Chapter 2

Sections 2-1 to 2-4: Transmission-Line Model

HW2:P1

Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2}\text{ m}) \times (20 \times 10^3\text{ Hz})}{3 \times 10^8\text{ m/s}} = 1.33 \times 10^{-5} \text{ (negligible)}.$$

(b) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(50 \times 10^{3}\text{ m}) \times (60 \times 10^0\text{ Hz})}{3 \times 10^8\text{ m/s}} = 0.01 \text{ (borderline)}.$$

(c) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2}\text{ m}) \times (600 \times 10^6\text{ Hz})}{3 \times 10^8\text{ m/s}} = 0.40 \text{ (nonnegligible)}.$$

(d) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(1 \times 10^{-3}\text{ m}) \times (100 \times 10^9\text{ Hz})}{3 \times 10^8\text{ m/s}} = 0.33 \text{ (nonnegligible)}.$$

HW2:P4 Part 1

Problem 2.2 Calculate the line parameters R', L', G', and C' for a coaxial line with an inner conductor diameter of 0.5 cm and an outer conductor diameter of 1 cm, filled with an insulating material where $\mu = \mu_0$, $\varepsilon_r = 4.5$, and $\sigma = 10^{-3}$ S/m. The conductors are made of copper with $\mu_c = \mu_0$ and $\sigma_c = 5.8 \times 10^7$ S/m. The operating frequency is 1 GHz.

Solution: Given

$$a = (0.5/2)\text{ cm} = 0.25 \times 10^{-2}\text{ m},$$
$$b = (1.0/2)\text{ cm} = 0.50 \times 10^{-2}\text{ m},$$

combining Eqs. (2.5) and (2.6) gives

$$\begin{aligned} R' &= \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a} + \frac{1}{b}\right) \\ &= \frac{1}{2\pi}\sqrt{\frac{\pi(10^9\text{ Hz})(4\pi \times 10^{-7}\text{ H/m})}{5.8 \times 10^7\text{ S/m}}}\left(\frac{1}{0.25 \times 10^{-2}\text{ m}} + \frac{1}{0.50 \times 10^{-2}\text{ m}}\right) \\ &= 0.788\ \Omega/\text{m}. \end{aligned}$$

From Eq. (2.7),

$$L' = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{4\pi \times 10^{-7} \text{ H/m}}{2\pi} \ln 2 = 139 \text{ nH/m}.$$

From Eq. (2.8),

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi \times 10^{-3} \text{ S/m}}{\ln 2} = 9.1 \text{ mS/m}.$$

From Eq. (2.9),

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln(b/a)} = \frac{2\pi \times 4.5 \times (8.854 \times 10^{-12} \text{ F/m})}{\ln 2} = 362 \text{ pF/m}.$$

HW2:P3

Problem 2.3 A 1-GHz parallel-plate transmission line consists of 1.2-cm-wide copper strips separated by a 0.15-cm-thick layer of polystyrene. Appendix B gives $\mu_c = \mu_0 = 4\pi \times 10^{-7}$ (H/m) and $\sigma_c = 5.8 \times 10^7$ (S/m) for copper, and $\varepsilon_r = 2.6$ for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume $\mu = \mu_0$ and $\sigma \simeq 0$ for polystyrene.

Solution:

$$R' = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{1.2 \times 10^{-2}} \left(\frac{\pi \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}\right)^{1/2} = 1.38 \quad (\Omega/\text{m}),$$

$$L' = \frac{\mu d}{w} = \frac{4\pi \times 10^{-7} \times 1.5 \times 10^{-3}}{1.2 \times 10^{-2}} = 1.57 \times 10^{-7} \quad (\text{H/m}),$$

$$G' = 0 \qquad \text{because } \sigma = 0,$$

$$C' = \frac{\varepsilon w}{d} = \varepsilon_0\varepsilon_r \frac{w}{d} = \frac{10^{-9}}{36\pi} \times 2.6 \times \frac{1.2 \times 10^{-2}}{1.5 \times 10^{-3}} = 1.84 \times 10^{-10} \quad (\text{F/m}).$$

HW2:P2

Problem 2.4 Show that the transmission line model shown in Fig. 2-37 (P2.4) yields the same telegrapher's equations given by Eqs. (2.14) and (2.16).

