ECE406/506 Real-Time Digital Signal Processing
Lecture 5 - Design of Digital Filters - FIR

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Overview

Review - Design structures

- Different representations of causal LTI systems
  - LCDE with initial rest condition
  - System function: $H(z)$ with $|z| > R_+$ and starts at $n = 0$
  - Unit sample response: $h[n]$
  - Frequency response: $H(e^{j\omega})$
  - Design structures: Block diagram vs. Signal flow graph and how to determine system function (or unit sample response) from the graphs
    - Direct form I (zeros first)
    - Direct form II (poles first) - Canonic structure
    - Transposed form (zeros first)
    - Cascade form
    - Parallel form

FIR filters with generalized linear phase

- General FIR filters

$$y[n] = \sum_{k=0}^{M-1} h[k]x[n-k], H(z) = \sum_{k=0}^{M-1} h[k]z^{-k}$$

  - All poles are at $z = 0$
  - ROC is the entire z-plane except for $z = 0$
- FIR filters with generalized linear phase

$$H(e^{j\omega}) = A(e^{j\omega})e^{j(\beta-\alpha\omega)}$$
$$arg[H(e^{j\omega})] = \beta - \omega\alpha, 0 < \omega < \pi$$
$$grd[H(e^{j\omega})] = -\frac{d}{d\omega}\{arg[H(e^{j\omega})]\} = \alpha$$

- Necessary condition on $h[n]$ to have constant group delay

$$\sum_{n=-\infty}^{\infty} h[n]\sin[\omega(n-\alpha)+\beta] = 0 \text{ for all } \omega$$

Four types of causal FIR systems with generalized linear phase

- Sufficient condition: $h[n]$ satisfies the symmetry and antisymmetry conditions

$$h[n] = \pm h[M-n], n = 0, 1, \cdots, M$$

- Type 1: $h[n] = h[M-n]$, M even. $\beta = 0$ or π. Delay is even as $\alpha = M/2$.

$$H(e^{j\omega}) = e^{-j\omega M/2}(h[M/2] + 2\sum_{k=1}^{M/2} h[M/2-k]\cos\omega k)$$

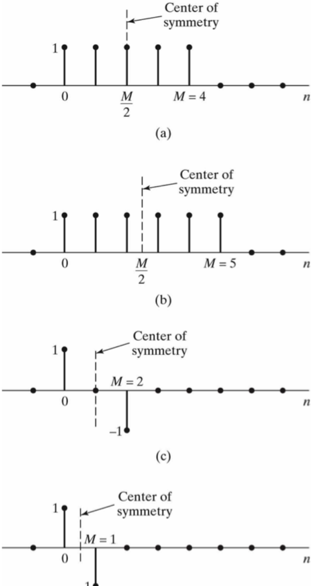
- Type 2: $h[n] = h[M-n]$, M odd. $\beta = 0$ or π. Delay is integer plus a half.

$$H(e^{j\omega}) = e^{-j\omega M/2}(2\sum_{k=1}^{(M+1)/2} h[(M+1)/2-k]\cos[\omega(k-1/2)])$$

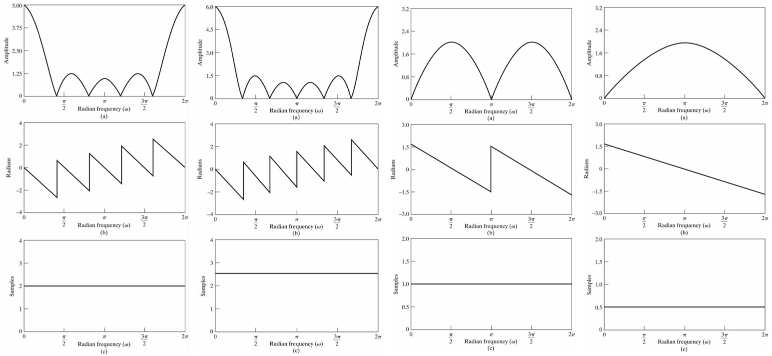
Four types of causal FIR systems (cont')

- Type 3: $h[n] = -h[M - n]$, M even. $\beta = \pi/2$ or $3\pi/2$. Delay is integer.
- Type 4: $h[n] = -h[M - n]$, M odd. $\beta = \pi/2$ or $3\pi/2$. Delay is integer plus a half.

