ECE 421/599
Electric Energy Systems
4 – Transmission Line Parameters

Instructor:

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Introduction

- Transmission lines
  - Overhead lines
  - Underground Cables (less than 1%)
- Properties
  - Series Resistance (stranding and skin effect)
  - Series Inductance (magnetic & electric fields; flux linkages within the conductor cross section and external flux linkages)
  - Shunt Capacitance (magnetic & electric fields; charge and discharge due to potential difference between conductors)
  - Shunt Conductance (due to leakage currents along insulators or corona discharge caused by ionization of air)
- Line-to-line voltage levels
  - 69kV, 115kV, 138kV and 161kV (sub-transmission)
  - 230kV, 345kV, and 500kV (EHV)
  - 765kV (UHV)

Corona discharge on insulator string of a 500 kV line (source: wikipedia.org)

Overhead Transmission Lines

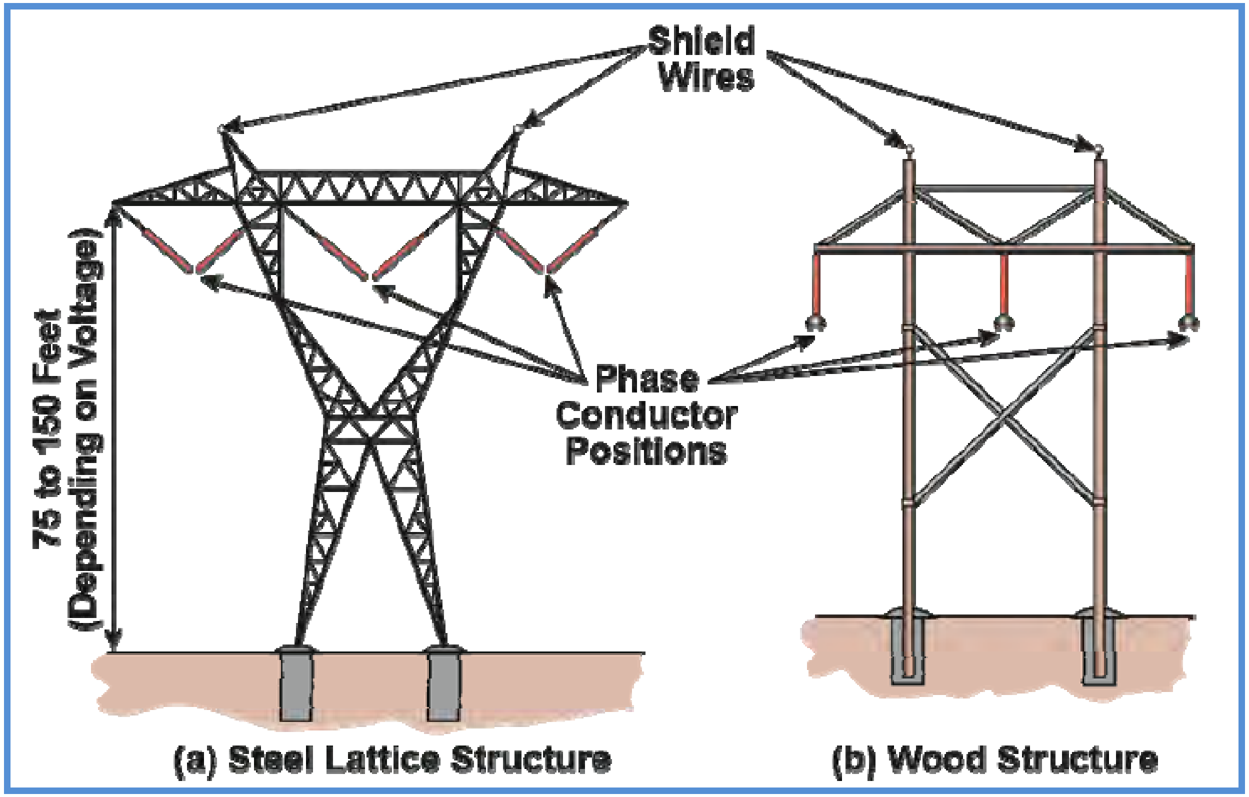


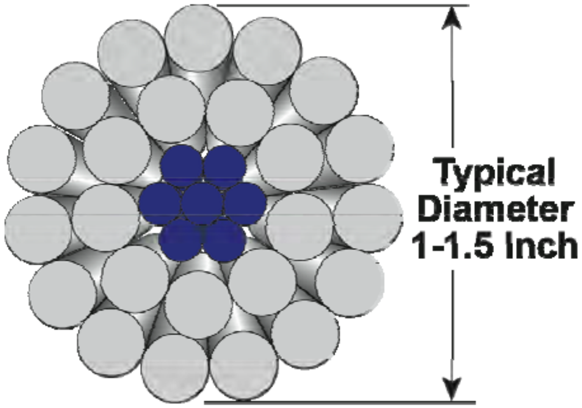
Figure 2-83. Transmission Line Structures

Shield wires (ground wires) are ground conductors used to protect the transmission lines from lightning strikes

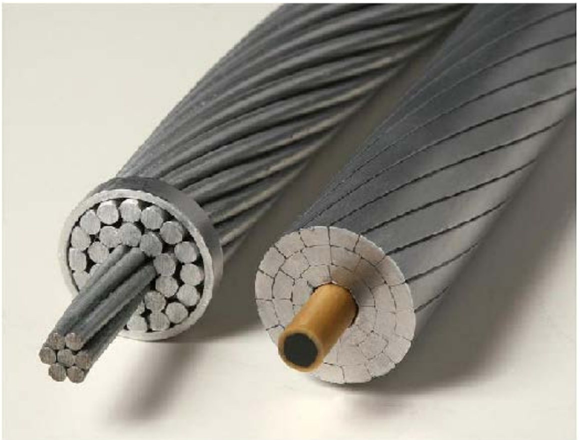
(Source: wikipedia.org and EPRI dynamic tutorial)

Overhead Transmission Lines

- Materials
  - AAC (All Aluminum Conductor),
  - AAAC (All Aluminum Alloy Conductor)
  - ACSR (Aluminum Conductor Steel Reinforced)
  - ACAR (Aluminum Conductor Alloy Reinforced)
  - ACCC (Aluminum Conductor Composite Core)
- Why not copper?
  - Relative lower costs and higher strength-to-weight ratios than copper
- Bundle conductors
  - Preferred for high voltages, e.g. 2-conductor bundles for 230kV, 3-4 for 345-500kV, and 6 for 765kV



ACSR (7 steel and 24 aluminum strands)



24/7 ACSR and modern ACCC conductors

A bundle of 4 conductor

Line Resistance

Consider a solid round conductor at a specific temperature:

- DC resistance

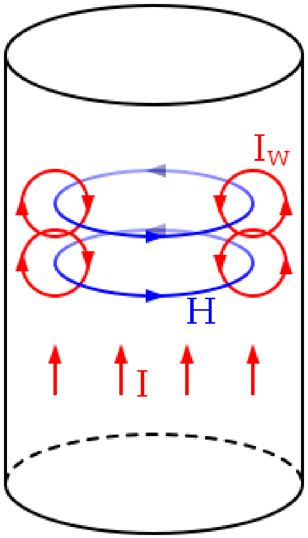
$$R_{dc} = \frac{\rho l}{A}$$

ρ = conductor resistivity

l = conductor length

A = conductor cross-sectional area

- AC resistance
  - Current is not uniformly distributed over the cross-sectional area; current density is greatest at the surface (skin effect)
  - 2% higher than DC resistance at 60Hz



Skin effect: circulating eddy current I_W cancelling the current flow in the center of the conductor.

