Quantum Mechanics Review/Overview

Quantum mechanics is a new way of representing the world:

- An amplitude for every "event"
- |the amplitude of an event|^2 = the probability of the event

In general, the amplitude is complex.

The amplitude is somewhat behind the scene. The probability is all that we can possibly know. This randomness is different from that in classic statistical mechanics.

Accept this as a basic assumption. The philosophical interpretation is up to you.

When the "event" is an object (electron) being at position $\boldsymbol{r}$ at time t, the "amplitude" $\psi(\boldsymbol{r}, t)$ is the wave function.

$\psi(\boldsymbol{r}, t)$ can often be viewed as a superposition of plane waves $e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$ (Fourier transform).

For the special case of plane waves,

energy $E = \hbar\omega$

momentum $\boldsymbol{p} = \hbar\boldsymbol{k}$

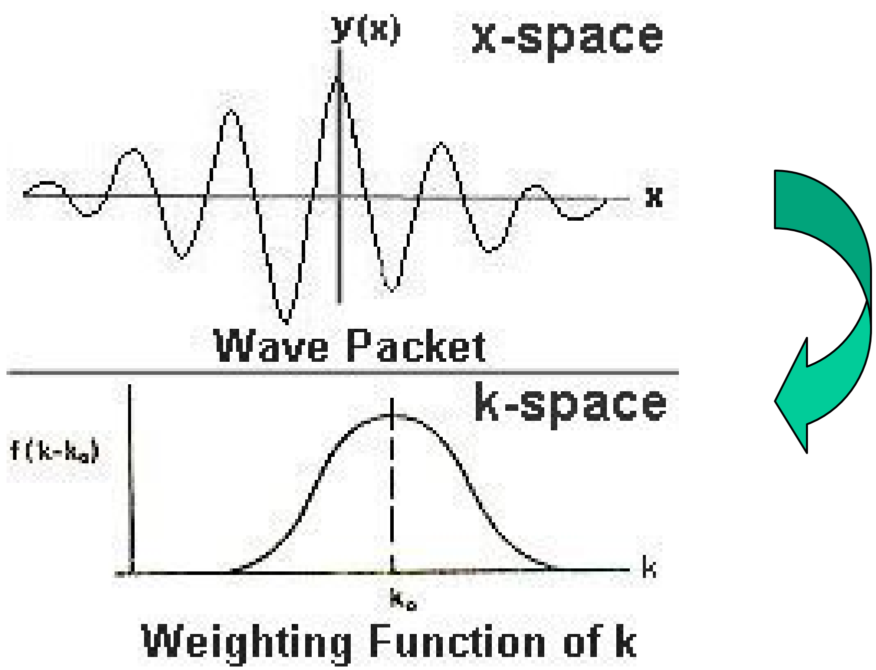
(momentum somehow related to space periodicity or translational symmetry)

We now have a problem.

For an electron in uniform motion (not a quantum way to say things) $e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$

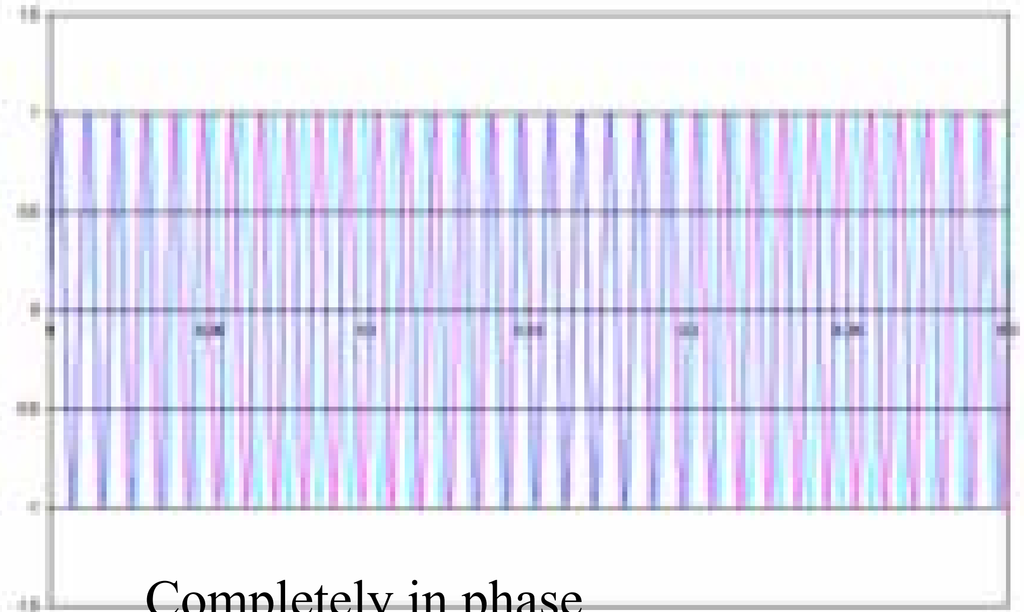
$$| e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)} |^2 = 1$$

A good representation of the particle (electron) is a wave packet (or short wave train).