Solution: The voltage at the central upper node is the same whether it is calculated from the left port or the right port:

$$\begin{aligned} v(z+\tfrac{1}{2}\Delta z,t) &= v(z,t) - \tfrac{1}{2}R'\Delta z\, i(z,t) - \tfrac{1}{2}L'\Delta z \frac{\partial}{\partial t} i(z,t) \\ &= v(z+\Delta z,t) + \tfrac{1}{2}R'\Delta z\, i(z+\Delta z,t) + \tfrac{1}{2}L'\Delta z \frac{\partial}{\partial t} i(z+\Delta z,t). \end{aligned}$$

Make sure you really go through the derivation. Then, make sure you can do it for the special case G'=0, R'=0(lossless).

The derivation is a good exercise, but there is a better, much easier, and more interesting way. Just cascade these units, then you will immediately find that the cascade is the same as the one using the repeating unit we used in class.

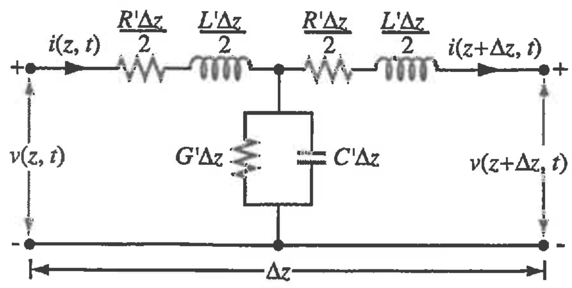

Figure P2.4: Transmission line model.

Recognizing that the current through the $G' \parallel C'$ branch is $i(z,t) - i(z+\Delta z, t)$ (from Kirchhoff's current law), we can conclude that

$$i(z,t) - i(z+\Delta z,t) = G'\Delta z\, v(z+\tfrac{1}{2}\Delta z,t) + C'\Delta z \frac{\partial}{\partial t} v(z+\tfrac{1}{2}\Delta z,t).$$

From both of these equations, the proof is completed by following the steps outlined in the text, ie. rearranging terms, dividing by Δz, and taking the limit as $\Delta z \to 0$.

HW2:P4 Part 2

Problem 2.5 Find α, β, u_p, and Z_0 for the coaxial line of Problem 2.2.

Solution: From Eq. (2.22),

$$\begin{aligned}\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ &= \sqrt{(0.788\ \Omega/\text{m}) + j(2\pi \times 10^9\ \text{s}^{-1})(139 \times 10^{-9}\ \text{H/m})} \\ &\quad \times \sqrt{(9.1 \times 10^{-3}\ \text{S/m}) + j(2\pi \times 10^9\ \text{s}^{-1})(362 \times 10^{-12}\ \text{F/m})} \\ &= (109 \times 10^{-3} + j44.5)\ \text{m}^{-1}.\end{aligned}$$

Thus, from Eqs. (2.25a) and (2.25b), $\alpha = 0.109$ Np/m and $\beta = 44.5$ rad/m.
From Eq. (2.29),

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{(0.788\ \Omega/\text{m}) + j(2\pi \times 10^9\ \text{s}^{-1})(139 \times 10^{-9}\ \text{H/m})}{(9.1 \times 10^{-3}\ \text{S/m}) + j(2\pi \times 10^9\ \text{s}^{-1})(362 \times 10^{-12}\ \text{F/m})}}$$
$$= (19.6 + j0.030)\ \Omega.$$

From Eq. (2.33),

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{44.5} = 1.41 \times 10^8\ \text{m/s}.$$

Section 2-5: The Lossless Line

HW2:P5 **Problem 2.6** In addition to not dissipating power, a lossless line has two important features: (1) it is dispertionless (μ_p is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad \text{(distortionless line).}$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the ~~loss~~ lossless line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \qquad \beta = \omega\sqrt{L'C'}, \qquad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}
\gamma = \alpha + j\beta &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{R'}{L'} + j\omega\right)} \\
&= \sqrt{L'C'}\left(\frac{R'}{L'} + j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.
\end{aligned}$$

Hence,

$$\alpha = \Re\mathfrak{e}(\gamma) = R'\sqrt{\frac{C'}{L'}}, \qquad \beta = \Im\mathfrak{m}(\gamma) = \omega\sqrt{L'C'}, \qquad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$

Problem 2.7 For a distortionless line with $Z_0 = 50\ \Omega$, $\alpha = 20$ (mNp/m), $u_p = 2.5 \times 10^8$ (m/s), find the line parameters and λ at 100 MHz.

