Real-Time Digital Signal Processing Lecture 5 - Design Structures for Digital Filters

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Overview

1 Roadmap

2 Representations

3 Structures

Recap

- Week 1: Background
- Week 2: DSK and Lab
- Week 3: I/O - Sampling and Reconstruction
  - Sampling: impulse-train vs. zero-order hold
  - Reconstruction: band-limited interpolation vs. zero-order hold interpolation vs. higher-order hold interpolation
  - Aliasing: the aliasing frequency vs. the folding frequency
  - The sampling theorem
  - The sigma-delta ADC
- Week 4: The z-transform and Design Structure

Recap

- The z-transform
  - Relationship to FT and Laplace transform
  - System function
  - Region of convergence (ROC)
  - Properties
- The inverse z-transform
- Useful filters

Different system representations

- LTI systems with initial rest condition (or causal LTI systems or ROC is outside the outmost pole)
- Using LCDE

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] + \sum_{k=1}^{N} a_k y[n-k]$$

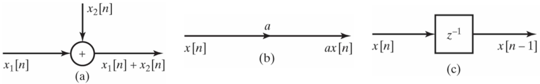
- Using system function

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

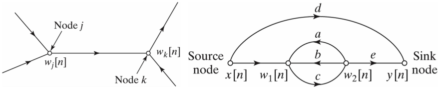
- Block diagram and signal flow graph

Block diagram vs. Signal flow graph

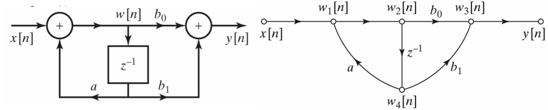
- Block diagram symbols: adder, multiplier, unit delay (memory)



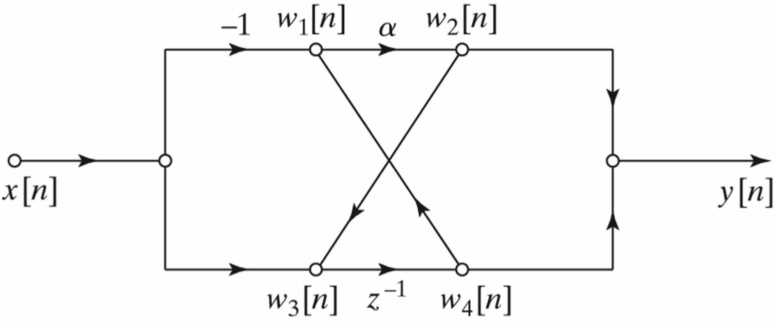
- Signal flow graph: directed branches (branch gain, delay branch), nodes (source node, sink node)



- A comparison: nodes in the flow graph represent both branching points and adders, whereas in the block diagram a special symbol is used for adders



Determination of the system function from a flow graph



Different design structures

- Direct form I
- Direct form II - Canonic structure
- Transposed form
- Cascade form
- Parallel form
- Why study design structure? (Performance metrics)
  - Computational resource (# of adders, multipliers, unit delays)
  - Precision

Direct form I and II

$$H(z)=\frac{\sum_{k=0}^{M} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}}$$

- Direct form I: implementing zeros first

$$H(z)=H_2(z)H_1(z)=\left(\frac{1}{1-\sum_{k=1}^{N} a_k z^{-k}}\right)\left(\sum_{k=0}^{M} b_k z^{-k}\right) \quad (1)$$

$$V(z)=H_1(z)X(z)=\left(\sum_{k=0}^{M} b_k z^{-k}\right)X(z) \quad (2)$$

$$Y(z)=H_2(z)V(z)=\left(\frac{1}{1-\sum_{k=1}^{N} a_k z^{-k}}\right)V(z) \quad (3)$$

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]+\sum_{k=1}^{N} a_k y[n-k] \quad (4)$$