(source: wikipedia.org)

- Temperature impact

$$\frac{R_1}{T + t_1} = \frac{R_2}{T + t_2}$$

t_1 and t_2 are in ^oC

$T \approx 228$ for aluminum

Inductance of a Single Conductor

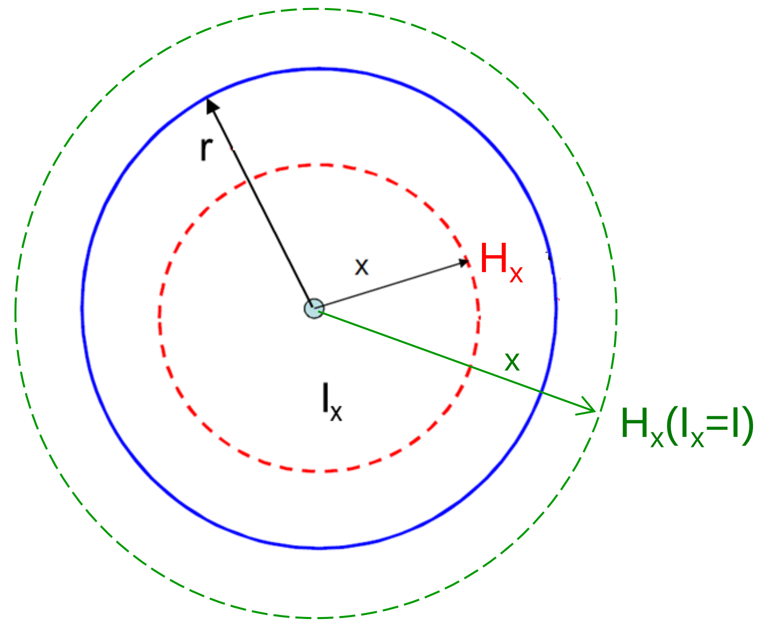
- Inductance for nonmagnetic material:

$$L = \lambda / I = L_{\text{int}} + L_{\text{ext}} = \lambda_{\text{int}} / I + \lambda_{\text{ext}} / I$$

- Ampere's law

$$\int_0^{2\pi x} H_x \cdot dl = I_x \quad \Rightarrow \quad \boxed{H_x = \frac{I_x}{2\pi x}}$$

$$\boxed{B_x = \mu_0 H_x = \frac{\mu_0 I_x}{2\pi x}}$$



I_x: current enclosed at radius x

H_x: magnetic field intensity

B_x: magnetic flux density

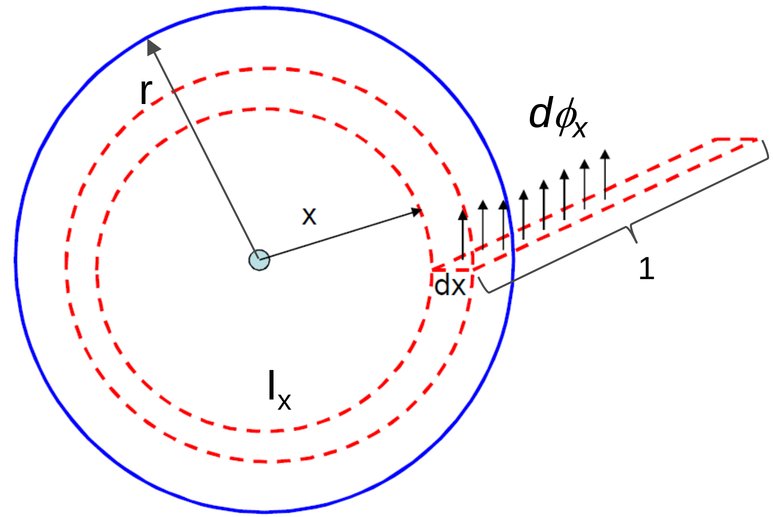
$\mu_0=4\pi\times10^{-7}$H/m: permeability of free space

Inductance due to the internal flux linkage

- Assume uniform current density

$$\frac{I_x}{\pi x^2} = \frac{I}{\pi r^2} \Rightarrow H_x = \frac{I_x}{2\pi x} = \frac{I}{2\pi r^2} x$$

$$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi r^2} x$$



- Differential flux and flux linkage:

$$d\phi_x = B_x \cdot (dx \cdot 1) = \frac{\mu_0 I}{2\pi r^2} x dx$$

For a small region of thickness *dx* and 1 meter length of the conductor

$$d\lambda_x = \frac{x^2}{r^2} \cdot d\phi_x = \frac{\mu_0 I}{2\pi r^4} x^3 dx$$

The portion of *I* is linked to $d\phi_x$

- Total internal flux linkage and the inductance:

$$\lambda_{int} = \int_0^r d\lambda = \frac{\mu_0 I}{2\pi r^4} \int_0^r x^3 dx = \frac{\mu_0 I}{8\pi} \quad Wb/m$$

$$L_{int} = \frac{\lambda_{int}}{I} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7} \quad H/m$$

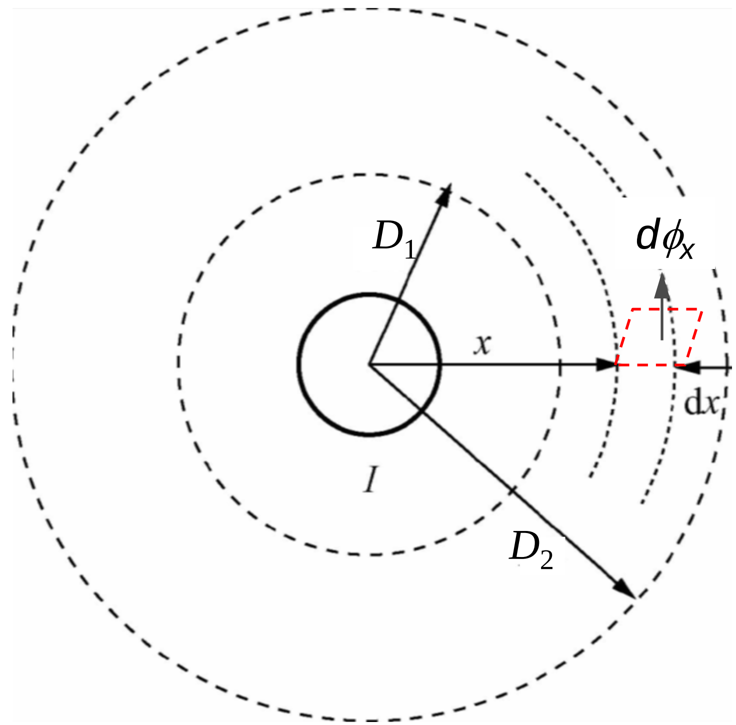
A constant independent of the conductor radius.

Inductance due to the external flux linkage

- $I_x = I$ for $x > r$

$$B_x = \mu_0 H_x = \frac{\mu_0 I_x}{2\pi x} = \frac{\mu_0 I}{2\pi x}$$

$$d\lambda_x = \frac{r^2}{r^2} d\phi_x = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi x} dx$$



- External flux linkage and inductance between two points:

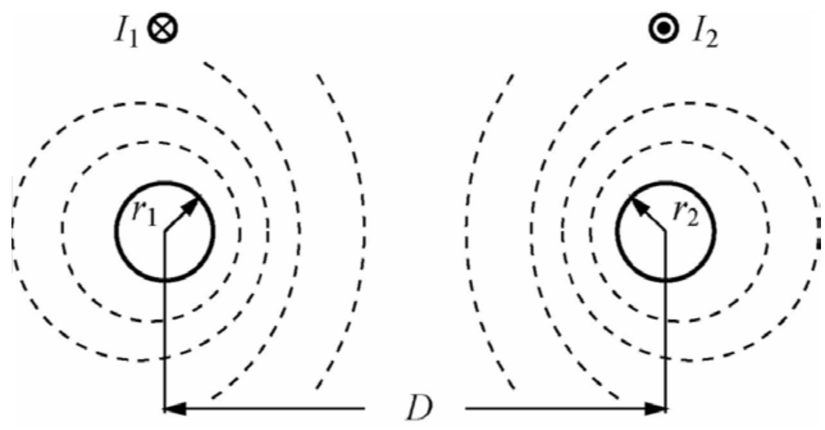
$$\lambda_{ext} = \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{1}{x} dx = 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \text{ Wb/m}$$

$$L_{ext} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m}$$

Inductance of Single-Phase Lines

- 1-meter length of a single-phase line with two solid round conductors

$I_1 = -I_2 \quad D_1 = r_1 \quad D_2 = D$ (why?)



$$L_1 = L_{1(\text{int})} + L_{1(ext)} = \frac{1}{2}\times 10^{-7} + 2\times 10^{-7}\ln\frac{D}{r_1} \text{ H/m}$$

$$r_1' \overset{\text{def}}{=} r_1 e^{-1/4}$$

$$L_1 = 2\times 10^{-7}\ln\frac{D}{r_1'} \quad \text{H/m}$$

$$L_2 = 2\times 10^{-7}\ln\frac{D}{r_2'} \quad \text{H/m}$$

- If two conductors are identical:

$r_1 = r_2 = r \qquad r' = r\,e^{-1/4}$

GMR (Geometric mean radius):

$D_S = r'$ for a single conductor

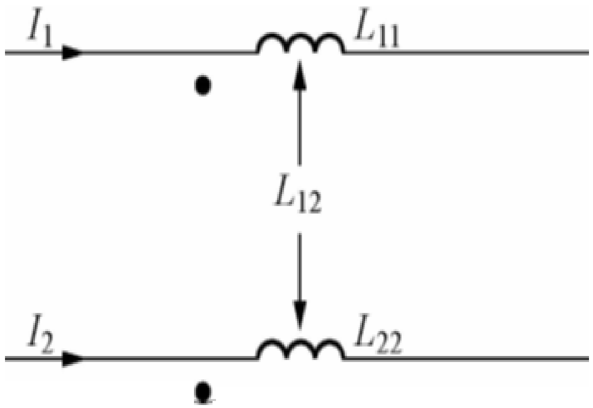
$$L = 2\times 10^{-7}\ln\frac{D}{r'} \text{ H/m}$$

$$L = 0.2\ln\frac{D}{D_s} \text{ mH/km}$$

Compared to $L_{ext} = 2\times 10^{-7}\ln\frac{D}{r}$ H/m

GMR is the radius of a fictitious conductor without internal flux but the same inductance as the actual conductor

Self- and Mutual Inductances



- Consider the flux linkages for 1-meter length of the single-phase circuit

$$I_1+I_2=0$$

$$\lambda_1 = L_{11}I_1 + L_{12}I_2 = (L_{11} - L_{12})I_1 = L_1 I_1$$

$$\lambda_2 = L_{21}I_1 + L_{22}I_2 = (-L_{21} + L_{22})I_2 = L_2 I_2$$

Compare to

$$L_1 = 2\times10^{-7}\ln\frac{D_{12}}{r_1'} = 2\times10^{-7}\ln\frac{1}{r_1'} - 2\times10^{-7}\ln\frac{1}{D_{12}} \quad \text{H/m}$$

$$L_2 = 2\times10^{-7}\ln\frac{D_{12}}{r_2'} = -2\times10^{-7}\ln\frac{1}{D_{12}} + 2\times10^{-7}\ln\frac{1}{r_2'} \quad \text{H/m}$$

$$L_{11} = 2\times10^{-7}\ln\frac{1}{r_1'}$$

$$L_{22} = 2\times10^{-7}\ln\frac{1}{r_2'}$$

$$L_{12} = L_{21} = 2\times10^{-7}\ln\frac{1}{D_{12}}$$

$$\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} = \begin{bmatrix}L_{11} & L_{12}\\ L_{21} & L_{22}\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix} = 2\times10^{-7}\begin{bmatrix}\ln\frac{1}{r_1'} & \ln\frac{1}{D_{12}}\\ \ln\frac{1}{D_{12}} & \ln\frac{1}{r_2'}\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}$$

$$\boldsymbol{\lambda} = \mathbf{LI}$$

- Consider *n* conductors

$$I_1+I_2=0$$

$$L_{11}=2\times10^{-7}\ln\frac{1}{r_1'}$$

$$L_{22}=2\times10^{-7}\ln\frac{1}{r_2'}$$

$$L_{12}=L_{21}=2\times10^{-7}\ln\frac{1}{D_{12}}$$

$$I_1+I_2+\cdots+I_i+\cdots+I_n=0$$

$$L_{ii}=2\times10^{-7}\ln\frac{1}{r_i'}$$

$$L_{ij}=L_{ji}=2\times10^{-7}\ln\frac{1}{D_{ij}}$$

$$\lambda_i=L_{ii}I_i+\sum_{\substack{j=1\\ j\neq i}}^{n}L_{ij}I_j=2\times10^{-7}\Big(I_i\ln\frac{1}{r_i'}+\sum_{\substack{j=1\\ j\neq i}}^{n}I_j\ln\frac{1}{D_{ij}}\Big)$$

$$\boldsymbol{\lambda}=\mathbf{LI}$$

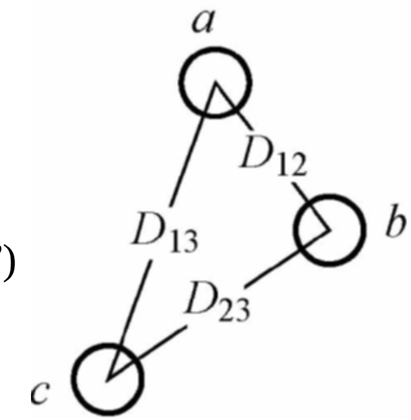
$$\mathbf{L}=2\times10^{-7}\begin{bmatrix}\ln\frac{1}{r_1'} & \ln\frac{1}{D_{12}} & \cdots & \ln\frac{1}{D_{1n}}\\ \ln\frac{1}{D_{12}} & \ln\frac{1}{r_2'} & & \ln\frac{1}{D_{2n}}\\ \vdots & & \ddots & \vdots\\ \ln\frac{1}{D_{1n}} & \ln\frac{1}{D_{2n}} & \cdots & \ln\frac{1}{r_n'}\end{bmatrix}$$

$$D_{ii}\overset{\Delta}{=}r_i'$$

$$=2\times10^{-7}\begin{bmatrix}\ln\frac{1}{D_{11}} & \ln\frac{1}{D_{12}} & \cdots & \ln\frac{1}{D_{1n}}\\ \ln\frac{1}{D_{12}} & \ln\frac{1}{D_{22}} & & \ln\frac{1}{D_{2n}}\\ \vdots & & \ddots & \vdots\\ \ln\frac{1}{D_{1n}} & \ln\frac{1}{D_{2n}} & \cdots & \ln\frac{1}{D_{nn}}\end{bmatrix}$$

Inductance of 3-Phase Transmission Lines (Asymmetric Spacing)

- Consider a three-phase line with 3 identical conductors



$$\boldsymbol{\lambda} = \mathbf{LI}$$

$$\mathbf{L} = 2\times10^{-7}\begin{bmatrix} \ln\frac{1}{r'} & \ln\frac{1}{D_{12}} & \ln\frac{1}{D_{13}} \\ \ln\frac{1}{D_{12}} & \ln\frac{1}{r'} & \ln\frac{1}{D_{23}} \\ \ln\frac{1}{D_{13}} & \ln\frac{1}{D_{23}} & \ln\frac{1}{r'} \end{bmatrix}$$

$(D_{11}=D_{22}=D_{33}=r')$

$$\lambda_a = 2\times10^{-7}(I_a\ln\frac{1}{r'} + I_b\ln\frac{1}{D_{12}} + I_c\ln\frac{1}{D_{13}})$$

$$\lambda_b = 2\times10^{-7}(I_a\ln\frac{1}{D_{12}} + I_b\ln\frac{1}{r'} + I_c\ln\frac{1}{D_{23}})$$

$$\lambda_c = 2\times10^{-7}(I_a\ln\frac{1}{D_{13}} + I_b\ln\frac{1}{D_{23}} + I_c\ln\frac{1}{r'})$$

$$L_a = \frac{\lambda_a}{I_a} = 2\times10^{-7}(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{12}} + a\ln\frac{1}{D_{13}})$$

$$L_b = \frac{\lambda_b}{I_b} = 2\times10^{-7}(a\ln\frac{1}{D_{12}} + \ln\frac{1}{r'} + a^2\ln\frac{1}{D_{23}})$$

$$L_c = \frac{\lambda_c}{I_c} = 2\times10^{-7}(a^2\ln\frac{1}{D_{13}} + a\ln\frac{1}{D_{23}} + \ln\frac{1}{r'})$$

For balanced three-phase current:

Note: L_a, L_b and L_c may have imaginary terms

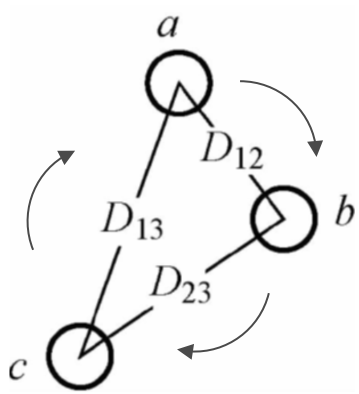
$$I_b = I_a\angle-120° = I_a\angle240° = a^2 I_a$$

