

ECE406/506 Real-Time Digital Signal Processing

Lecture 1 - Introduction

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Overview

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1 Introduction

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Basic building blocks in DSP

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- Frequency analysis
- Sampling
- Filtering

Three-step teaching approach

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- Review DSP theory
- Explain and demonstrate how MATLAB realizes it
- Implementation on DSP board with C

Clarification of terminologies (1)

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- Discrete vs. Digital
 - Continuous-time vs. **Discrete-time signal**
 - Continuous-valued vs. Discrete-valued signal
 - Digital signal
- Deterministic vs. Random signal

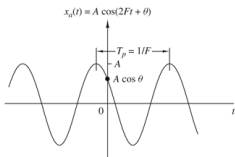


Figure 1.3.1 Example of an analog sinusoidal signal.
(a) Analog signal.

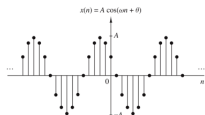


Figure 1.3.3 Example of a discrete-time sinusoidal signal ($\omega = \pi/6$ and $\theta = \pi/3$).

(b) Discrete-time signal.

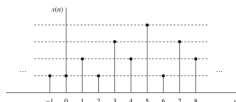


Figure 1.2.5 Digital signal with four different amplitude values.

(c) Digital signal.

Clarifications on terminologies (2)

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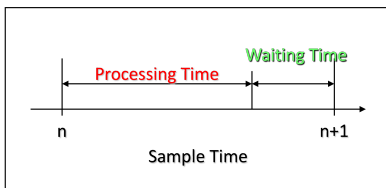
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- What is real-time processing?
 - The processing of a particular sample must occur within a given time period or the system will not operate properly.
 - When waiting time ≥ 0 , we have a real-time application



- Hard real-time system: The system will fail if the processing is not done in a timely manner
- Soft real-time system: The system will tolerate some failures to meet real-time targets and still continue to operate, but with some degradation in performance.

Signal processing courses at UT

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- ECE 315 - Signals and Systems I
- ECE 316 - Signals and Systems II
- ECE 505 - Digital Signal Processing
- **ECE 406/506 - Real-Time Digital Signal Processing**

Examples

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- Performance analysis
- Quantization noise
- DTMF decoder

Sinusoid

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$$x_a(t) = A \cos(\Omega t + \theta), -\infty < t < \infty$$

or

$$x_a(t) = A \cos(2\pi Ft + \theta), -\infty < t < \infty$$

where

- A : amplitude
- θ : phase (radians) or phase shift
- $\Omega = 2\pi F$: radian frequency (radians per second, rad/s)
- F : cyclic frequency (cycles per second, herz, Hz)
- $T_p = 1/F$: fundamental period (sec) such that $x_a(t + T_p) = x_a(t)$

More on frequency

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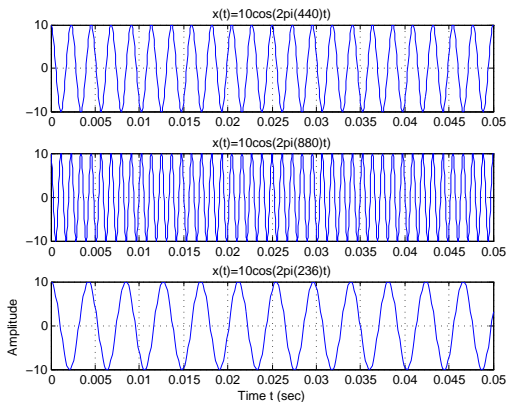


Figure: Sinusoids with different frequencies.

What if $F = 0$?

More on frequency - How does it sound?¹

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- A440
- A880
- C236
- A tuning fork demo

¹The multimedia materials are from McClellan, Schafer and Yoder, *DSP FIRST: A Multimedia Approach*. Prentice Hall, Upper Saddle River, New Jersey, 1998. Copyright (c) 1998 Prentice Hall

