ECE 421/599
Electric Energy Systems
6 – Power Flow Analysis

Instructor:

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Fall 2014

Introduction

- Power flow (load flow) analysis
  - Steady-state analysis of an interconnected power system
  - To solve the power flow equations for V_i (i.e. $|V_i|$ and δ_i) and S_i (i.e. P_i and Q_i)
  - Basis of power system analysis and design

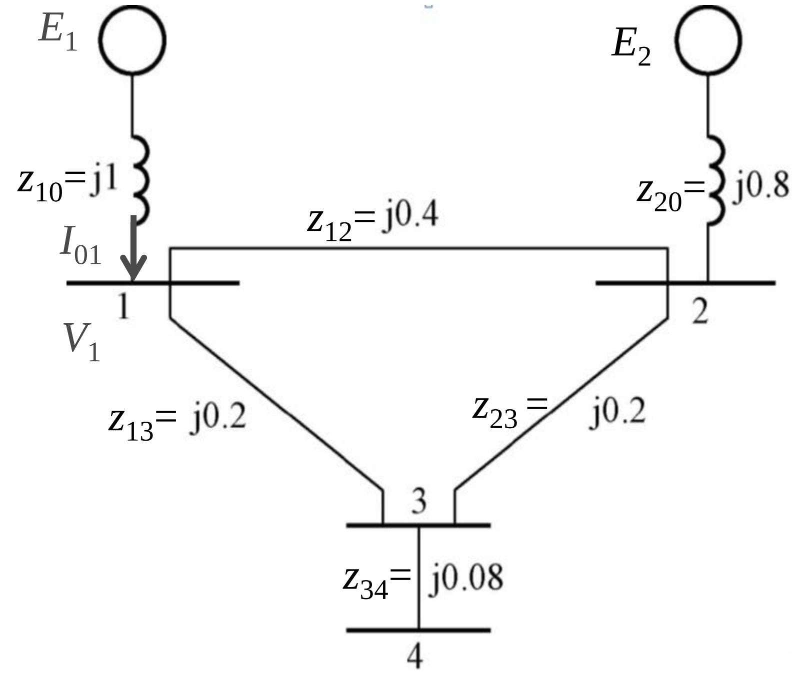
- Assumptions for a power system to be studied
  - Balanced conditions
  - Represented by a single-phase network
  - Modeled by nodes (buses) and branches
  - All impedances/admittances are in per unit on a common MVA base (typically 100MVA)

Bus Admittance Matrix

- Branch admittance $$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$

- Apply KCL to nodes (buses)1-4

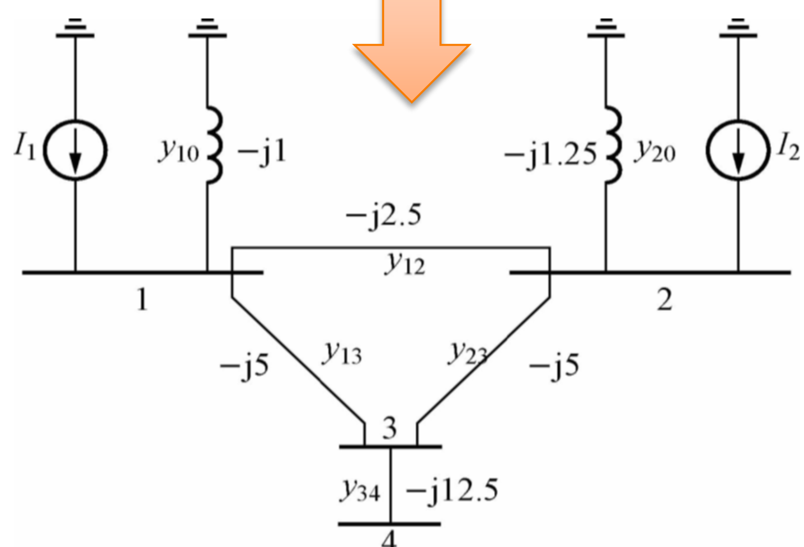
$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$
$$I_2 = y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3)$$
$$0 = y_{23}(V_3 - V_2) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4)$$
$$0 = y_{34}(V_4 - V_3)$$



I_i : current injected by an equivalent current source

$$I_{01} = (E_1 - V_1)y_{10} = E_1 y_{10} + (0 - V_1)y_{10} \overset{\Delta}{=} I_1 + (0 - V_1)y_{10}$$

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 \quad - y_{12}V_2 - y_{13}V_3$$
$$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3$$
$$0 = -y_{13}V_1 - y_{23}V_2 \quad + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4$$
$$0 = \qquad - y_{34}V_3 + y_{34}V_4$$



$$\boxed{I_i = V_i \sum_{j=0,\, j\neq i}^{n} y_{ij} - \sum_{j=1,\, j\neq i}^{n} y_{ij} V_j}$$

$Y_{11} = y_{10} + y_{12} + y_{13}$ $\qquad Y_{12} = Y_{21} = -y_{12}$ $\qquad Y_{14} = Y_{41} = 0$

