Real-Time Digital Signal Processing Lecture 6 - Finite Impulse Response Filter - I

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Overview

Recap

- Week 1: Background
- Week 2: DSK and Lab
- Week 3: I/O - Sampling and Reconstruction
- Week 4: The z-transform and Design Structure
  - The z-transform
    - Relationship to FT and Laplace transform
    - System function
    - Region of convergence (ROC)
    - Properties
  - The inverse z-transform
  - Useful filters
    - L-point running sum
    - Bandpass filter with complex coefficient
    - Bandpass filter with real coefficient

Review - Design structures

- Different representations of causal LTI systems
  - LCDE with initial rest condition
  - $H(z)$ with $|z| > R_+$ and starts at $n = 0$
  - Block diagram vs. Signal flow graph: how to determine system function (or unit sample response) from the graphs
- Design structures
  - Direct form I (zeros first)
  - Direct form II (poles first) - Canonic structure
  - Transposed form (zeros first) - Canonic structure
  - Cascade form
  - Parallel form
  - Why different design structures? (Metrics)
    - computational resource, precision, and execution time

FIR filters with generalized linear phase

- General FIR filters (Finite Impulse Response)

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k], H(z) = \sum_{k=0}^{M} h[k]z^{-k}$$

  - All poles are at $z = 0$
  - ROC is the entire z-plane except for $z = 0$
- FIR filters with generalized linear phase

$$H(e^{j\omega}) = A(e^{j\omega})e^{j(\beta-\alpha\omega)}$$
$$\sphericalangle H(e^{j\omega}) = arg[H(e^{j\omega})] = \beta - \omega\alpha, 0 < \omega < \pi$$
$$\tau(\omega) = grd[H(e^{j\omega})] = -\frac{d}{d\omega}\{arg[H(e^{j\omega})]\} = \alpha$$

- Necessary condition on $h[n]$ to have constant group delay

$$\sum_{n=-\infty}^{\infty} h[n]\sin[\omega(n-\alpha)+\beta] = 0 \text{ for all } \omega$$

Four types of causal FIR systems with generalized linear phase

- Sufficient condition: $h[n]$ satisfies the symmetry and antisymmetry conditions

$$h[n] = \pm h[M-n], n = 0, 1, \cdots, M$$

- Type 1: $h[n] = h[M-n]$, M even. $\beta = 0$ or π. Delay is even as $\alpha = M/2$.

$$H(e^{j\omega}) = e^{-j\omega M/2}(h[M/2] + 2\sum_{k=1}^{M/2} h[M/2-k]\cos\omega k)$$

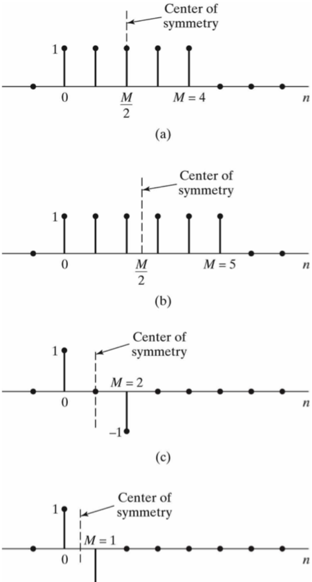
- Type 2: $h[n] = h[M-n]$, M odd. $\beta = 0$ or π. Delay is integer plus a half.

$$H(e^{j\omega}) = e^{-j\omega M/2}(2\sum_{k=1}^{(M-1)/2} h[(M-1)/2-k]\cos[\omega(k+1/2)])$$

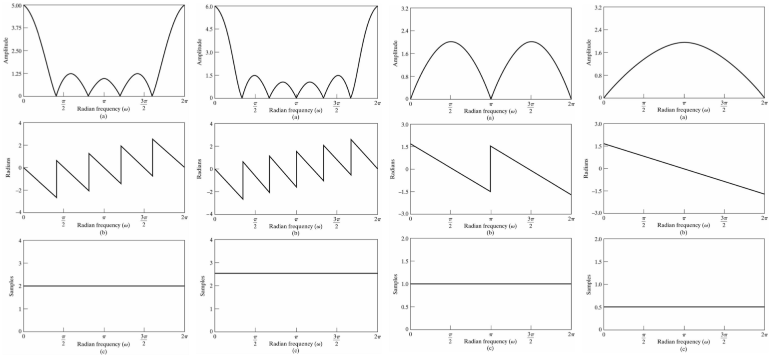
Four types of causal FIR systems (cont')

- Type 3: $h[n] = -h[M-n]$, M even. $\beta = \pi/2$ or $3\pi/2$. Delay is integer. $h[M/2]$ must be zero.
- Type 4: $h[n] = -h[M-n]$, M odd. $\beta = \pi/2$ or $3\pi/2$. Delay is integer plus a half.