More on frequency - The MATLAB code

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```
1 % Lecture 1 - Sinusoid
2 % plot a sinusoidal signal and listen to it
3 % 440Hz is the frequency of A above middle C on a musical scale
4 % it is often used as the reference note for tuning purpose
5 %
6 clear buffer
7 clear all;
8 clf;
9
10 % specify parameters
11 F = 440;
12 t = 0:1/F/30:1/F*5;
13 x = 10*cos(2*pi*F*t - 0.4*pi);
14
15 % plot the signal
16 plot(t,x);
17 title('Sinusoidal signal x(t)');
18 xlabel('Time t (sec)');
19 ylabel('Amplitude');
20 grid on;
21
22 % play the signal
23 sound(x)
```

Complex exponential signals

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- According to Euler's formula

$$\begin{aligned}x_a(t) &= A \cos(\Omega t + \theta) = \Re\{Ae^{j(\Omega t + \theta)}\} \\ &= \Re\{Ae^{j\theta} e^{j\Omega t}\} = \Re\{Xe^{j\Omega t}\}\end{aligned}$$

- The rotating phasor interpretation
 - Complex amplitude (or Phasor): $X = e^{j\theta}$
 - Rotating phasor: multiplying the fixed phasor X by $e^{j\Omega t}$ causes the phasor to rotate. If Ω is positive, the direction of rotation is counterclockwise; when Ω is negative, clockwise.
 - The phase shift θ defines where the phasor is pointing when $t = 0$
- A rotating phasor demo²

²The multimedia materials are from McClellan, Schafer and Yoder, *DSP FIRST: A Multimedia Approach*. Prentice Hall, Upper Saddle River, New Jersey, 1998. Copyright (c) 1998 Prentice Hall

Spectrum and Time-frequency spectrum

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- Spectrum: frequency domain representation of the signal that reveals the frequency content of the signal
- Two-sided spectrum: According to inverse Euler's formula

$$x_a(t) = A \cos(\Omega t + \theta) = \frac{A}{2} e^{j\theta} e^{j\Omega t} + \frac{A}{2} e^{-j\theta} e^{-j\Omega t}$$

such that the sinusoid can be interpreted as made up of 2 complex phasors

$$\left\{ \left(\frac{1}{2} X, F \right), \left(\frac{1}{2} X^*, -F \right) \right\}$$

- Spectrogram: frequency changes over time

Application 1: Phasor addition

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- When adding several sinusoids having the same frequency but different amplitudes and phases, the resulting signal is a complex exponential signal with the same frequency

$$\sum_{k=1}^N A_k \cos(\Omega t + \theta_k) = A \cos(\Omega t + \theta)$$

- Proof
- Exercise:

$$1.7 \cos(2\pi(10)t + 70\pi/180) + 1.9 \cos(2\pi(10)t + 200\pi/180)$$

Application 2: Producing new signals from sinusoids

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■ Additive linear combination

$$\begin{aligned}x_a(t) &= A_0 + \sum_{k=1}^N A_k \cos(2\pi F_k t + \theta_k) \\&= X_0 + \sum_{k=1}^N \Re\{X_k e^{j2\pi F_k t}\} \\&= X_0 + \sum_{k=1}^N \left\{ \frac{X_k}{2} e^{j2\pi F_k t} + \frac{X_k^*}{2} e^{-j2\pi F_k t} \right\}\end{aligned}$$

where $X_k = A e^{j\theta_k}$.

■ $2N + 1$ complex phasors

$$\{(X_0, 0), (\frac{1}{2}X_1, F_1), (\frac{1}{2}X_1^*, -F_1), (\frac{1}{2}X_2, F_2), (\frac{1}{2}X_2^*, -F_2), \dots\}$$

■ Exercise

$$x_a(t) = 10 + 14 \cos(200\pi t - \pi/3) + 8 \cos(500\pi t + \pi/2)$$

Application 3: Adding two sinusoids with nearly identical frequencies - Beat notes