$Y_{22} = y_{20} + y_{12} + y_{23}$ $\qquad Y_{13} = Y_{31} = -y_{13}$ $\qquad Y_{42} = Y_{24} = 0$

$Y_{33} = y_{13} + y_{23} + y_{34}$ $\qquad Y_{23} = Y_{32} = -y_{23}$

$Y_{44} = y_{34}$ $\qquad Y_{34} = Y_{43} = -y_{34}$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$
$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$
$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$

$$\begin{bmatrix} I_1 \\ I_2 \\ : \\ I_i \\ : \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12}\cdots & Y_{1i}\cdots & Y_{1n} \\ Y_{21} & Y_{22}\cdots & Y_{2i}\cdots & Y_{2n} \\ : & : & : & : \\ Y_{i1} & Y_{i2}\cdots & Y_{ii}\cdots & Y_{in} \\ : & : & : & : \\ Y_{n1} & Y_{n2}\cdots & Y_{in}\cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ : \\ V_i \\ : \\ V_n \end{bmatrix}$$

$$\boxed{I_i = V_i \sum_{j=0,\, j\neq i}^{n} y_{ij} - \sum_{j=1,\, j\neq i}^{n} y_{ij}V_j = \sum_{j=1}^{n} Y_{ij}V_j}$$

$$\boxed{\mathbf{I}_{\mathbf{bus}} = \mathbf{Y}_{\mathbf{bus}}\mathbf{V}_{\mathbf{bus}}}$$

- $\mathbf{I}_{\mathbf{bus}}$: currents injected by external current sources
- $\mathbf{V}_{\mathbf{bus}}$: bus voltages relative to a reference (not included), usually the ground
- $\mathbf{Y}_{\mathbf{bus}}$: bus admittance matrix (symmetric and sparse)
  - Diagonal elements: called self-admittances or driving point admittances

$$Y_{ii} = \sum_{j=0,\, j\neq i}^{n} y_{ij}$$

  - Off-diagonal elements: called mutual admittances or transfer admittances

$$Y_{ij} = Y_{ji} = -y_{ij} \qquad j \neq i$$

- $\mathbf{Z}_{\mathbf{bus}}=\mathbf{Y}^{-1}_{\mathbf{bus}}$: bus impedance matrix

$$\mathbf{V}_{bus} = \mathbf{Y}_{bus}^{-1}\mathbf{I}_{bus} = \mathbf{Z}_{bus}\mathbf{I}_{bus}$$

$$\mathbf{Y}_{\mathbf{bus}} = \begin{bmatrix} -j8.50 & j2.50 & j5.00 & 0 \\ j2.50 & -j8.75 & j5.00 & 0 \\ j5.00 & j5.00 & -j22.50 & j12.50 \\ 0 & 0 & j12.50 & -j12.50 \end{bmatrix}$$

How many non-zero elements for a system with N buses (not including the ground) and M branches? 2M+N

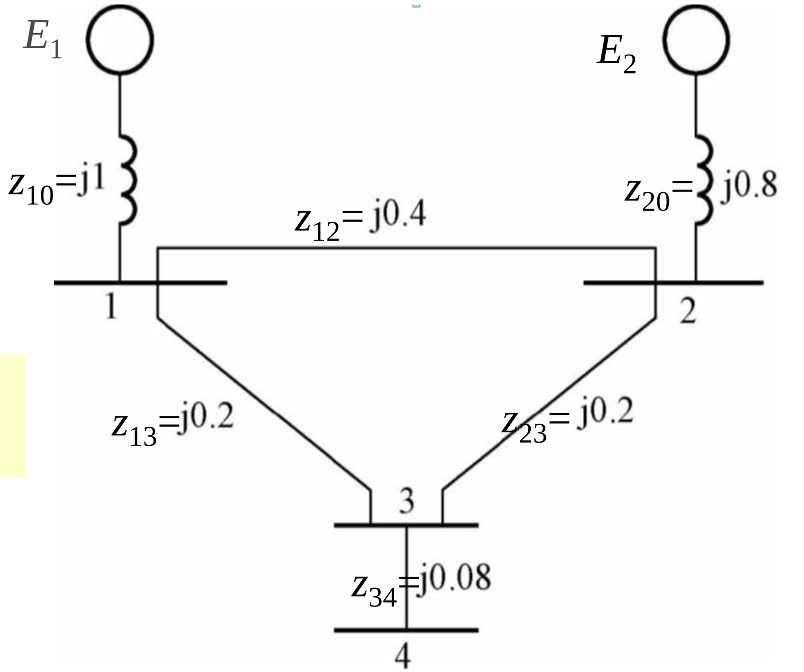
$$\mathbf{Z}_{\mathbf{bus}} = \mathbf{Y}_{\mathbf{bus}}^{-1} = \begin{bmatrix} j0.50 & j0.40 & j0.45 & j0.45 \\ j0.40 & j0.48 & j0.44 & j0.44 \\ j0.45 & j0.44 & j0.545 & j0.545 \\ j0.45 & j0.44 & j0.545 & j0.625 \end{bmatrix}$$