- Direct form II: implementing poles first

$$H(z)=H_1(z)H_2(z)=\left(\sum_{k=0}^{M} b_k z^{-k}\right)\left(\frac{1}{1-\sum_{k=1}^{N} a_k z^{-k}}\right) \quad (5)$$

$$W(z)=H_2(z)X(z)=\left(\frac{1}{1-\sum_{k=1}^{N} a_k z^{-k}}\right)X(z) \quad (6)$$

$$Y(z)=H_1(z)W(z)=\left(\sum_{k=0}^{M} b_k z^{-k}\right)W(z) \quad (7)$$

$$y[n]=\sum_{k=1}^{N} a_k y[n-k]+\sum_{k=0}^{M} b_k x[n-k] \quad (8)$$

Comparison

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

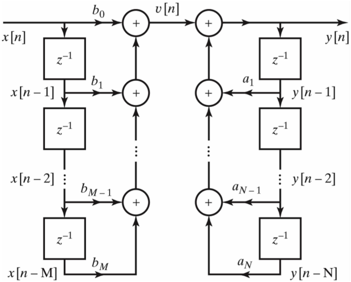


Figure : IIR direct form I.

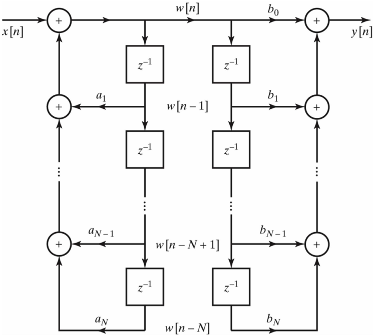


Figure : IIR direct form II.

Comparison (cont')

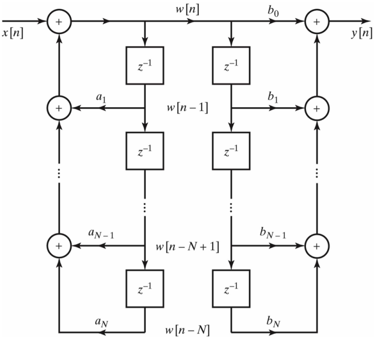


Figure : IIR direct form II (a).

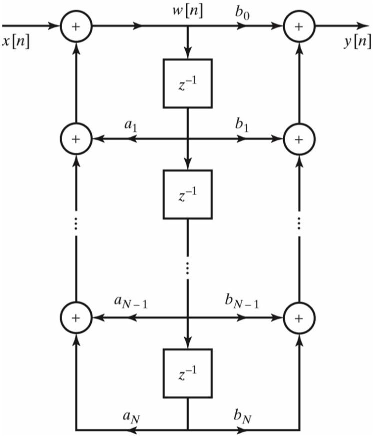


Figure : IIR direct form II (b).

Comparison (cont')

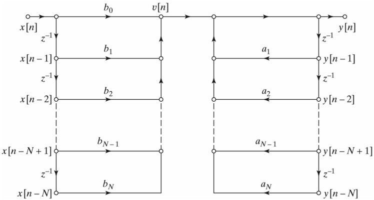


Figure : IIR direct form I.

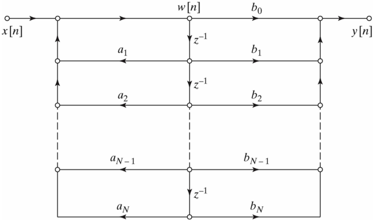
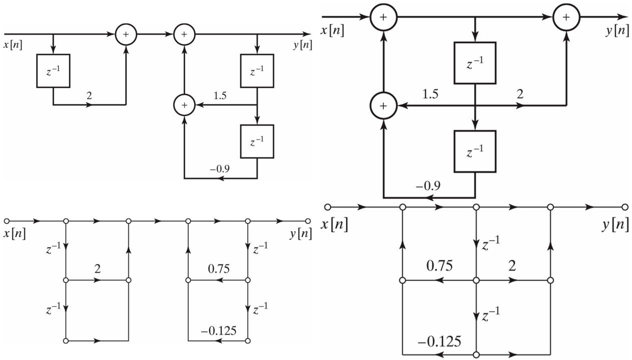


Figure : IIR direct form II.

