) part, we are done.

Distance we need to move to hit this circle:
\[ 0.178\lambda - 0.115\lambda = 0.063\lambda \]
Normalized admittance at the cross point:
\[ y(0.063\lambda) = 1 + 1.58j \]
Therefore, \( Y(0.063\lambda) = Y_0 + j1.58Y_0 \)

Is the susceptance inductive or capacitive?
Let’s keep in mind that we are dealing with admittance!

For a capacitor $C$, $Y = j\omega C$

For an inductor $L$, $Y = \frac{1}{j\omega L} = -j \frac{1}{\omega L}$

So, the susceptance is capacitive. **How to cancel it?**

**Impedance Matching Using Lumped Elements**

Let’s say $Z_0 = 50 \, \Omega$. Then the susceptance we need to cancel is $1.58 \times (1/50) \, \Omega^{-1}$.

We can use an inductor $L$ to cancel it: $\frac{1}{\omega L} = 1.58 \times \frac{1}{50} \, \Omega^{-1} \quad \Rightarrow \quad L = (50 \, \Omega) / (1.58 \omega)$

Given the frequency, we can calculate the needed $L$.

**To keep in mind:**

- Any matching is frequency specific
- Not just $L$, but also distance $d$, because $d$ is in terms of $\lambda$, which is frequency dependent
- We are dealing with admittance. The impedance is the diametrical point
- If you want to find out what the SWR is at $d$ (where you put your inductor) in absence of the inductor, what do you do? (Or, how do you find $z(d)$ of the line in absence of the inductor?) – Refer to the chart.
You can also use a capacitor to achieve matching.

You rotate further (i.e., moving further along the constant-SWR circle), and you will hit the $g = 1$ circle again.

That means, you move further along the transmission line, until you have $G = Y_0$.

Apparently, this point is symmetric with that point of the last solution (where you used an inductor).

So, the admittance here (of the main branch) is $y(d) = 1 - 1.58j$

$$d = 0.322\lambda - 0.115\lambda = 0.207\lambda$$

Again, we are dealing with admittance (the impedance is the point diametrical to the admittance).

So, the reactive part of $y(d)$ is inductive.

You need a capacitor $C$ to cancel it.

If $Z_0 = 50\ \Omega$,

$$\omega C = 1.58 \times (1/50)\ \Omega^{-1}$$

$$C = (1.58/\omega) \times (1/50)\ \Omega^{-1}$$

Again, matching is frequency specific.
To keep in mind:

Here, we deal with admittance. The circle we call the $r = 1$ circle (when talking about impedance) is now the “$g = 1$” circle.

However, if you are talking about impedance (i.e. using the Smith chart as the “$z$-chart,” the $g = 1$ circle is the green circle:

Each point on this circle is diametrical to a point on the “$g = 1$” circle of the “$y$-chart.”

**Question**: where is the “$r = 1$” circle on the $y$-chart?

In the lab, you will use the “network analyzer,” which measures **impedance** and displays it on the Smith Chart (as the $z$-chart).

But you deal with admittance, because you will connect something in parallel with the main branch. You need to first get $z(d)$ onto the $g = 1$ circle (on the $z$-chart). (Where is it?)

To help you, the TAs cover the screen with a transparent sheet on which the $g = 1$ circle is printed.

**Finish HW4: Problems 4 & 5**
\[ z_L = 0.5 - j \quad y_L = 0.4 + 0.8j \]

On this circle, \( g = \)

\[ y = 1 + jb, \text{ i.e.,} \]
\[ Y = Y_0 + jB \]

\[ d = 0.178 \lambda - 0.115 \lambda \]
\[ = 0.063 \lambda \]

\[ y(d) = 1 + 1.58j \]