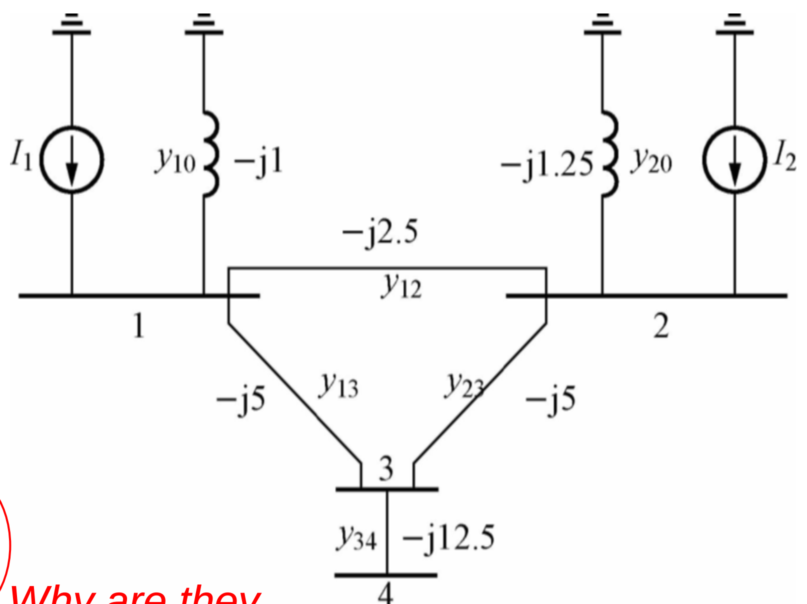
- If it is known that $E_1=1.1\angle 0^{o}$ pu and $E_2=1.0\angle 0^{o}$ pu

$$I_1 = \frac{E_1}{z_{10}} = \frac{1.1}{j1.0} = -j1.1 \text{ pu}$$

$$I_2 = \frac{E_2}{z_{20}} = \frac{1.0}{j0.8} = -j1.25 \text{ pu}$$

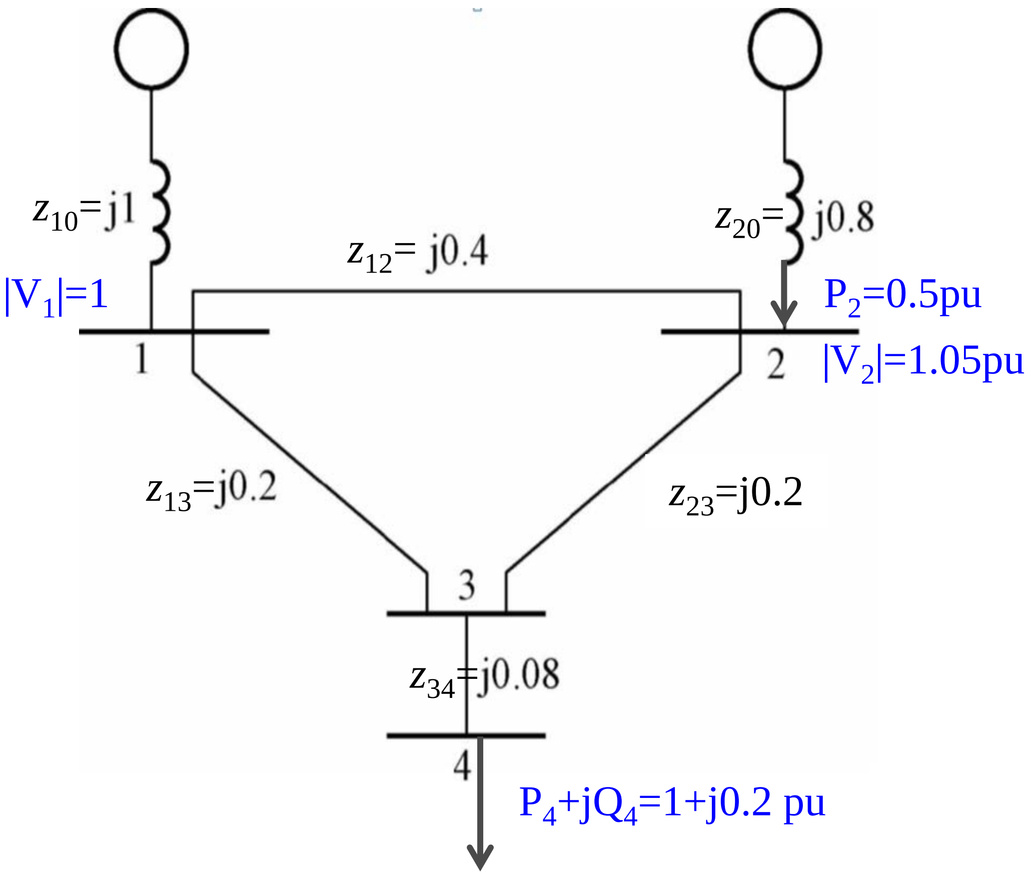
$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \mathbf{Z}_{\mathbf{bus}} \begin{bmatrix} -j1.1 \\ -j1.25 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.050 \\ 1.040 \\ 1.045 \\ 1.045 \end{bmatrix}$$