Solution: The product of the expressions for α and Z_0 given in Problem 2.6 gives

$$R' = \alpha Z_0 = 20 \times 10^{-3} \times 50 = 1 \quad (\Omega/\text{m}),$$

and taking the ratio of the expression for Z_0 to that for $u_p = \omega/\beta = 1/\sqrt{L'C'}$ gives

$$L' = \frac{Z_0}{u_p} = \frac{50}{2.5 \times 10^8} = 2 \times 10^{-7} \text{ (H/m)} = 200 \quad \text{(nH/m)}.$$

With L' known, we use the expression for Z_0 to find C':

$$C' = \frac{L'}{Z_0^2} = \frac{2 \times 10^{-7}}{(50)^2} = 8 \times 10^{-11} \text{ (F/m)} = 80 \quad \text{(pF/m)}.$$

The distortionless condition given in Problem 2.6 is then used to find G'.

$$G' = \frac{R'C'}{L'} = \frac{1 \times 80 \times 10^{-12}}{2 \times 10^{-7}} = 4 \times 10^{-4} \text{ (S/m)} = 400 \quad (\mu\text{S/m}),$$

and the wavelength is obtained by applying the relation

$$\lambda = \frac{\mu_p}{f} = \frac{2.5 \times 10^8}{100 \times 10^6} = 2.5 \text{ m}.$$

Problem 2.8 Find α and Z_0 of a distortionless line whose $R' = 2$ Ω/m and $G' = 2 \times 10^{-4}$ S/m.

Solution: From the equations given in Problem 2.6,

$$\alpha = \sqrt{R'G'} = [2 \times 2 \times 10^{-4}]^{1/2} = 2 \times 10^{-2} \quad \text{(Np/m)},$$

$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{R'}{G'}} = \left(\frac{2}{2 \times 10^{-4}}\right)^{1/2} = 100\ \Omega.$$

HW2:P6 **Problem 2.9** A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R', L', G', and C'.

Solution: Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40\ \Omega$, $\alpha = 0.02$ Np/m, and $\beta = 0.75$ rad/m. Since $\boxed{Z_0 \text{ is real and } \alpha \neq 0,}$ the line is $\boxed{\text{distortionless.}}$ From Problem 2.6, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'}\sqrt{\frac{R'}{G'}} = \sqrt{R'G'}\sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \text{ Np/m} \times 40\ \Omega = 0.6\ \Omega\text{/m}$$ typo

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \text{ Np/m})^2}{0.8\ \Omega\text{/m}} = 0.5 \text{ mS/m}.$$

HW2:P8

Problem 2.10 Using a slotted line, the voltage on a lossless transmission line was found to have a maximum magnitude of 1.5 V and a minimum magnitude of 0.6 V. Find the magnitude of the load's reflection coefficient.

Solution: From the definition of the Standing Wave Ratio given by Eq. (2.59),

$$S = \frac{|\tilde{V}|_{\max}}{|\tilde{V}|_{\min}} = \frac{1.5}{0.6} = 2.5.$$

Solving for the magnitude of the reflection coefficient in terms of S, as in Example 2-4,

$$|\Gamma| = \frac{S-1}{S+1} = \frac{2.5-1}{2.5+1} = 0.43.$$

Problem 2.11 Polyethylene with $\varepsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with characteristic impedance of 50 Ω. The radius of the inner conductor is 1.2 mm.

(a) What is the radius of the outer conductor?

(b) What is the phase velocity of the line?

