## Impedance Matching

Generally, $Z_{L}=R_{L}+j X_{L}, X_{L} \neq 0$. You need to "turn two knobs" to achieve match.


This time, we do not want to cut the line to insert a matching network. Instead, we tap something at some location to achieve matching. Therefore, we work with admittance.

First, locate this load on the chart.
Then, find $y_{L}$ using the chart.

$$
z_{L}=0.5-j
$$

$$
y_{L}=0.4+0.8 j
$$



$$
z_{L}=0.5-j \quad y_{L}=0.4+0.8 j
$$

Move away from the load, i.e., along the constant-SWR circle clockwise.

We will hit this circle:

$$
g=1
$$

When we use the chart for $z \leftrightarrow \Gamma$ mapping, we called this circle the $r=1$ circle.

The Smith chart is whichever you consider it to be (a $z$ chart or a $y$-chart).

$$
z_{L}=0.5-j \quad y_{L}=0.4+0.8 j
$$

Move away from the load, i.e., along the constant-SWR circle clockwise.

We will hit this circle:

$$
g=1
$$

On this circle,
$y=1+j b$, i.e.,
$Y=Y_{0}+j B$
The real part (conductance) is already matched to $Y_{0}$.
If we can somehow cancel the $j B$ part, we are done.

Now, read one of the two sets of ticks of the outer scale.

How far do we need to move to reach this point?

$$
z_{L}=0.5-j \quad y_{L}=0.4+0.8 j
$$

On this circle, $\quad g=1$
$y=1+j b$, i.e.,
$Y=Y_{0}+j B$
The real part (conductance) is already matched to $Y_{0}$.
If we can somehow cancel the $j B$ part, we are done.

Distance we need to move to hit this circle:
$0.178 \lambda-0.115 \lambda=0.063 \lambda$
Normalized admittance at the cross point:

$$
y(0.063 \lambda)=1+1.58 j
$$

Therefore, $Y(0.063 \lambda)=Y_{0}+j 1.58 Y_{0}$
Is the susceptance inductive or capacitive?

Let's keep in mind that we are dealing with admittance!
For a capacitor $C, Y=j \omega C$

$$
\text { For an inductor } L, Y=\frac{1}{j \omega L}=-j \frac{1}{\omega L}
$$

So, the susceptance is capacitive. How to cancel it?

## Impedance Matching Using Lumped Elements

Let's say $Z_{0}=50 \Omega$. Then the susceptance we need to cancel is $1.58 \times(1 / 50) \Omega^{-1}$.
We can use an inductor $L$ to cancel it: $\frac{1}{\omega L}=1.58 \times \frac{1}{50} \Omega^{-1} \quad \Rightarrow \quad L=(50 \Omega) /(1.58 \omega)$
Given the frequency, we can calculate the needed $L$.
To keep in mind:
We stopped here on Thu 9/23/2021.

- Any matching is frequency specific
- Not just $L$, but also distance $d$, because $d$ is in terms of $\lambda$, which is frequency dependent
- We are dealing with admittance. The impedance is the diametrical point
- If you want to find out what the SWR is at $d$ (where you put your inductor) in absence of the inductor, what do you do? (Or, how do you find $z(d)$ of the line in absence of the inductor?) - Refer to the chart.


## Quiz 3

A transmission line of characteristic impedance $Z_{0}=50 \Omega$ is terminated in a $\operatorname{load} Z_{\mathrm{L}}=19 \Omega$. Is the load at a voltage maximum or a voltage minimum? (Not asking yes or no, but which.) Why? What is the equivalent impedance 1.25 wavelengths away from the load (i.e. towards the generator)? Is this location a voltage maximum or a voltage minimum? Why?

Answers and grading guidelines (50 points for submission) Minimum at load. (10 points, even if wrong reason given for next question.) $Z_{\mathrm{L}}<Z_{0}$, destructive interference therefore minimum. (10 points. If correct reasons are given among wrong ones, count only the right ones; see below)
$\sim 132 \Omega$ (10 points), corresponding to a voltage maximum (10 points) 1.25 wavelengths away from load, since this location is the same as 0.25 wavelengths away from the load and 0.25 wavelengths away from a minimum is a maximum. ( 10 points if student answers with largely right reasons but with inaccuracy in the language or math or terminology.)

You can also use a capacitor to achieve matching. See Smith chart on next slide.
You rotate further (i.e., moving further along the constant-SWR circle), and you will hit the $g=1$ circle again.

That means, you move further along the transmission line, until you have $G=Y_{0}$.
Apparently, this point is symmetric with that point of the last solution (where you used an inductor).

So, the admittance here (of the main branch) is $y(d)=1-1.58 j$

$$
d=0.322 \lambda-0.115 \lambda=0.207 \lambda
$$

Again, we are dealing with admittance (the impedance is the point diametrical to the admittance).
So, the reactive part of $y(d)$ is inductive.
You need a capacitor $C$ to cancel it.
If $Z_{0}=50 \Omega$,

$$
\begin{aligned}
\omega C & =1.58 \times(1 / 50) \Omega^{-1} \\
C & =(1.58 / \omega) \times(1 / 50) \Omega^{-1}
\end{aligned}
$$

Again, matching is frequency specific.

$$
z_{L}=0.5-j \quad y_{L}=0.4+0.8 j
$$

On this circle, $\quad g=1$ $y=1+j b$, i.e.,
$Y=Y_{0}+j B$
The real part (conductance) is already matched to $Y_{0}$.
If we can somehow cancel the $j B$ part, we are done.