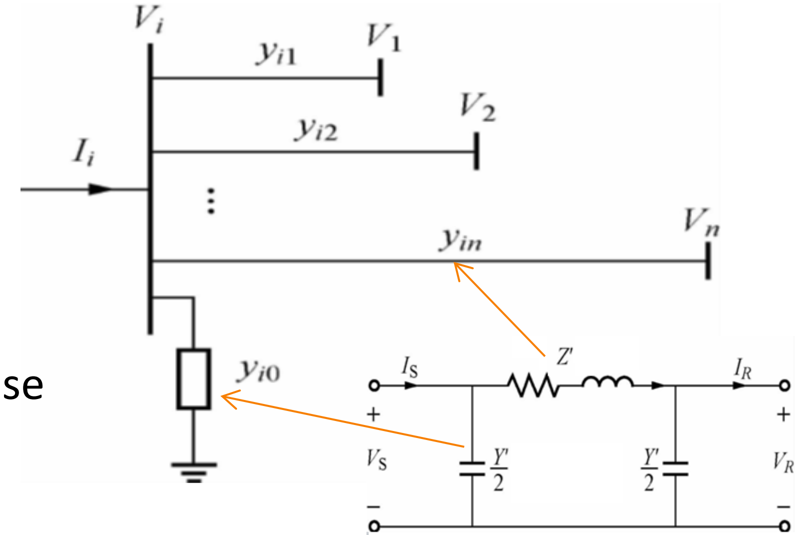
Why are they same?

A more general power flow study



- How to solve all bus voltages and line real and reactive power flows?

Power Flow Equation



- Consider a typical bus of an *n*-bus system
  - All lines represented by equivalent π models
  - Admittances are in pu on a common MVA base

- Apply KCL

$$I_i = y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \ldots + y_{ij}(V_i - V_j) + \ldots + y_{in}(V_i - V_n)$$

$$= (y_{i0} + y_{i1} + y_{i2} + \ldots y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \ldots - y_{ij}V_j - \ldots - y_{in}V_n \qquad j \neq i$$

$$\boxed{I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i}$$

$$\Rightarrow$$

$$\boxed{\frac{P_i - \mathrm{j}Q_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i}$$

$$S_i = P_i + \mathrm{j}Q_i = V_i I_i^* \qquad I_i = \frac{P_i - \mathrm{j}Q_i}{V_i^*}$$

$$\frac{P_i - \mathrm{j}Q_i}{|V_i|\angle -\delta_i} = |V_i|\angle\delta_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}|V_j|\angle\delta_j \qquad j \neq i$$

- Solve $|V_i|$, δ_i, P_i, Q_i and then calculate P_{ij}, Q_{ij}
  - *n* complex nonlinear algebraic equations (2×*n* real equations) with 4×*n* real quantities
  - can be solved by iterative techniques

Power Flow Solution

- Determining:
  - $|V_i|$ and δ_i (magnitude and phase angle of each bus voltage)
  - P_{ij} and Q_{ij} (real and reactive power flows in each line)
- The system is assumed to be operating under balanced conditions and a single-phase model is used
- 4 quantities, i.e. $|V_i|$, δ_i, P_i and Q_i, are associated with each bus
- System buses are usually classified into three types

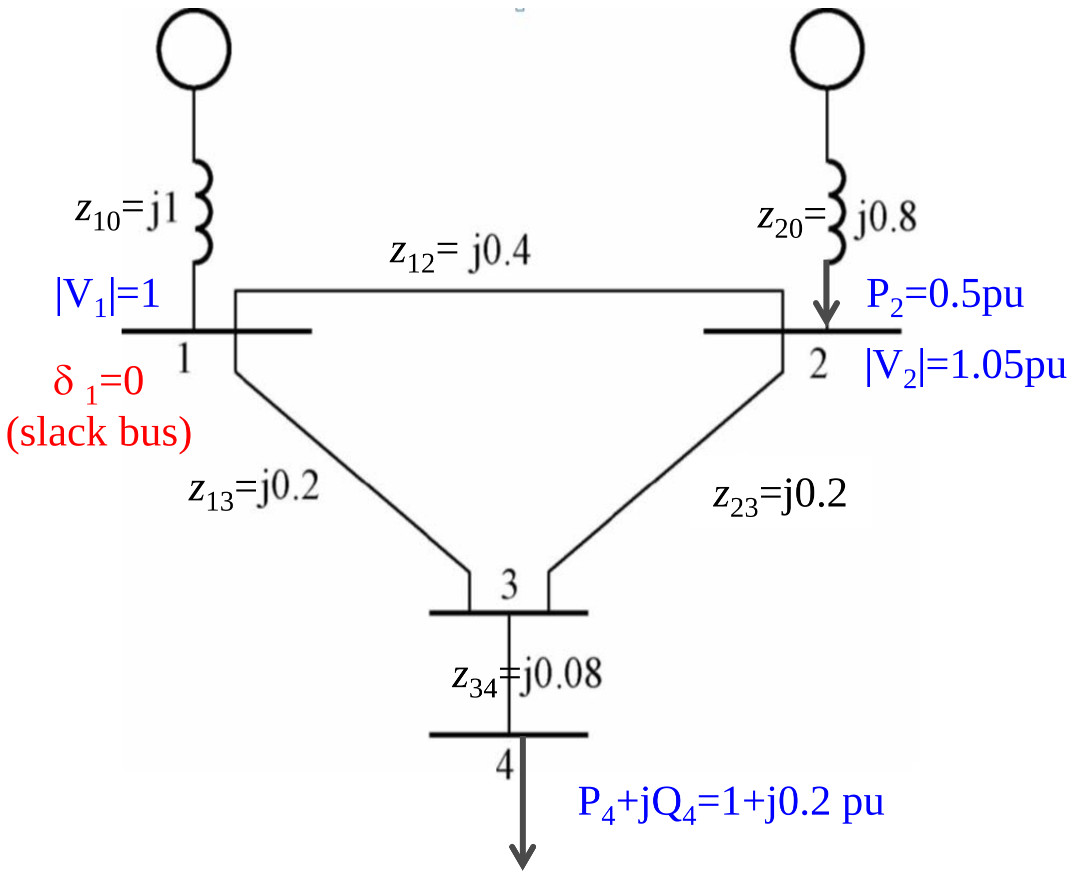
Slack bus (swing bus or V-δ bus)	• Selected as the reference having $\lvert V_i \rvert$ and δ_i fixed • P_i and Q_i are usually unlimited and can take any values to make up the gap between system generation and load
Load buses (P-Q buses)	• P_i and Q_i are specified
Regulated buses (generator buses or P-V buses)	• P_i and V_i are specified. • Limits of Q_i are also specified