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- Adding two sinusoids with frequencies, F_1 and F_2 , very close to each other

$$x_a(t) = \cos(2\pi F_1 t) + \cos(2\pi F_2 t)$$

where

- $F_1 = F_c - F_\Delta$ and $F_2 = F_c + F_\Delta$.
 - $F_c = \frac{1}{2}(F_1 + F_2)$ is the *center frequency*
 - $F_\Delta = \frac{1}{2}(F_2 - F_1)$ is the *deviation frequency*
 - In general, $F_\Delta \ll F_c$
- Two-sided spectrum representation,

$$\left\{ \left(\frac{1}{2}, F_1 \right), \left(\frac{1}{2}, -F_1 \right), \left(\frac{1}{2}, F_2 \right), \left(\frac{1}{2}, -F_2 \right) \right\}$$

Adding two sinusoids with nearly identical frequencies - Beat notes (cont')

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- Rewrite $x_a(t)$ as a product of two cosines

$$\begin{aligned}x_a(t) &= \Re\{e^{j2\pi F_1 t}\} + \Re\{e^{j2\pi F_2 t}\} \\&= \Re\{e^{j2\pi(F_c - F_\Delta)t} + e^{j2\pi(F_c + F_\Delta)t}\} \\&= \Re\{e^{j2\pi F_c t}(e^{-j2\pi F_\Delta t} + e^{j2\pi F_\Delta t})\} \\&= \Re\{e^{j2\pi F_c t}(2 \cos(2\pi F_\Delta t))\} \\&= 2 \cos(2\pi F_\Delta t) \cos(2\pi F_c t)\end{aligned}$$

- Adding two sinusoids with nearly identical frequencies = Multiplying two sinusoids with frequencies far apart
- What is the effect of multiplying a higher-frequency sinusoid (e.g., 2000 Hz) by a lower-frequency sinusoid (e.g., 20 Hz)? The “beating” phenomenon.

Adding two sinusoids with nearly identical frequencies - Beat notes (cont')

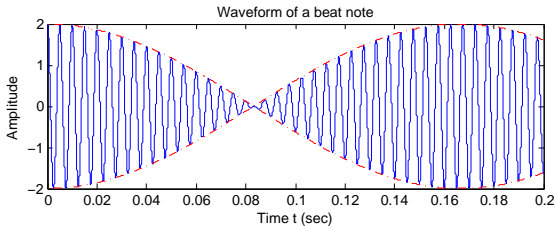
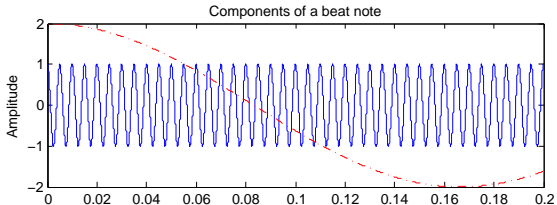
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■ A demo

Adding two sinusoids with nearly identical frequencies: Beat notes (cont')

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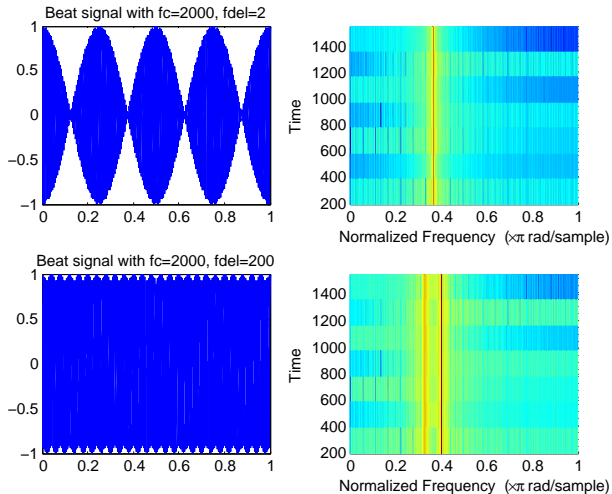


Figure: Beat notes and the spectrogram.

Application 4: Multiplying sinusoids - Amplitude modulation

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- Modulation for communication systems: multiplying a low-frequency signal by a high-frequency sinusoid

$$x_a(t) = v_a(t) \cos(2\pi F_c t)$$

- $v_a(t)$: the modulation signal to be transmitted, must be a sum of sinusoids
 - $\cos(2\pi F_c t)$: the carrier signal
 - F_c : the carrier frequency
 - F_c should be much higher than any frequencies contained in the spectrum of $v_a(t)$.
- Exercise:

$$v_a(t) = 5 + 2 \cos(40\pi t), F_c = 200 \text{ Hz}$$

- Difference between a beat note and an AM signal?

Multiplying sinusoids - Amplitude modulation (cont')

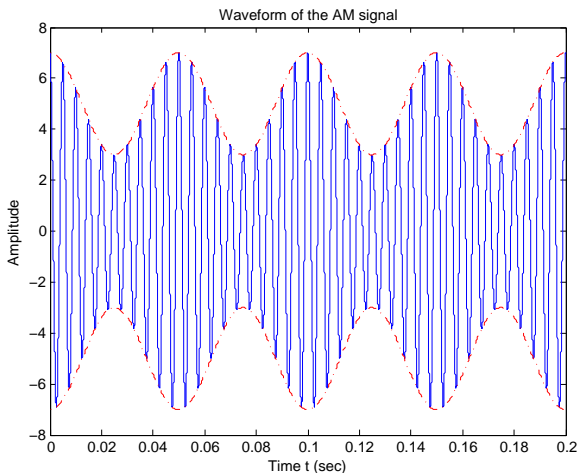
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■ A demo

Application 5: Adding cosine waves with harmonically related frequencies - Periodic waveforms

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- Fourier Series Theorem: Any **periodic** signal can be approximated with a sum of **harmonically** related sinusoids, although the sum may need an infinite number of terms.

$$\begin{aligned}x_a(t) &= A_0 + \sum_{k=1}^N A_k \cos(2\pi kF_0 t + \theta_k) \\ &= X_0 + \Re\{\sum_{k=1}^N X_k e^{j2\pi kF_0 t}\}\end{aligned}$$

- $F_k = kF_0$: the **harmonic** of F_0
- F_0 : the fundamental frequency
- Estimate interesting waveforms by clever choice of $X_k = A_k e^{j\theta_k}$

Adding cosine waves with harmonically related frequencies - Periodic waveforms (cont')