Examples $h[n]$


Center of symmetry
1
0
M/2
M = 4
n
(a)
Center of symmetry
1
0
M/2
M = 5
n
(b)
Center of symmetry
1
M = 2
0
–1
n
(c)
Center of symmetry
1
M = 1
0
n

Examples $H(e^{j\omega})$, α, and group delay



Zeros of FIR filters with generalized linear phase

- The roots of $H(z)$ must occur in reciprocal pairs, and
- If $h[n]$ is real, the complex-valued roots must occur in complex-conjugate pairs

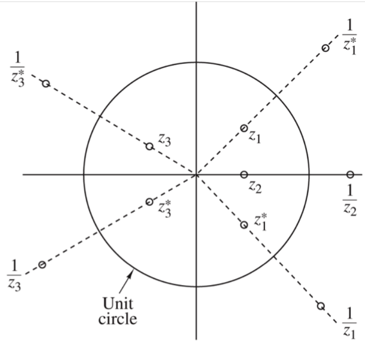
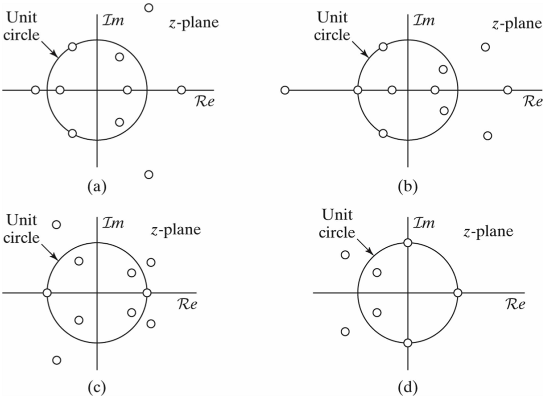


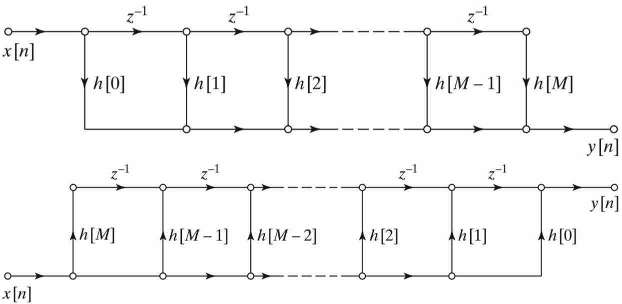
Figure 10.2.1 Symmetry of zero locations for a linear-phase FIR filter.

Zeros of the four types of FIR systems

- Type I and II: $z = -1$
- Type III and IV: $z = \pm 1$



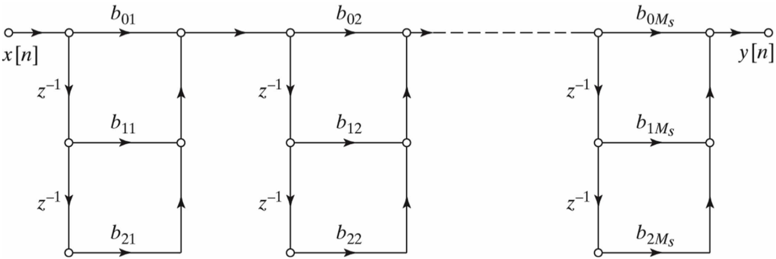
FIR - Direct and transposed direct form



- tapped delay line structure (transversal filter structure)
- discrete convolution

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

Cascade form

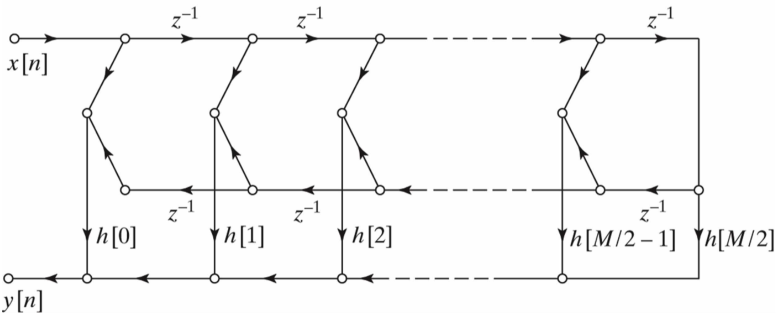


$$H(z) = \sum_{n=0}^{M} h[n]z^{-n} = \prod_{k=1}^{M_s} (b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2})$$

$$M_s = \lfloor (M+1)/2 \rfloor$$

Linear phase FIR systems

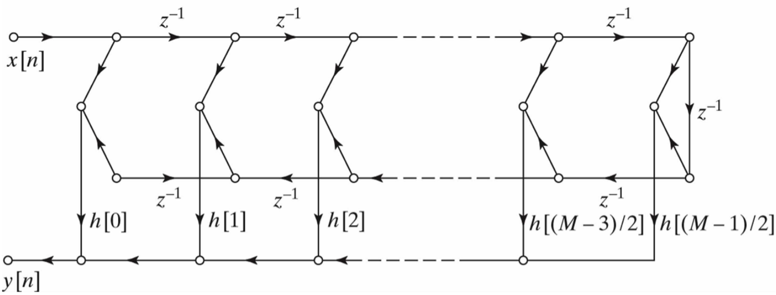
$$h[M-n] = h[n], n = 0, 1, \cdots, M$$



When M is even

Linear phase FIR systems (cont')

$$h[M-n] = h[n], n = 0, 1, \cdots, M$$



When M is odd

When to use what?

- Constraints on the zeros of $H(z)$ at $z = 1$ and $z = -1$ (What type of filters need to be designed?)
- Integer/non-integer group delay
- Phase shift