More Thinking on Types of Buses

- Need to know two of $|V_i|$, δ_i, P_i and Q_i (either constant or observed)
- Relax the other two within upper and lower limits
- May assume more types of buses for a variety of natures of buses

	$\|V_i\|$	δ_i	P_i	Q_i
V-δ	X	X		
P-Q			X	X
P-V	X		X	
Q-V	X			X
P-δ		X	X	
Q-δ		X		X

A more general power flow study



Solution of Nonlinear Algebraic Equations

- Gauss-Seidel Method

- Newton-Raphson Method

Gauss-Seidel Method: Example 6.2

$$f(x) = x^3 - 6x^2 + 9x - 4 = 0$$

3 roots: $x_{1,2}$=1 and x_3=4

$$x = -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} = g(x)$$

$$x^{(0)} = 2$$

$$x^{(1)} = g(x^{(0)}) = -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.2222$$ $|x^{(1)}-x^{(0)}|=0.2222$

$x^{(2)}= g(x^{(1)})$ = 2.5173 $|x^{(2)}-x^{(1)}|$ =0.2951

$x^{(3)}= g(x^{(2)})$ = 2.8966 $|x^{(3)}-x^{(2)}|$ = 0.3793

$x^{(4)}= g(x^{(3)})$ = 3.3376 $|x^{(4)}-x^{(3)}|$ = 0.4410

$x^{(5)}= g(x^{(4)})$ = 3.7398 $|x^{(5)}-x^{(4)}|$ = 0.4022

$x^{(6)}= g(x^{(5)})$ = 3.9568 $|x^{(6)}-x^{(5)}|$ = 0.2170

$x^{(7)}= g(x^{(6)})$ = 3.9988 $|x^{(7)}-x^{(6)}|$ = 0.0420

$x^{(8)}= g(x^{(7)})$ = 4.0000 $|x^{(8)}-x^{(7)}|$ = 0.0012

$x^{(9)}= g(x^{(8)})$ = 4.0000 $|x^{(9)}-x^{(8)}|$ = <0.0001

Gauss-Seidel Method

- To solve nonlinear equation $f(x)=0$

- Re-write $x=g(x)$

- Start iteration from an initial estimate $x^{(0)}$

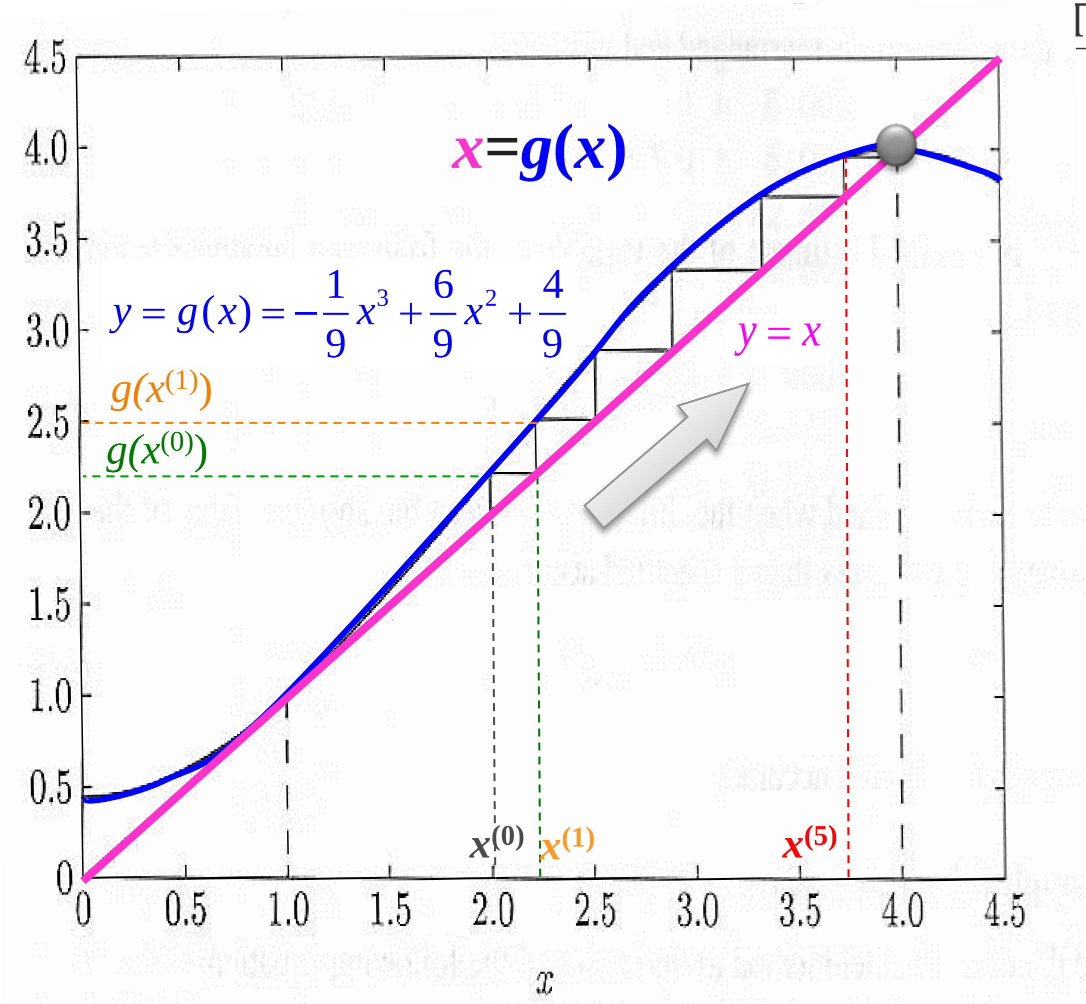
$$x^{(1)}=g(x^{(0)})$$

$$x^{(2)}=g(x^{(1)})$$

$$\dots$$

$$x^{(k+1)}=g(x^{(k)})$$

- Stop when $|x^{(k+1)}-x^{(k)}|\leq\varepsilon$. Solution: $x=x^{(k+1)}$



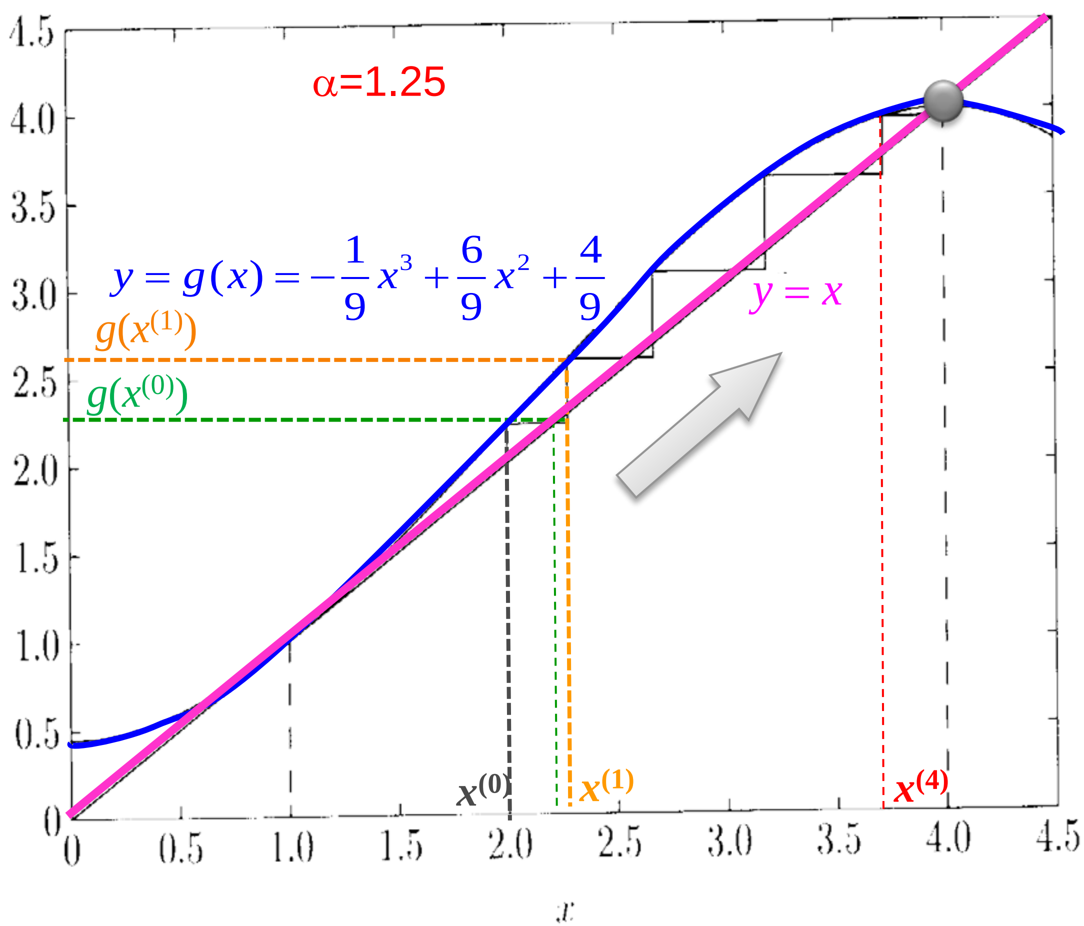
- "Zigzag" graphical illustration
  - Can the iteration be faster?
  - How to find all roots?
  - Does the iteration always converge to a root?