Example

- $H(z) = \frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$
- $H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$

Solution

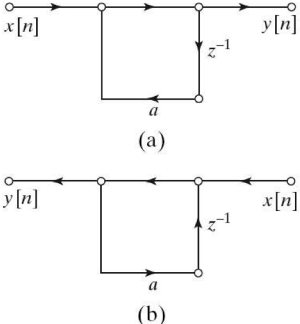


Canonic vs. Noncanonic structures

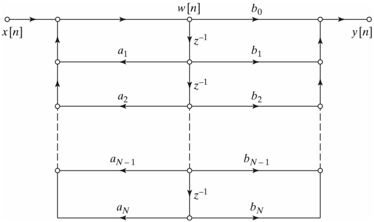
- A digital filter structure is said to be *canonic* if the number of delays is equal to the order of the difference equation. Otherwise, it is a *noncanonic* structure. That is, minimum number of delays required is $max(N, M)$.

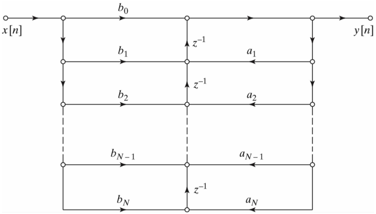
Transposed form

- The transposition theorem: For single-input, single-output systems, the resulting flow graph has the same system function as the original graph if the input and output nodes are interchanged.
    - reverse direction of all brances
    - interchange input and output
- Implement zeros first, then poles as compared to the direct II form



Comparison - Direct form II (L) vs. Transposed direct form II (R)



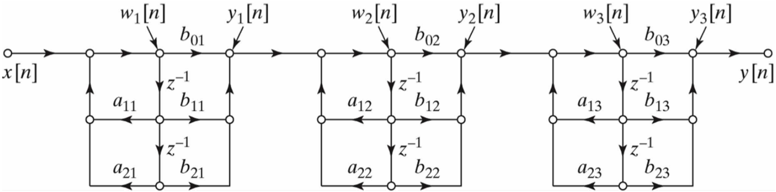


Cascade form

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

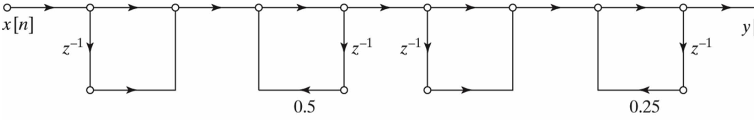
$$H(z) = \prod_{k=1}^{N_s} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

$$N_s = \lfloor (N+1)/2 \rfloor$$

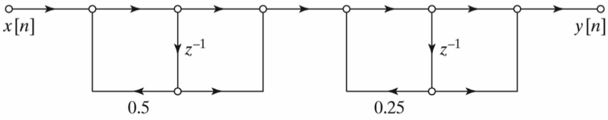


Example

- Ex: $H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}} = \frac{(1+z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})}$



(a)



(b)

Why cascading?

- Use of computational resource
  - Direct form II structure: $2N+1$ constant multipliers

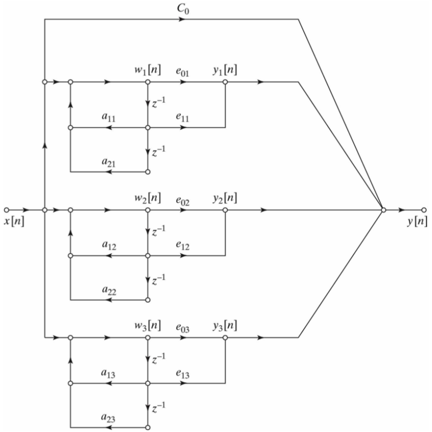
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

  - Cascade form structure: $5N/2$ constant multipliers (assume $M = N$ and N is even)

$$H(z) = \prod_{k=1}^{N_s} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

- Precision: Sensitivity of poles to error in coefficient is lower for lower-order systems and higher for higher-order systems

Parallel form



$$H(z) = \sum_{k=0}^{N_p} C_k z^{-k} + \sum_{k=1}^{N_s} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

Example

- Ex: $H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}} = 8 + \frac{-7+8z^{-1}}{1-0.75z^{-1}+0.125z^{-2}}$