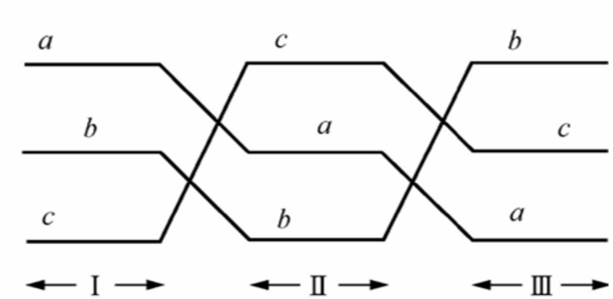
$$I_c = I_a\angle-240° = I_a\angle120° = aI_a$$

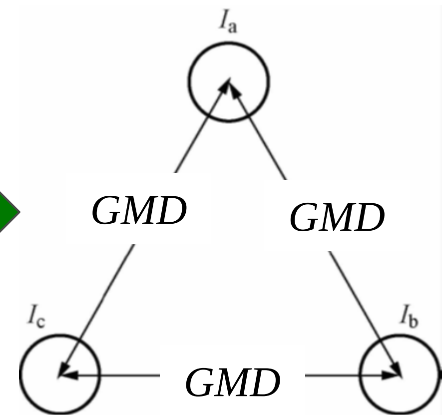
$$a = 1\angle120°$$

$$a + a^2 = 1\angle120° + 1\angle240° = -1$$

Transpose Line: Mitigation of Asymmetry







GMD: geometric mean distance

$$L = \frac{L_a + L_b + L_c}{3}$$

$$= \frac{2\times 10^{-7}}{3}\left(3\ln\frac{1}{r'} - \ln\frac{1}{D_{12}} - \ln\frac{1}{D_{23}} - \ln\frac{1}{D_{13}}\right) = 2\times 10^{-7}\ln\frac{\sqrt[3]{D_{12}D_{23}D_{13}}}{r'}$$

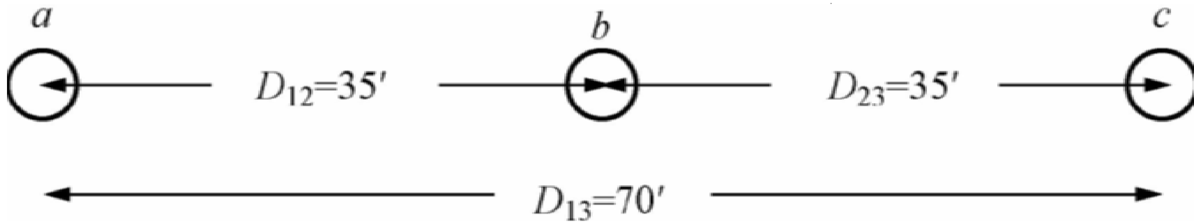
$\sqrt[3]{D_{12}D_{23}D_{13}}$ **=GMD**

$$L = 0.2\ln\frac{GMD}{D_s}\ \text{mH/km}$$

For symmetric spacing:

$D_{12}=D_{23}=D_{13}=D$

$$L = 0.2\ln\frac{D}{D_s}\ \text{mH/km}$$



A three-phase line has three conductors with *r*=1.345 in. Determine the inductance per phase. What if transposition is adopted?

$D_s = r' = 1.345e^{-1/4} = 1.0475\text{in} = 0.0266\text{m}$ $\quad D_{12} = D_{23} = 0.889\text{m} \quad D_{13} = 1.778\text{m}$

$$L_a = 0.2\times(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{12}} + a\ln\frac{1}{D_{13}}) = 0.7711 - j0.1201 = 0.7804\angle\text{-}8.85° \text{ mH/km}$$

$$L_b = 0.2\times(a\ln\frac{1}{D_{12}} + \ln\frac{1}{r'} + a^2\ln\frac{1}{D_{23}}) = 0.7018 \text{ mH/km}$$

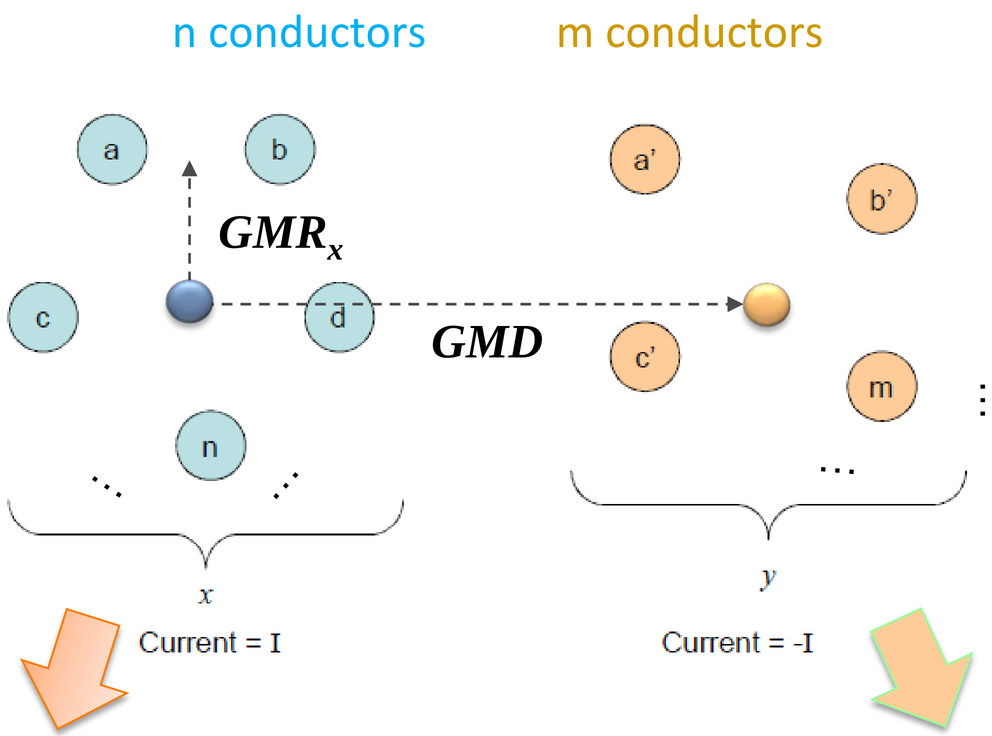
$$L_c = 0.2\times(a^2\ln\frac{1}{D_{13}} + a\ln\frac{1}{D_{23}} + \ln\frac{1}{r'}) = 0.7711 + j0.1201 = 0.7804\angle 8.85° \text{ mH/km}$$

With transposition:

$$L = \frac{L_a + L_b + L_c}{3} = 0.2\ln\frac{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}{D_s} = 0.7480 \text{ mH/km}$$

Inductance of Bundled Conductors

- Single-phase with two bundled conductors



$$L_x = 2\times10^{-7}\ln\frac{GMD}{GMR_x}\ \text{H/m}$$

$$GMD = \sqrt[mn]{\prod_{i\in x,\, j\in y} D_{ij}}$$

$$GMR_x = \sqrt[n^2]{\prod_{i\in x}\prod_{j\in x} D_{ij}} = \sqrt[n^2]{\prod_{i\in x} D_{ii} \prod_{i,j\in x, i\neq j} D_{ij}^2}$$

$$\lambda_a = 2\times10^{-7}\frac{I}{n}\left(\ln\frac{1}{r_x'} + \ln\frac{1}{D_{ab}} + \ln\frac{1}{D_{ac}} + \cdots + \ln\frac{1}{D_{an}}\right) + 2\times10^{-7}\frac{-I}{m}\left(\ln\frac{1}{D_{aa'}} + \ln\frac{1}{D_{ab'}} + \ln\frac{1}{D_{ac'}} + \cdots + \ln\frac{1}{D_{am}}\right)$$

$$\lambda_a = 2\times10^{-7} I \ln \frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}\cdots D_{am}}}{\sqrt[n]{r_x' D_{ab}D_{ac}\cdots D_{an}}}$$

$$L_a = \frac{\lambda_a}{I_a} = \frac{\lambda_a}{I/n} = 2n\times10^{-7} \ln \frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}\cdots D_{am}}}{\sqrt[n]{r_x' D_{ab}D_{ac}\cdots D_{an}}} \quad \cdots \quad L_n = 2n\times10^{-7} \ln \frac{\sqrt[m]{D_{na'}D_{nb'}D_{nc'}\cdots D_{nm}}}{\sqrt[n]{r'_x D_{nb}D_{nc}\cdots D_{nc}}}$$