Faster iteration?

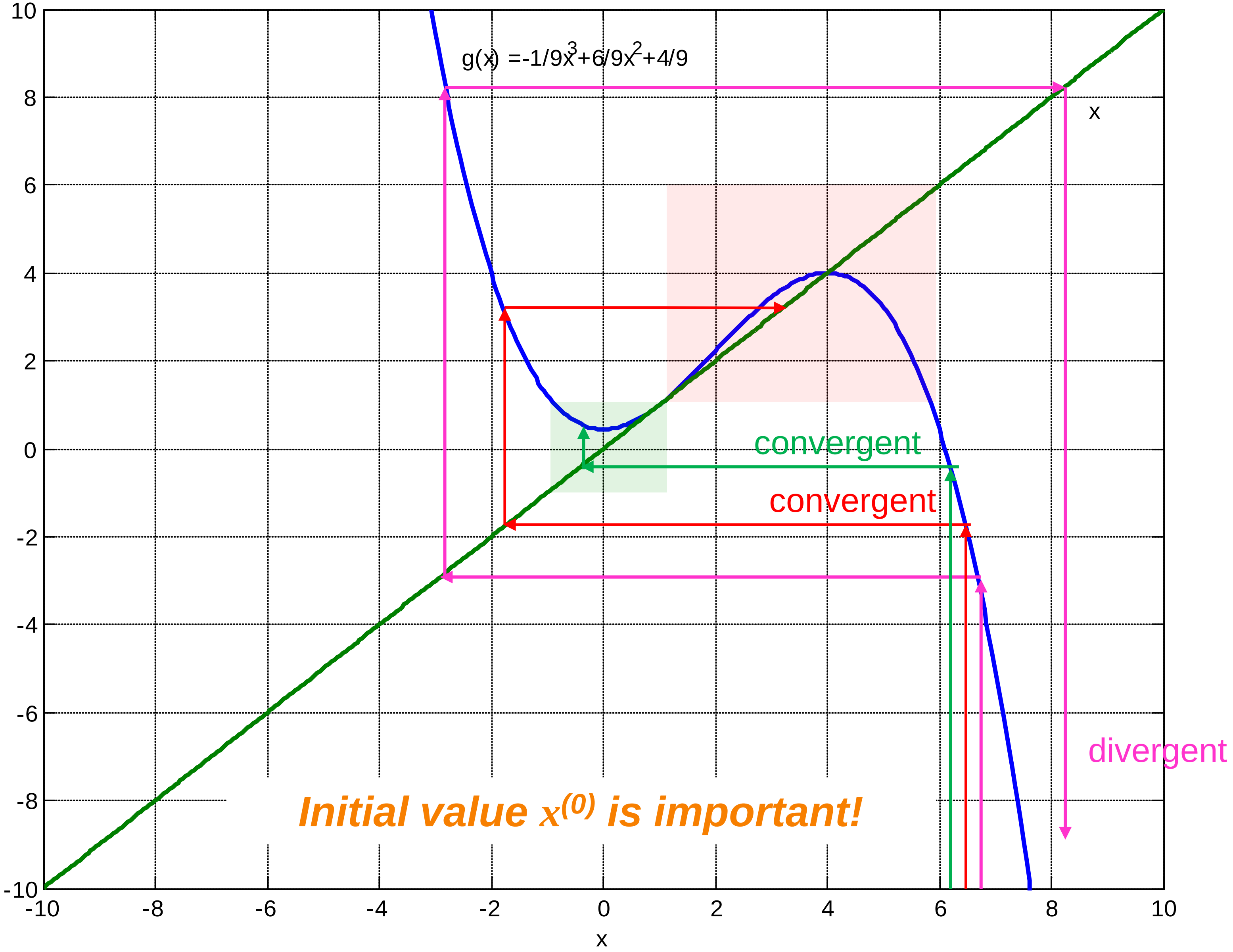
$$x^{(k+1)} = g(x^{(k)}) = x^{(k)} + \boxed{[g(x^{(k)}) - x^{(k)}]}$$ Adjustment on x