Solution: Given a lossless coaxial line, $Z_0 = 50\ \Omega$, $\varepsilon_r = 2.25$, $a = 1.2$ mm:

(a) From Table 2-2, $Z_0 = (60/\sqrt{\varepsilon_r}) \ln(b/a)$ which can be rearranged to give

$$b = ae^{Z_0\sqrt{\varepsilon_r}/60} = (1.2 \text{ mm})e^{50\sqrt{2.25}/60} = 4.2 \text{ mm}.$$

(b) Also from Table 2-2,

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25}} = 2.0 \times 10^8 \text{ m/s}.$$

HW2:P7 →

Problem 2.12 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)$ Ω. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,
(b) the standing-wave ratio on the line,
(c) the position of the voltage maximum nearest the load,
(d) the position of the current maximum nearest the load.

Solution:

(a) From Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57e^{-j79.8^\circ}.$$

(b) From Eq. (2.59),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.56)

$$l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8 \text{ cm}}{4\pi} \frac{\pi \text{ rad}}{180^\circ} + \frac{n \times 8 \text{ cm}}{2}$$
$$= -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}.$$

$n \neq 0$ because l_{max} not negative.

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.58),

$$l_{min} = l_{max} - \lambda/4 = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$

HW2:P9 →

Problem 2.13 On a 150-Ω lossless transmission line, the following observations were noted: distance of first voltage minimum from the load = 3 cm; distance of first voltage maximum from the load = 9 cm; $S = 3$. Find Z_L.

Solution: Distance between a minimum and an adjacent maximum = $\lambda/4$. Hence,

$$9 \text{ cm} - 3 \text{ cm} = 6 \text{ cm} = \lambda/4,$$

$\theta_r = d_{min}\frac{4\pi}{\lambda} - \frac{(2n+1)\lambda}{4}\frac{4\pi}{\lambda}$

$\boxed{\theta_r = d_{min}\frac{4\pi}{\lambda} - \frac{(2n+1)\pi\lambda}{\lambda}}$

or $\lambda = 24$ cm. Accordingly, the first voltage minimum is at $\ell_{min} = 3$ cm $= \frac{\lambda}{8}$. Application of Eq. (2.57) with $n = 0$ gives

$$\theta_r - 2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = -\pi,$$

which gives $\theta_r = -\pi/2$.

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5.$$

Hence, $\Gamma = 0.5e^{-j\pi/2} = -j0.5$.

Finally,

$$Z_L = Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right] = 150\left[\frac{1-j0.5}{1+j0.5}\right] = (90 - j120)\ \Omega.$$

$d_{min} = \frac{\theta_r \lambda}{4\pi} + \frac{(2n+1)\lambda}{4}$, $n = 0, 1, 2, \ldots$

$= \frac{\theta_r}{2\beta} + \frac{n\lambda}{2}$

$d_{max} = \frac{\theta_r \lambda}{4\pi}$

d_{max} $\begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0 \\ n = 0, 1, \ldots & \theta_r > 0 \end{cases}$

Problem 2.14 Using a slotted line, the following results were obtained: distance of first minimum from the load = 4 cm; distance of second minimum from the load = 14 cm, voltage standing-wave ratio = 1.5. If the line is lossless and $Z_0 = 50\ \Omega$, find the load impedance.

Solution: Following Example 2.5: Given a lossless line with $Z_0 = 50\ \Omega$, $S = 1.5$, $l_{min(0)} = 4$ cm, $l_{min(1)} = 14$ cm. Then

$$l_{min(1)} - l_{min(0)} = \frac{\lambda}{2}$$

or

$$\lambda = 2 \times (l_{min(1)} - l_{min(0)}) = 20 \text{ cm}$$

and

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad/cycle}}{20 \text{ cm/cycle}} = 10\pi \text{ rad/m}.$$

From this we obtain

$$\theta_r = 2\beta l_{min(n)} - (2n+1)\pi \text{ rad} = 2 \times 10\pi \text{ rad/m} \times 0.04 \text{ m} - \pi \text{ rad}$$
$$= -0.2\pi \text{ rad} = -36.0°.$$

Also,

$$|\Gamma| = \frac{S-1}{S+1} = \frac{1.5-1}{1.5+1} = 0.2.$$

From Eq. (2.63)

$$Z_{\rm in} = Z_0 \left(\frac{Z_{\rm L} + jZ_0 \tan\beta l}{Z_0 + jZ_{\rm L} \tan\beta l} \right)$$
$$= 100 \left(\frac{(60+j30) + j100\tan\left(\frac{2\pi\ {\rm rad}}{\lambda} 0.35\lambda\right)}{100 + j(60+j30)\tan\left(\frac{2\pi\ {\rm rad}}{\lambda} 0.35\lambda\right)} \right) = (64.8 - j38.3)\ \Omega.$$

Problem 2.19 Show that the input impedance of a quarter-wavelength long lossless line terminated in a short circuit appears as an open circuit.