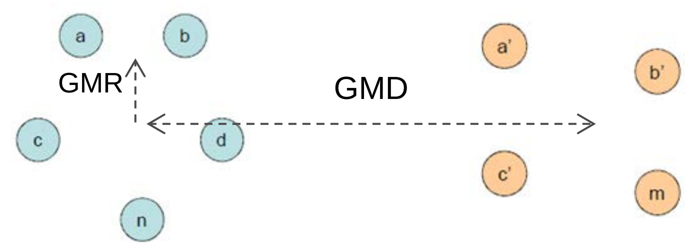
Average inductance of each conductor: $L_{av} = \dfrac{L_a + L_b + L_c + \cdots + L_n}{n}$

- Inductance of bundle x:

Since all conductors are connected in parallel,

$$L_x = \frac{L_{av}}{n} = \frac{L_a + L_b + L_c + \cdots + L_n}{n^2}$$

$$\boxed{L_x = 2\times10^{-7} \ln \frac{GMD}{GMR_x} \text{ H/m}}$$



for m=1, n=1:

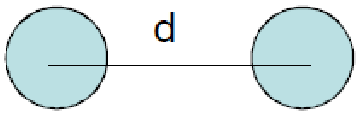
$GMR=r'$, $GMD=D$

$$GMD = \sqrt[mn]{(D_{aa'}D_{ab'}D_{ac'}\cdots D_{am})\cdots(D_{na'}D_{nb'}\cdots D_{nm})}$$

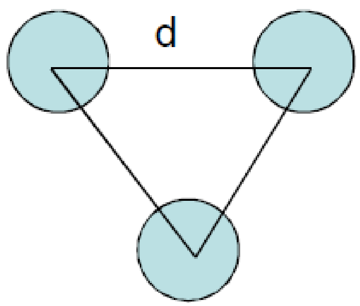
$$GMR_x = \sqrt[n^2]{(D_{aa}D_{ab}\cdots D_{an})\cdots(D_{na}D_{nb}\cdots D_{nn})} \qquad D_{aa} = D_{bb} = D_{nn} \overset{\Delta}{=} r_x' = D_{s,x}$$

GMR of Typical Bundled Conductors

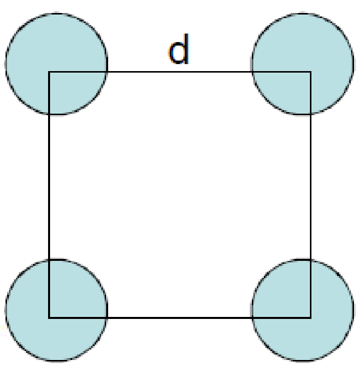
$$GMR_x = \sqrt[n^2]{\prod_{i\in x}\prod_{j\in x} D_{ij}} = \sqrt[n^2]{(D_{aa}D_{ab}\cdots D_{an})\cdots(D_{na}D_{nb}\cdots D_{nn})}$$



$$GMR = \sqrt[4]{(D_s\times d)^2} = \sqrt{D_s\times d}$$



$$GMR = \sqrt[9]{(D_s\times d\times d)^3} = \sqrt[3]{D_s\times d^2}$$

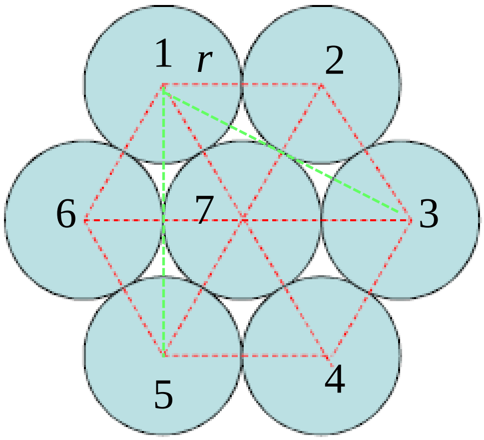


$$GMR = \sqrt[16]{(D_s\times d\times d\times d\times 2^{\frac{1}{2}})^4} = 1.09\sqrt[4]{D_s\times d^3}$$

Inductance of Stranded Conductors

- Example 4.1: Determine GMR

  A special case of the bundled conductors:



$$D_{12} = D_{16} = D_{17} = 2r$$

$$D_{14} = 4r$$

$$D_{13} = D_{15} = \sqrt{D_{14}^2 - D_{45}^2} = 2\sqrt{3}r$$

$$GMR = \sqrt[49]{(r' \cdot 2r \cdot 2\sqrt{3}r \cdot 4r \cdot 2\sqrt{3}r \cdot 2r \cdot 2r)^6}$$

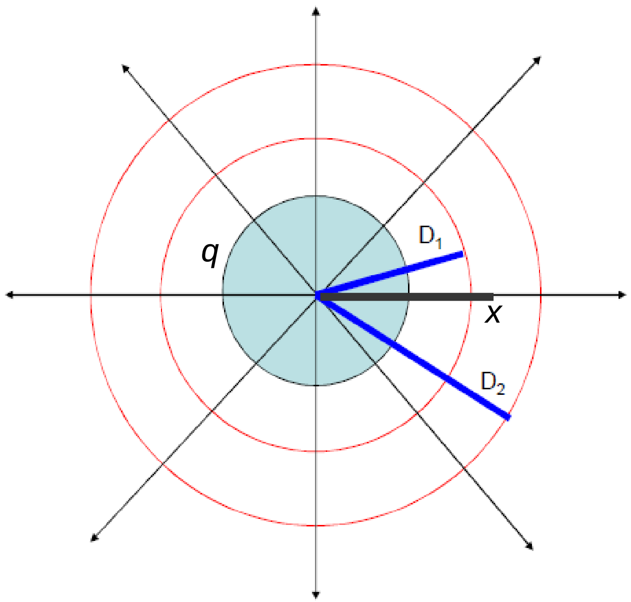
$$= r\sqrt[7]{(e)^{-\frac{1}{4}}(2)^6(3)^6(3)^{\frac{6}{7}}(2)^{\frac{6}{7}}}$$

$$= 2.1767r$$

Line Capacitance

- Consider a long round conductor carrying a charge of q (c/m)
  - Capacitance:

$$C = \frac{q}{V}$$



  - Electric flux density at a cylinder of radius x:

$$D = \frac{q}{A} = \frac{q}{2\pi x \times 1}$$

  - Electric field intensity

$$E = \frac{D}{\varepsilon_0} = \frac{q}{2\pi \varepsilon_0 x} \qquad \varepsilon_0 = 8.85 \times 10^{-12}\ F/m$$

Permittivity of free space

  - Electric potential difference between cylinders at D_1 and D_2

Defined as the work done in moving a unit charge from D_1 to D_2

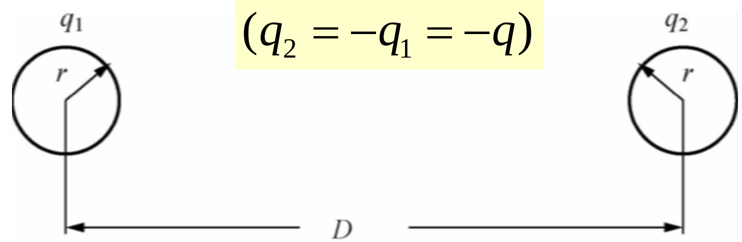
$$V_{12} = \int_{D_1}^{D_2} E dx = \int_{D_1}^{D_2} \frac{q}{2\pi \varepsilon_0 x} dx = \frac{q}{2\pi \varepsilon_0} \ln \frac{D_2}{D_1}$$

(Voltage drop from D_1 to D_2)

Capacitance of Single Phase Lines

- Consider 1-meter length of a single-phase line

$$V_{12} = \frac{q}{2\pi\varepsilon_0}\ln\frac{D_2}{D_1}$$



$$(q_2 = -q_1 = -q)$$

– Conductor 1 carries a charge of q_1(c/m)

$$V_{12(q_1)} = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{D}{r}$$

– Conductor 2 carries a charge of q_2(c/m)

$$V_{21(q_2)} = \frac{q_2}{2\pi\varepsilon_0}\ln\frac{D}{r} \Rightarrow V_{12(q_2)} = -\frac{q_2}{2\pi\varepsilon_0}\ln\frac{D}{r}$$