Examples $h[n]$


Center of symmetry
1
0
M
2
M = 4
n
(a)
Center of symmetry
1
0
M
2
M = 5
n
(b)
Center of symmetry
1
M = 2
0
n
−1
(c)
Center of symmetry
1
M = 1
0
n

Examples $H(e^{j\omega})$, α, and group delay



Zeros of FIR filters with generalized linear phase

- The roots of $H(z)$ must occur in reciprocal pairs, and
- If $h[n]$ is real, the complex-valued roots must occur in complex-conjugate pairs

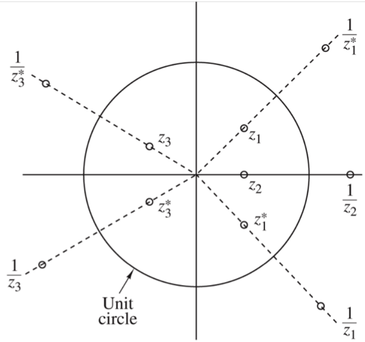
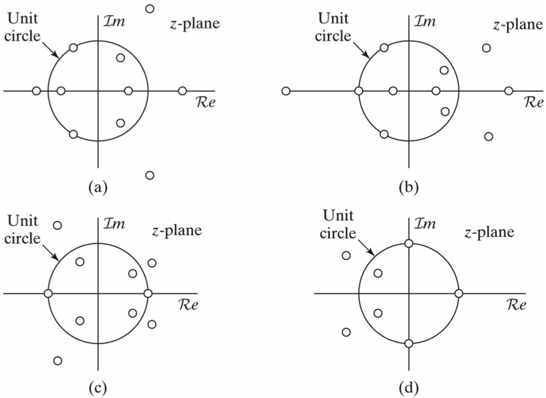


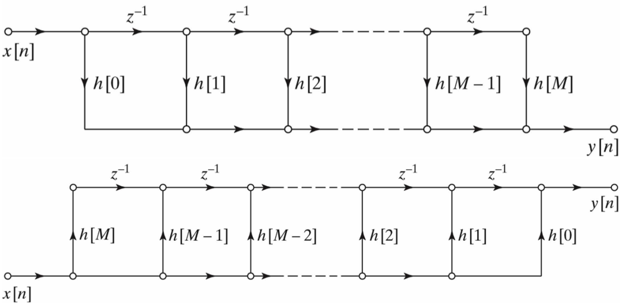
Figure 10.2.1 Symmetry of zero locations for a linear-phase FIR filter.

Zeros of the four types of FIR systems

- Type I: most versatile
- Type II: $z = -1$
- Type III: $z = \pm 1$
- Type IV: $z = 1$



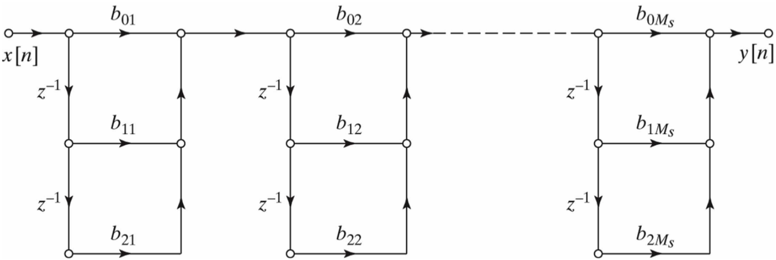
FIR - Direct and transposed direct form



- tapped delay line structure (transversal filter structure)
- discrete convolution

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

Cascade form

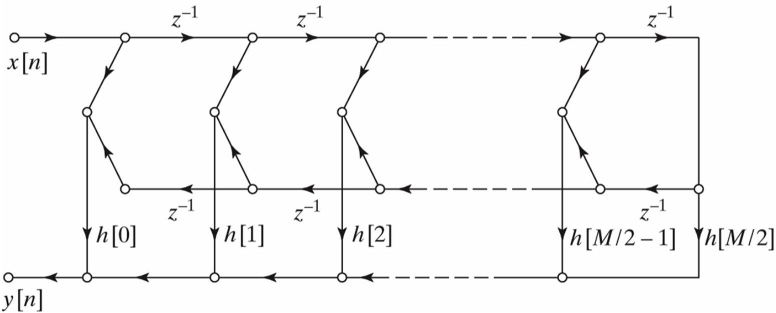


$$H(z) = \sum_{n=0}^{M} h[n]z^{-n} = \prod_{k=1}^{M_s} (b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2})$$

$$M_s = \lfloor (M+1)/2 \rfloor$$

Linear phase FIR systems

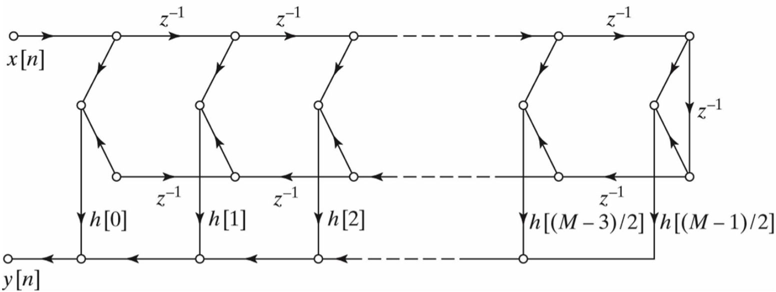
$$h[M-n] = h[n], n = 0, 1, \cdots, M$$



When M is even

Linear phase FIR systems (cont')

$$h[M-n] = h[n], n = 0, 1, \cdots, M$$



When M is odd

When to use what?

- Constraints on the zeros of $H(z)$ at $z = 1$ and $z = -1$ (What type of filters need to be designed?)
- Integer/non-integer group delay
- Phase shift

Examples

- $H(z) = 1 - z^{-2}$
- $H(z) = 1 - z^{-1}$
- $H(z) = 1 + z^{-1}$
- $H(z) = 1 + z^{-2}$