There is a second solution.

## Solution 1

## To keep in mind:

Here, we deal with admittance. The circle we call the $r=1$ circle (when talking about impedance) is now the " $g=1$ " circle.

However, if you are talking about impedance (i.e. using the Smith chart as the " $z$-chart," the $g=1$ circle is the green circle:

Each point on this circle is diametrical to a point on the " $g=1$ " circle of the " $y$-chart." See chart on next slide.

## Carefully review textbook Section 2-11 overview and 2-11.1, Problems 4 \& 5 of HW5

> Questions: Where is the " $r=1$ " circle on the $y$-chart?
> Where is the " $g=1$ " circle on the $z$-chart?

In the lab, you will use the "network analyzer," which measures impedance and displays it on the Smith Chart (as the $z$-chart).

But you deal with admittance, because you will connect something in parallel with the main branch. You need to first get $z(d)$ onto the $g=1$ circle (on the $z$-chart). (Where is it?)

To help you, the TAs cover the screen with a transparent sheet on which the $g=1$ circle is printed.

$$
z_{L}=0.5-j \quad y_{L}=0.4+0.8 j
$$

On this circle, $\quad g=1$ $y=1+j b$, i.e.,
$Y=Y_{0}+j B$
$d=0.178 \lambda-0.115 \lambda$
$=0.063 \lambda$
$y(d)=1+1.58 j$

When $y(d)$ is on $g=1$ circle of the
$y$-chart, $z(d)$ is on $g=1$ circle of the
$z$-chart, i.e. the $r=1$ circle of the $y$-chart.

## Single-Stub Matching

We just showed "lumped element

## matching."

But the desired lumped element may not be available, esp. at very high frequencies.
What can we do?


Then, use a reactive element with a susceptance $-1.58 j$ to cancel $1.58 j$. Match accomplished!

## Single-Stub Matching

We just showed "lumped element

## matching."

But the desired lumped element may not be available, esp. at very high frequencies.

## What can we do?

Recall that

- we can make any arbitrary reactance out of a transmission line terminated in a short circuit
- just by adjusting the length $l$



Then, use a reactive element with a susceptance $-1.58 j$ to cancel $1.58 j$. Match accomplished!

So, use a shorted line in place of the lumped reactive element.


Use a shorted line in place of the lumped reactive element.
The shorted line is called a "stub."
Thus the method "single-stub matching."


Locate the point on the $y$-chart corresponding to the short circuit. Where?

Move from there along the constant-SWR circle that corresponds to purely reactive impedance values.

## Which circle?

Reach the point $y_{\text {stub }}=-1.58 j$. Read the chart to find $l$.

- The point on the y-chart corresponding to the short circuit.
$y_{\text {stub }}=-1.58 j$

$y_{\text {stub }}=-1.58 j$
$l=0.34 \lambda-0.25 \lambda=0.09 \lambda$
or

$$
l=0.25 \lambda-0.16 \lambda=0.09 \lambda
$$

We may use either scale; just need to be consistent.


Recall that there is a second solution: at a larger distance $d$, $y(d)=1-1.58 j$.

At that $d$, you may use a stub with a different $l$ to cancel that $j b(d)=-1.58 j$.
Work out the second solution on your own.

To "tap" the stub onto the main line may not be as convenient as it sounds.
You need Tees.
Not that convenient to adjust $d$.
Therefore the "double-stub" method was developed.

We need to adjust two parameters to get a match.

https://www.moonraker.eu/antennas/antenna-hardware/ribbon-feeder/75-m-75-ohm-twin-feeder


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https://www.moonraker.eu/antennas/antenna-hardware/ribbon-feeder/75-m-75-ohm-twin-feeder

A few words about lab:
We have two lab tasks on impedance matching.
Single stub: We can't "slide" the stub on the main line to adjust $d$. Therefor we "adjust" the frequency.


Double stub: Two stub lengths are adjusted.

## Lab

- Documentation on course website.
- Sign up


## Test 1

- Fall break coming Thu.
- Tue $10 / 12 / 2021$, in-class. Will give extra time. When will be your next commitment?
- Covers contents about transmission lines.
- Concerns and suggestions?


## Chapter 3 of Textbook

- Math knowledge \& skills needed for the second half of the semester
- Read at an effort level suitable to you
- Start now
- Pay attention to notations: vectors, scalars, functions
- Homework 8


## About the Course Website

- Lots of information
- ADS tutorial by the TAs
- A link to download the Smith chart
- All notes, homework \& answers, and some quizzes with answers with grading guideline
- In the future, Tests and answers with grading guidelines
- Subject to frequent changes. Please check often
- Web address: http://web.eecs.utk.edu/~ggu1/files/UGHome.html