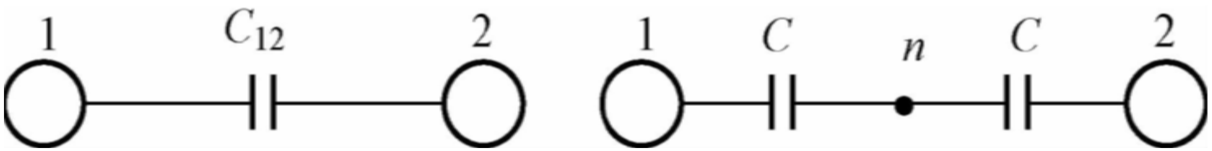
$$V_{12} = V_{12(q_1)} + V_{12(q_2)} = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{D}{r} + \frac{-q_2}{2\pi\varepsilon_0}\ln\frac{D}{r} = \frac{q}{\pi\varepsilon_0}\ln\frac{D}{r}$$

– Line-to-line capacitance between the conductors

$$C_{12} = \frac{\pi\varepsilon_0}{\ln\frac{D}{r}} \text{ F/m}$$

Compare to the inductance per conductor: $L = 2\times10^{-7}\ln\frac{D}{r'}$ H/m

- Define C, the capacitance per conductor, as the capacitance between each conductor and a neutral



$$C = \frac{q}{V_{12}/2} = 2C_{12} = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}} \text{ F/m} \qquad C = \frac{0.0556}{\ln\frac{D}{r}} \ \mu\text{F/km}$$

Compared to $L = 0.2\ln\frac{D}{r'} \quad = 0.05 + 0.2\ln\frac{D}{r} \text{ mH/km}$

- For an all aluminum conductor *r*=1.345 in (0.0342m) and D=35 in (0.889m)
  $\rho_{Al}(20^{o}\text{C}) = 2.82\times10^{-8}\ \Omega\cdot\text{m}$

  $C = 0.0171\ \mu\text{F/km} \ \rightarrow\ 1/(\omega C) = 0.155\text{M}\Omega/\text{km}$

  $L = 0.7018\ \text{mH/km} \ \rightarrow\ \omega L \ = 0.2646\Omega/\text{km}\ \sim 34.4\times R$

  $R = \rho_{Al}(20^{o}\text{C})\times1000/(\pi r^2) \quad = 0.00769\Omega/\text{km}$

Multi-Conductors

- Consider n parallel long conductors with charges of q_k c/m

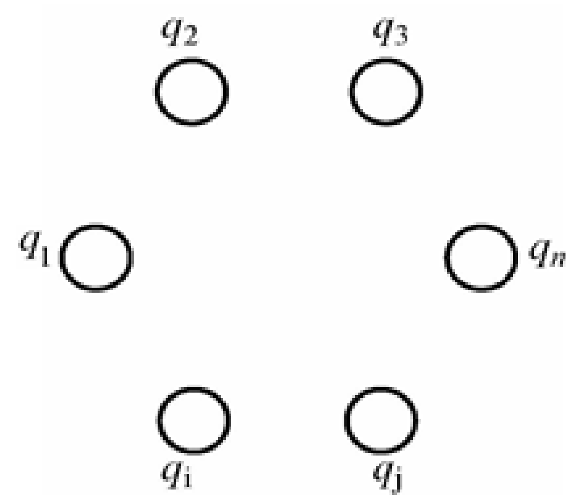
Since for one single conductor with q_k:

$$V_{ij} = \frac{q_k}{2\pi\varepsilon_0} \ln \frac{D_{kj}}{D_{ki}}$$

$D_{ii}=D_{jj}=r$

for *n* conductors:

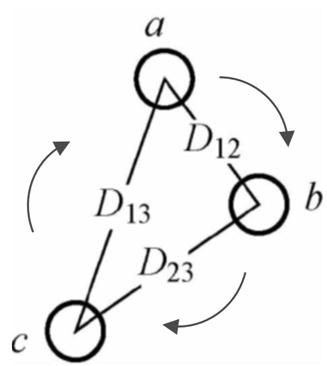
$$V_{ij} = \frac{1}{2\pi\varepsilon_0} \sum_{k=1}^{n} q_k \ln \frac{D_{kj}}{D_{ki}}$$

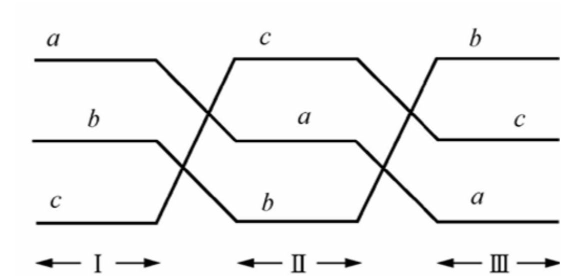


$$q_1 + q_2 + \cdots + q_n = 0$$

Capacitance of Three-Phase Lines

$$V_{ij} = \frac{1}{2\pi\varepsilon_0}\sum_{k=1}^{n} q_k \ln\frac{D_{kj}}{D_{ki}}$$





$$q_a + q_b + q_c = 0$$

$$V_{ab(I)} = \frac{1}{2\pi\varepsilon_0}(q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{13}})$$

$$V_{ab(II)} = \frac{1}{2\pi\varepsilon_0}(q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{13}}{D_{12}})$$

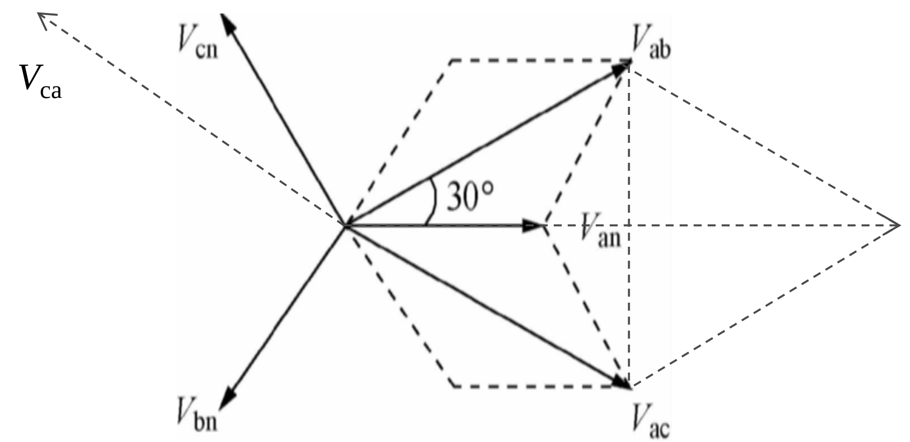
$$V_{ab(III)} = \frac{1}{2\pi\varepsilon_0}(q_a \ln\frac{D_{13}}{r} + q_b \ln\frac{r}{D_{13}} + q_c \ln\frac{D_{12}}{D_{23}})$$

$=0$

$$V_{ab} = \frac{1}{3\times 2\pi\varepsilon_0}(q_a \ln\frac{D_{12}D_{23}D_{13}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{13}} + q_c \ln\frac{D_{12}D_{23}D_{13}}{D_{12}D_{23}D_{13}})$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}(q_a \ln\frac{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}{r} + q_b \ln\frac{r}{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}) = \frac{1}{2\pi\varepsilon_0}(q_a \ln\frac{GMD}{r} + q_b \ln\frac{r}{GMD})$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}(q_a \ln\frac{GMD}{r} + q_b \ln\frac{r}{GMD})$$



How to calculate the capacitance per phase?

$$V_{ac} = \frac{1}{2\pi\varepsilon_0}(q_a \ln\frac{GMD}{r} + q_c \ln\frac{r}{GMD})$$

$$\begin{aligned} V_{ab} + V_{ac} &= 3V_{an} \\ &= \frac{1}{2\pi\varepsilon}[2q_a \ln\frac{GMD}{r} + (q_b + q_c)\ln\frac{r}{GMD}] \\ &= \frac{1}{2\pi\varepsilon}(2q_a \ln\frac{GMD}{r} - q_a \ln\frac{r}{GMD}) \\ &= \frac{3q_a}{2\pi\varepsilon_0}\ln\frac{GMD}{r} \end{aligned}$$

$$q_b + q_c = -q_a$$

$$C = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon_0}{\ln\frac{GMD}{r}} \text{ F/m} \quad = \frac{0.0556}{\ln\frac{GMD}{r}} \ \mu\text{F/km}$$