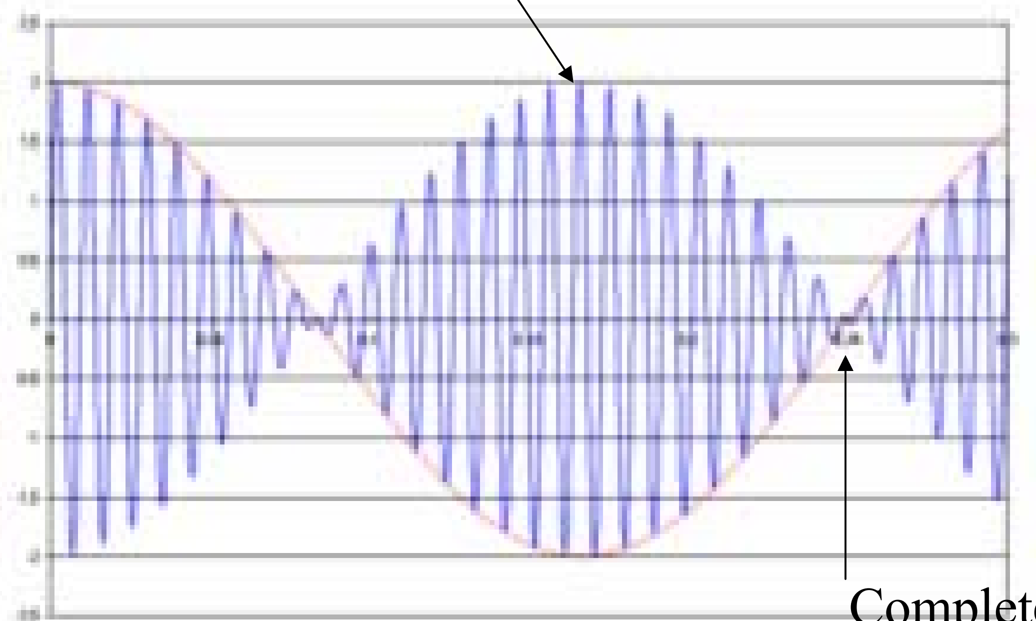


Fourier transform

Taken from http://universe-review.ca/R01-04-diffeq02.htm

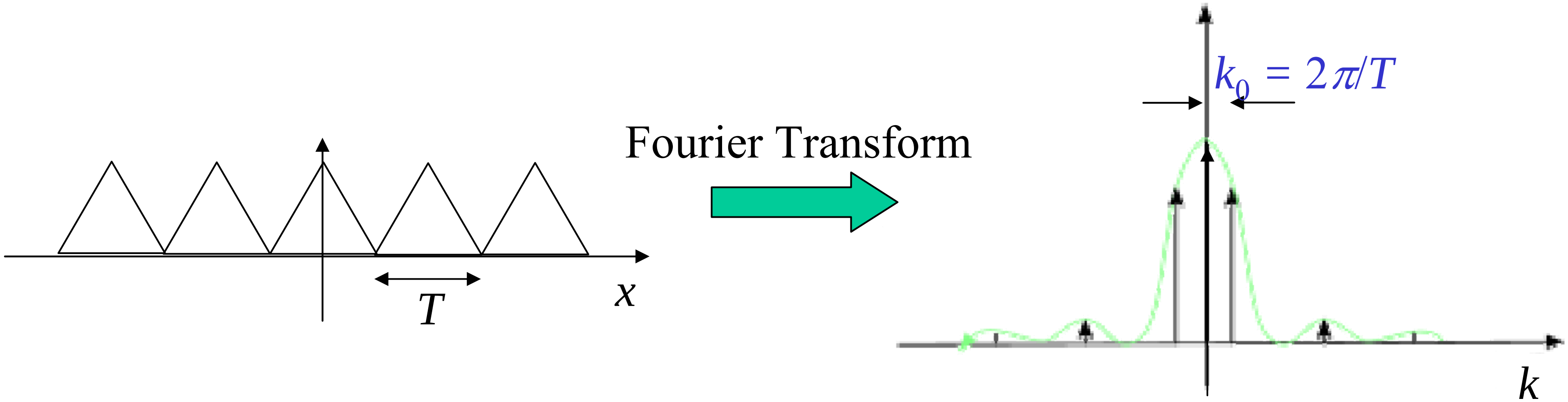


If there are only two wavelengths



Constructive interference will occur in regular intervals.