Solution:

$$Z_{\rm in} = Z_0 \left(\frac{Z_{\rm L} + jZ_0 \tan\beta l}{Z_0 + jZ_{\rm L} \tan\beta l} \right).$$

For $l = \frac{\lambda}{4}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$. With $Z_{\rm L} = 0$, we have

$$Z_{\rm in} = Z_0 \left(\frac{jZ_0 \tan\pi/2}{Z_0} \right) = j\infty \quad \text{(open circuit)}.$$

Problem 2.20 Show that at the position where the magnitude of the voltage on the line is a maximum the input impedance is purely real.

Solution: From Eq. (2.56), $l_{\rm max} = (\theta_{\rm r} + 2n\pi)/2\beta$, so from Eq. (2.61), using polar representation for Γ,

$$Z_{\rm in}(-l_{\rm max}) = Z_0 \left(\frac{1 + |\Gamma| e^{j\theta_{\rm r}} e^{-j2\beta l_{\rm max}}}{1 - |\Gamma| e^{j\theta_{\rm r}} e^{-j2\beta l_{\rm max}}} \right)$$
$$= Z_0 \left(\frac{1 + |\Gamma| e^{j\theta_{\rm r}} e^{-j(\theta_{\rm r}+2n\pi)}}{1 - |\Gamma| e^{j\theta_{\rm r}} e^{-j(\theta_{\rm r}+2n\pi)}} \right) = Z_0 \left(\frac{1+|\Gamma|}{1-|\Gamma|} \right),$$

which is real, provided Z_0 is real.

HW2:P10 **Problem 2.21** A voltage generator with $v_{\rm g}(t) = 5\cos(2\pi \times 10^9 t)$ V and internal impedance $Z_{\rm g} = 50\ \Omega$ is connected to a 50-Ω lossless air-spaced transmission line. The line length is 5 cm and it is terminated in a load with impedance $Z_{\rm L} = (100 - j100)\ \Omega$. Find

(a) Γ at the load.

(b) $Z_{\rm in}$ at the input to the transmission line.

(c) the input voltage $\widetilde{V}_{\rm i}$ and input current $\widetilde{I}_{\rm i}$.

Solution:

(a) From Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62e^{-j29.7^\circ}.$$

(b) All formulae for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9$ rad/s. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}.$$

Then, using Eq. (2.63),

$$\begin{aligned}
Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \\
&= 50 \left(\frac{(100 - j100) + j50 \tan \left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)}{50 + j(100 - j100) \tan \left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)} \right) \\
&= 50 \left(\frac{(100 - j100) + j50 \tan \left(\frac{\pi}{3} \text{ rad} \right)}{50 + j(100 - j100) \tan \left(\frac{\pi}{3} \text{ rad} \right)} \right) = (12.5 - j12.7)\ \Omega.
\end{aligned}$$

An alternative solution to this part involves the solution to part (a) and Eq. (2.61).

(c) In phasor domain, $\widetilde{V}_g = 5 \text{ V } e^{j0^\circ}$. From Eq. (2.64),

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40e^{-j34.0^\circ} \quad \text{(V)},$$

and also from Eq. (2.64),

$$\widetilde{I}_i = \frac{\widetilde{V}_i}{Z_{in}} = \frac{1.4e^{-j34.0^\circ}}{(12.5 - j12.7)} = 78.4e^{j11.5^\circ} \quad \text{(mA)}.$$

Problem 2.22 A 6-m section of 150-Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30^\circ) \quad \text{(V)}$$

and $Z_g = 150\ \Omega$. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)\ \Omega$, find

(a) λ on the line,

(b) the reflection coefficient at the load,

(c) the input impedance,