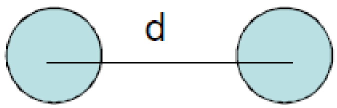
Compared to

$$C = \frac{0.0556}{\ln\frac{D}{r}} \ \mu\text{F/km}$$ (for single-phase line)

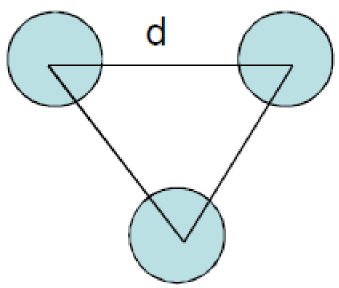
$$L = 0.2\ln\frac{GMD}{r'} \text{ mH/km}$$ (for 3-phase line)

Effect of Bundling

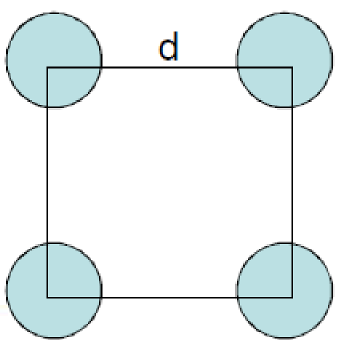
$$L = 0.2 \ln \frac{GMD}{D_s^b} \text{ mH/km} \qquad C = \frac{0.0556}{\ln \frac{GMD}{r^b}} \ \mu\text{F/km}$$



$$D_s^b = \sqrt{D_s \times d} \qquad r^b = \sqrt{r \times d}$$



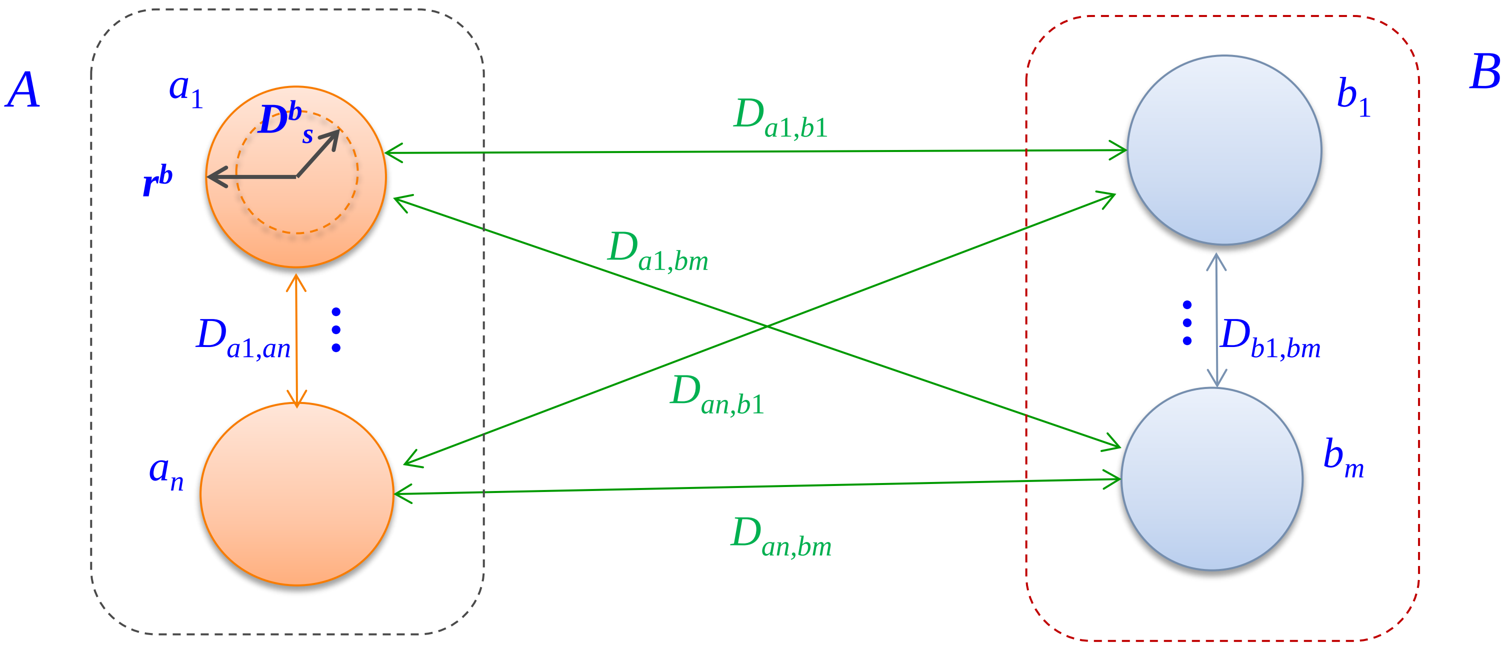
$$D_s^b = \sqrt[3]{D_s \times d^2} \qquad r^b = \sqrt[3]{r \times d^2}$$



$$D_s^b = 1.09\sqrt[4]{D_s \times d^3} \qquad r^b = 1.09\sqrt[4]{r \times d^3}$$

GMR_L $\qquad$ GMR_C

A Summary: GMD, GMR_L and $GMR_C \rightarrow$ L and C



- $r^b = r$ and $D^b_s = D_s$ for single conductors

$$GMD = \sqrt[nm]{D_{a_1b_1} D_{a_1b_2} \ldots D_{a_nb_{m-1}} D_{a_nb_m}}$$

$$GMR_{L,A} = \sqrt[n^2]{(D_s^b D_{a_1a_2} \ldots D_{a_1a_n}) \ldots (D_{a_na_1} \ldots D_{a_na_{n-1}} D_s^b)}$$

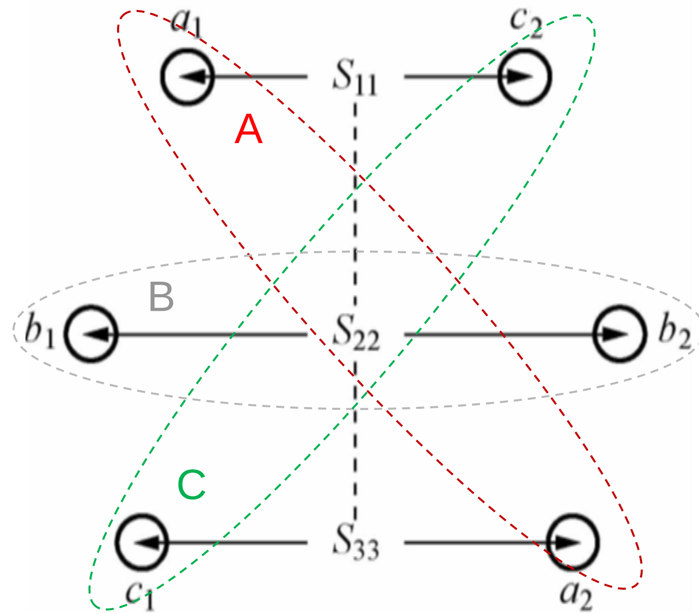
$$GMR_{C,A} = \sqrt[n^2]{(r^b D_{a_1a_2} \ldots D_{a_1a_n}) \ldots (D_{a_na_1} \ldots D_{a_na_{n-1}} r^b)}$$

$$L_A = 0.2 \ln \frac{GMD}{GMR_{L,A}} \text{ mH/km}$$

$$C_A = \frac{0.0556}{\ln \dfrac{GMD}{GMR_{C,A}}} \ \mu\text{F/km}$$

L↓ if GMR_L↑. C↑ if GMR_C↑

Three-Phase Double-Circuit Lines



- GMD between 3 phase groups

$$D_{AB} = \sqrt[4]{D_{a_1b_1}D_{a_1b_2}D_{a_2b_1}D_{a_2b_2}}$$

$$D_{BC} = \sqrt[4]{D_{b_1c_1}D_{b_1c_2}D_{b_2c_1}D_{b_2c_2}}$$

$$D_{AC} = \sqrt[4]{D_{a_1c_1}D_{a_1c_2}D_{a_2c_1}D_{a_2c_2}}$$

- GMD per phase (consider transposition)