If there are lots of **discrete** wavelengths

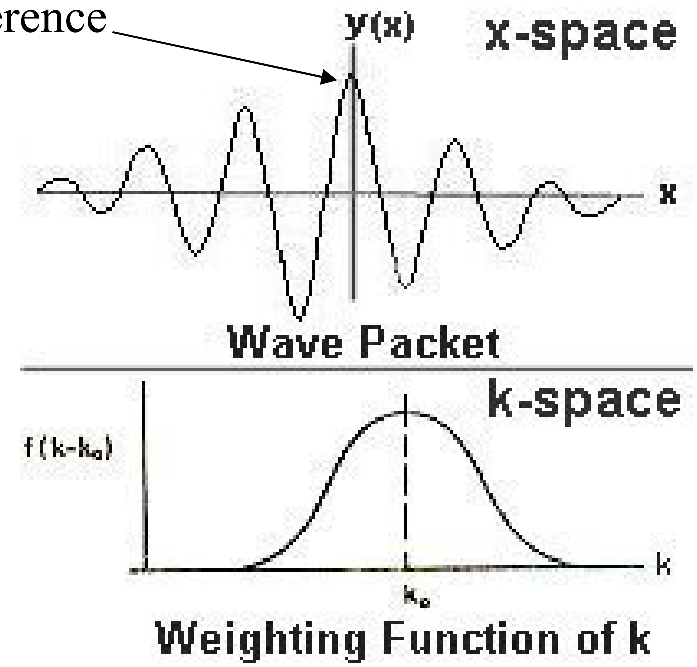


At an infinite number of locations, all waves of different wavelengths are in phase.

Therefore, periodic.