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- Fourier analysis: starting from $x_a(t)$ and calculate X_k . X_k can be calculated using the Fourier integral

$$X_k = \frac{2}{T_0} \int_0^{T_0} x_a(t) e^{-j2\pi kt/T_0} dt, X_0 = \frac{1}{T_0} \int_0^{T_0} x_a(t) dt$$

- T_0 : the fundamental period of $x_a(t)$
- X_0 : the DC component
- Fourier synthesis: starting from X_k and calculate $x_a(t)$
- Demo: synthetic vowel ('ah'), $F_0 = 100$ Hz

$$x_a(t) = \Re\{X_2 e^{j2\pi 2F_0 t} + X_4 e^{j2\pi 4F_0 t} + X_5 e^{j2\pi 5F_0 t} + X_{16} e^{j2\pi 16F_0 t} + X_{17} e^{j2\pi 17F_0 t}\}$$

- Exercise: How to approximate a square wave?

Application 6: Frequency modulation - the Chirp signal

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- A “chirp” signal is a swept-frequency signal whose frequency changes **linearly** from some low value to a high one.
- How to generate it?
 - concatenate a large number of short constant-frequency sinusoids, whose frequencies step from low to high
 - time-varying phase $\psi(t)$ as a function of time

$$x_a(t) = \Re\{Ae^{j\psi(t)}\} = A \cos(\psi(t))$$

- **instantaneous frequency**: the derivative (slope) of the phase

$$\Omega(t) = \frac{d}{dt}\psi(t), F(t) = \Omega(t)/(2\pi)$$

- **Frequency modulation**: frequency variation produced by the time-varying phase. Signals of this class are called **FM signals**

Frequency modulation - the Chirp signal (cont')

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- Linear FM signal: chirp signal
- Exercise: quadratic phase

$$\psi(t) = 2\pi\mu t^2 + 2\pi F_0 t + \theta, F(t) = 2\mu t + F_0$$

- Reverse process: If a certain linear frequency sweep is desired, the actual phase can be obtained from the integral of $\Omega(t)$.
- Exercise: synthesize a frequency sweep from $F_1 = 220$ Hz to $F_2 = 2320$ Hz over the time interval $t = 0$ to $t = T_2 = 3$ sec.

Frequency modulation - the Chirp signal

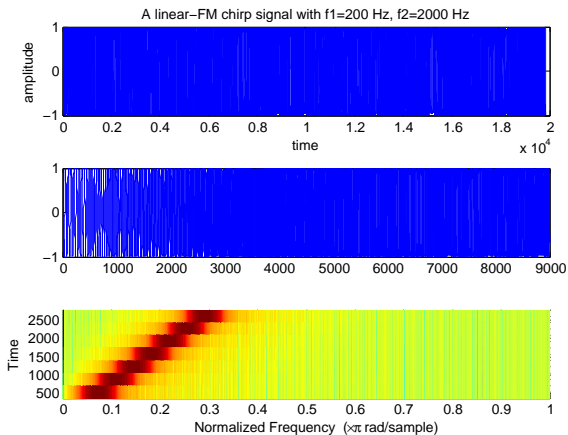
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■ A demo

Euler's formula and Inverse Euler's formula

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■ Euler's formula

$$e^{j\theta} = \cos \theta + j \sin \theta$$

■ Inverse Euler's formula

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Basic trigonometric identities

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$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

Basic properties of the sine and cosine functions

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■ Equivalence

$$\sin \theta = \cos(\theta - \pi/2) \text{ or } \cos \theta = \sin(\theta + \pi/2)$$

■ Periodicity

$$\cos(\theta + 2k\pi) = \cos \theta, \text{ when } k \text{ is an integer}$$

■ Evenness of cosine

$$\cos(-\theta) = \cos \theta$$

■ Oddness of sine

$$\sin(-\theta) = -\sin \theta$$

Basic properties of the sine and cosine functions (cont')

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■ Zeros of sine

$$\sin(\pi k) = 0, \text{ when } k \text{ is an integer}$$

■ Ones of cosine

$$\cos(2\pi k) = 1, \text{ when } k \text{ is an integer}$$

■ Minus ones of cosine

$$\cos\left[2\pi\left(k + \frac{1}{2}\right)\right] = -1, \text{ when } k \text{ is an integer}$$

■ Derivatives

$$\frac{d \sin \theta}{d\theta} = \cos \theta, \frac{d \cos \theta}{d\theta} = -\sin \theta$$