$$GMD = \sqrt[3]{D_{AB}D_{BC}D_{AC}}$$

- GMR_L of each phase group

$$D_{SA} = \sqrt[4]{(D_s^b D_{a_1a_2})^2} = \sqrt{D_s^b D_{a_1a_2}}$$

$$D_{SB} = \sqrt[4]{(D_s^b D_{b_1b_2})^2} = \sqrt{D_s^b D_{b_1b_2}}$$

$$D_{SC} = \sqrt[4]{(D_s^b D_{c_1c_2})^2} = \sqrt{D_s^b D_{c_1c_2}}$$

- Equivalent GMR_L

$$GMR_L = \sqrt[3]{D_{SA}D_{SB}D_{SC}}$$

- GMR_C of each phase group

$$r_A = \sqrt{r^b D_{a_1a_2}} \qquad r_C = \sqrt{r^b D_{c_1c_2}}$$

$$r_B = \sqrt{r^b D_{b_1b_2}}$$

- Equivalent GMR_C

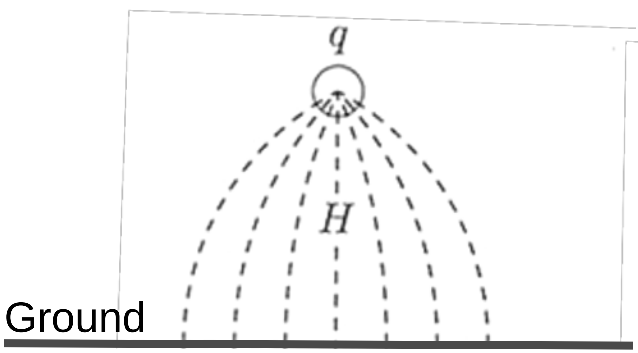
$$GMR_C = \sqrt[3]{r_A r_B r_C}$$

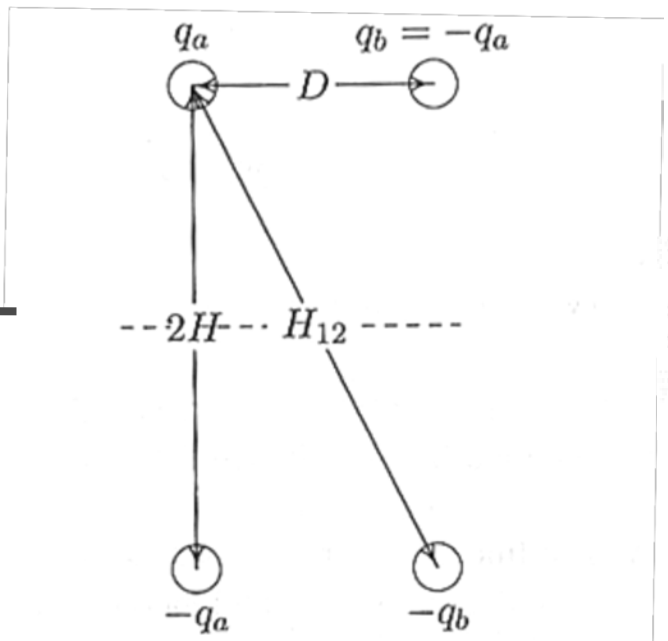
- Inductance and Capacitance:

$$C = \frac{0.0556}{\ln \frac{GMD}{GMR_C}} \ \mu\text{F/km} \qquad L = 0.2 \ln \frac{GMD}{GMR_L} \ \text{mH/km}$$

Effect of Earth on the Capacitance

- The presence of earth alters the distribution of electric flux lines and equipotential surfaces
  - The earth level is an equipotential surface → *Image Charges* Method
  - The effect of the earth is to increase the capacitance
- Negligible for balanced steady-state analysis if conductors are high





(b) Single-phase line and its image

$$C_{an} = C_{bn} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\frac{2H}{H_{12}}\right)}$$

Problem 4.15

Example 4.2

A 500-kV three-phase transposed line is composed of one $ACSR$ 1,272,000–cmil, 45/7 Bittern conductor per phase with horizontal conductor configuration as shown in Figure 4.19. The conductors have a diameter of 1.345 in and a GMR of 0.5328 in. Find the inductance and capacitance per phase per kilometer of the line.

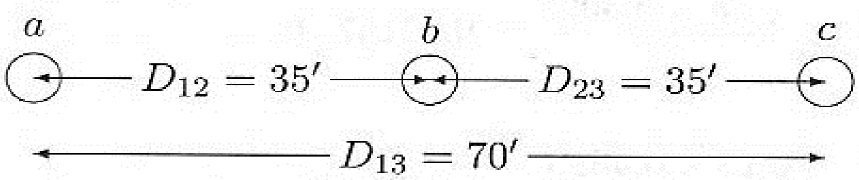


FIGURE 4.19
Conductor layout for Example 4.2.

1 mil = 0.001 inch = 0.0254 mm

1 cmil (circular mil, i.e. the area of a circle with a diameter of 1 mil)

= $\pi/4$ x mil^2 =5.067 x 10^{-10} m^2 = 5.067 x 10^{-4} mm^2

Conductor radius is $r = \frac{1.345}{2\times 12} = 0.056$ ft, and $GMR_L = 0.5328/12 = 0.0444$ ft. GMD is obtained using (4.42)

$$GMD = \sqrt[3]{35 \times 35 \times 70} = 44.097 \quad \text{ft}$$

From (4.58) the inductance per phase is

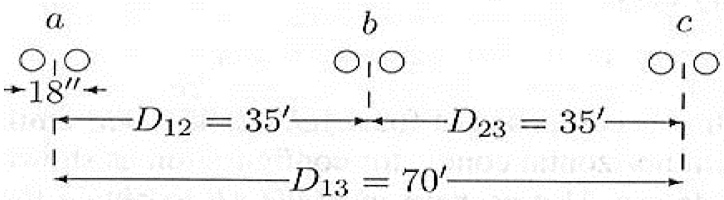
$$L = 0.2 \ln \frac{44.097}{0.0444} = 1.38 \quad \text{mH/km}$$

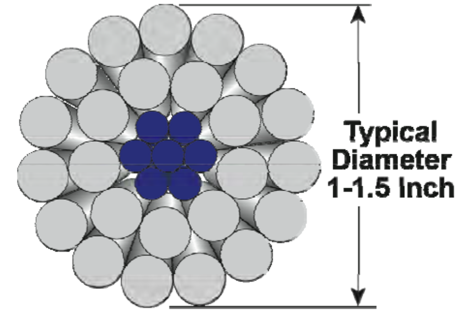
and from (4.92) the capacitance per phase is

$$C = \frac{0.0556}{\ln \frac{44.097}{0.056}} = 0.0083 \quad \mu\text{F/km}$$

Example 4.3

The line in Example 4.2 is replaced by two *ACSR* 636,000-cmil, 24/7 Rook conductors which have the same total cross-sectional area of aluminum as one Bittern conductor. The line spacing as measured from the center of the bundle is the same as before and is shown in Figure 4.20.





ACSR (7 steel and 24 aluminum strands)

The conductors have a diameter of 0.977 in and a *GMR* of 0.3924 in. Bundle spacing is 18 in. Find the inductance and capacitance per phase per kilometer of the line and compare it with that of Example 4.2.

Conductor radius is $r = \frac{0.977}{2} = 0.4885$ in, and from Example 4.2 $GMD =$ 44.097 ft. The equivalent geometric mean radius with two conductors per bundle, for calculating inductance and capacitance, are given by (4.51) and (4.88)

$$GMR_L = \frac{\sqrt{d \times D_s}}{12} = \frac{\sqrt{18 \times 0.3924}}{12} = 0.22147 \quad \text{ft}$$

and

$$GMR_c = \frac{\sqrt{d \times r}}{12} = \frac{\sqrt{18 \times 0.4885}}{12} = 0.2471 \quad \text{ft}$$

From (4.58) the inductance per phase is

$$L = 0.2 \ln \frac{44.097}{0.22147} = 1.0588 \quad \text{mH/km}$$

and from (4.92) the capacitance per phase is

$$C = \frac{0.0556}{\ln \frac{44.097}{0.2471}} = 0.0107 \quad \mu\text{F/km}$$

Comparing with the results of Example 4.2, there is a 23.3 percent reduction in the inductance and a 28.9 percent increase in the capacitance.