If there are a continuum of wavelengths

Constructive interference only at one point



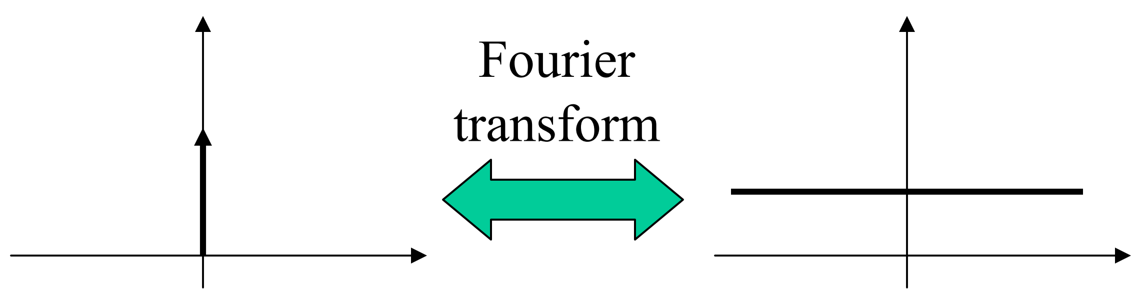
Fourier transform

There's no common multiple of all these wavelengths

$$\Delta x \Delta k \approx 1$$

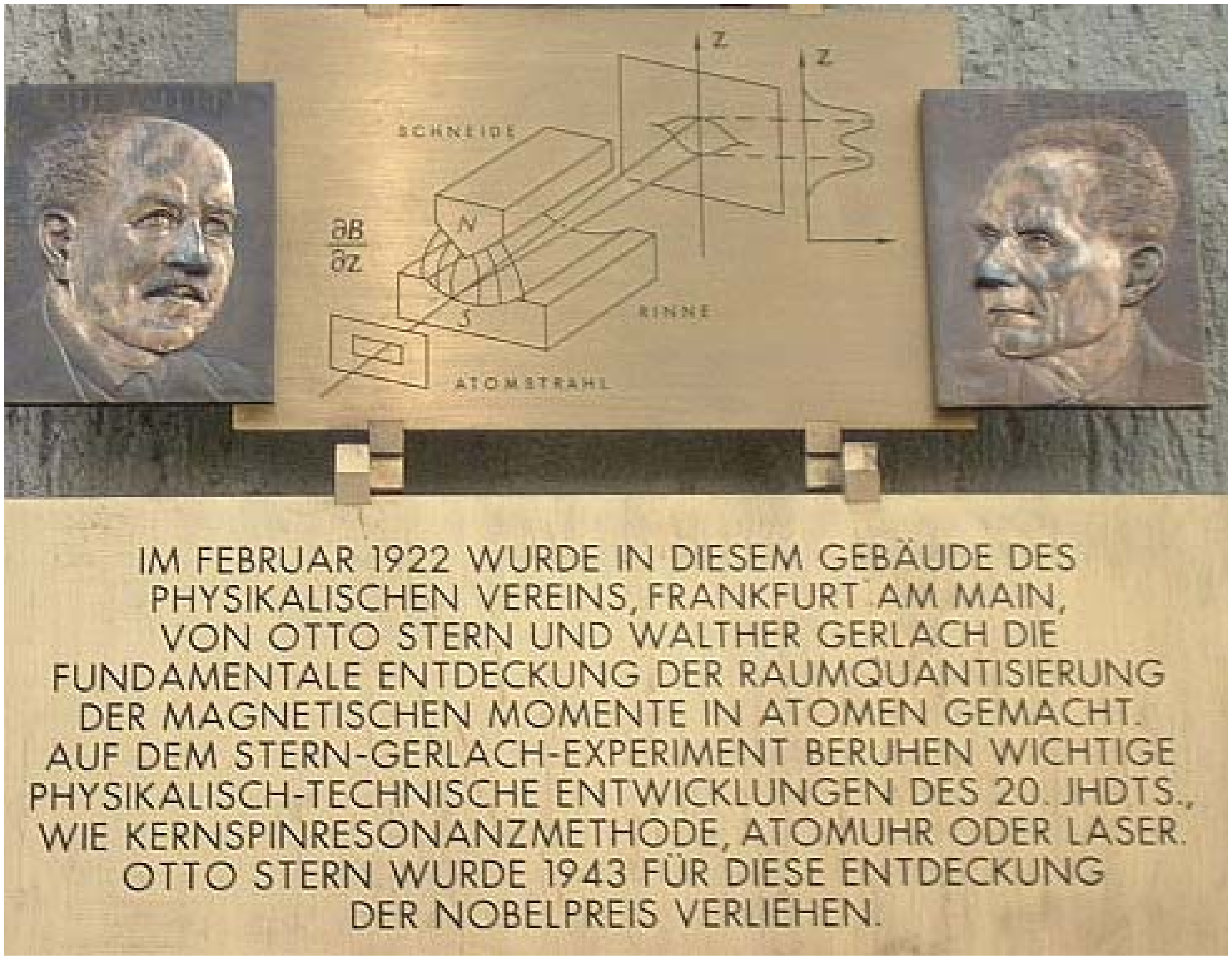
$$\Delta x(\ \ \Delta k) \approx$$

$$\Delta x \Delta p \approx$$



Quantum Mechanics Illustrated in a Simple System

(before we go into Shrödinger equation)



Stern-Gerlach Experiment

Energy of magnet in a magnetic field

$$U = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu B \cos\theta$$

Force on the magnet

$$F = -\frac{\partial U}{\partial z} = \mu \frac{\partial B}{\partial z} \cos\theta$$

The apparatus ***measures*** the projection of spin in a given direction (// ***B***).

Result:
Only two outcomes.

Watch the video at http://en.wikipedia.org/wiki/Stern%E2%80%93Gerlach_experiment

- The apparatus ***<u>measures</u>*** the projection of spin in a given direction (// ***B***); lets call that *z*.
- Spin is the intrinsic magnetic moment (or angular momentum; a factor between the two quantities) of the electron. Call that ***S***; it's a vector. Its projection along the *z* direction is S_z.
- S_z has only two possible values. Spin up or spin down. These are called eigenvalues.
- Eigenstates are the electron's states in which the physical quantity (spin here) takes the eigenvalue.
- The electron is not necessarily in any eigenstate. In general, the electron is in a state that is a linear combination of the eigenstates.
- **When a measurement is performed, the electron "collapses" to one of the eigenstates.**
- The probability of the electron collapsing onto an eigenstate (i.e., the measurement yielding the corresponding eigenvalue) is
  |amplitude of the eigenstate in the linear combination|^2.

What if we have another Stern-Gerlach apparatus after one?

If you have further interest (esp. based on your previous exposure to quantum mechanics), please refer to my on line content for my Nanoelectronics course (same webpage as this course; just scroll down)

Reading assignment 2 (book online)
Course note 3: “The Quantum World” and further reading assignment therein