- Using an acceleration factor when updating $x^{(k)}$

$$x^{(k+1)} = x^{(k)} + \boxed{\alpha[g(x^{(k)}) - x^{(k)}]}$$



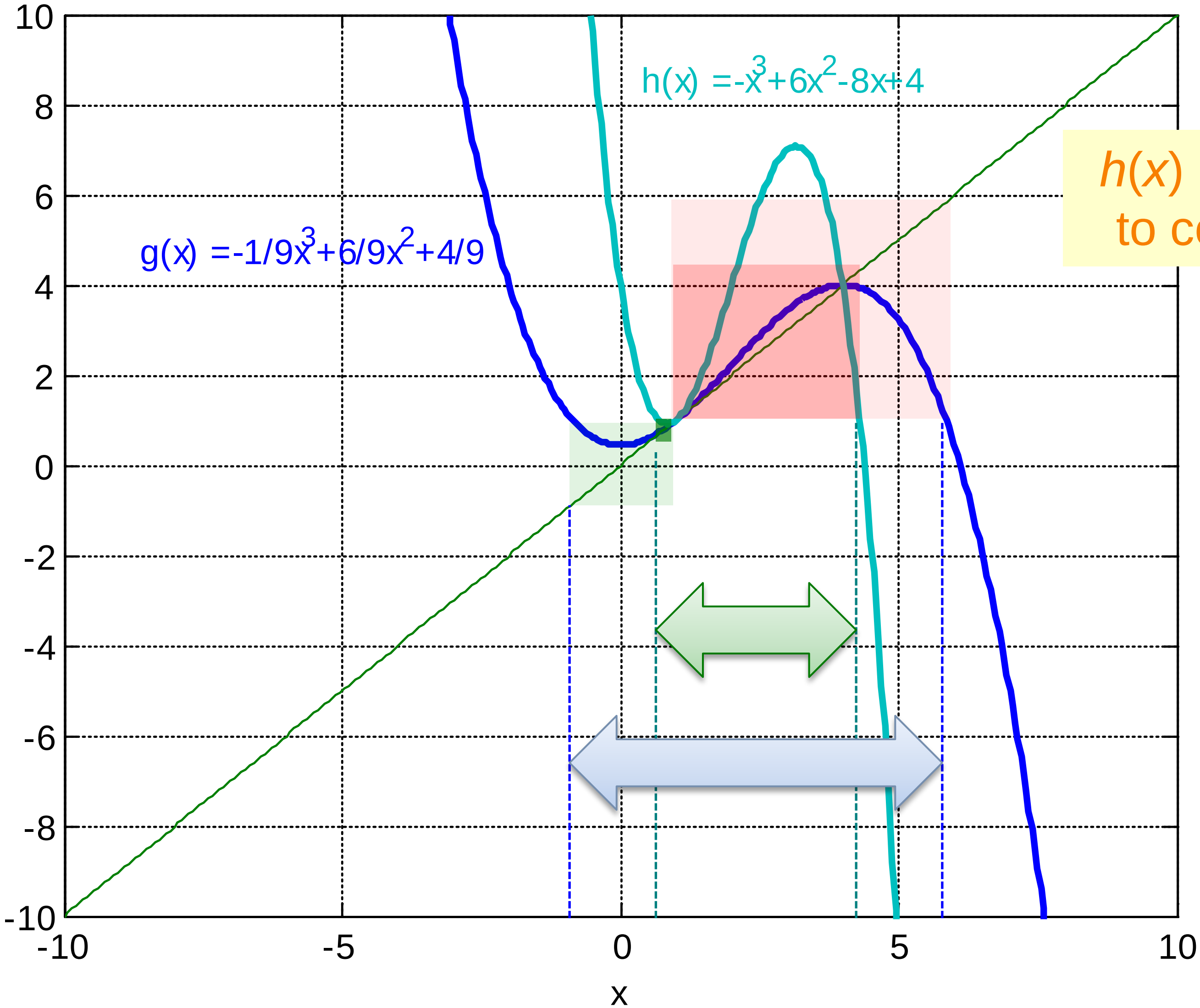
All roots? Always convergent?



Initial value $x^{(0)}$ is important!

$$f(x) = x^3 - 6x^2 + 9x - 4 = 0$$

$$x = -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} = g(x) \qquad x = -x^3 + 6x^2 - 8x + 4 = h(x)$$



h(*x*) is harder to converge

A system of *n* equations in *n* variables

$$
\begin{aligned}
&f_1(x_1, x_2, x_3, \ldots x_n) = 0 \\
&f_2(x_1, x_2, x_3, \ldots x_n) = 0 \\
&f_3(x_1, x_2, x_3, \ldots x_n) = 0 \\
&\cdots \\
&f_n(x_1, x_2, x_3, \ldots x_n) = 0
\end{aligned}
$$

$$
\begin{aligned}
&x_1^{(k+1)} = g_1\left(x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \ldots, x_n^{(k)}\right) \\
&x_2^{(k+1)} = g_2\left(x_1^{(k+1)}, x_2^{(k)}, x_3^{(k)}, \ldots, x_n^{(k)}\right) \\
&x_3^{(k+1)} = g_3\left(x_1^{(k+1)}, x_2^{(k+1)}, x_3^{(k)}, \ldots, x_n^{(k)}\right) \\
&\cdots \\
&x_n^{(k+1)} = g_n\left(x_1^{(k+1)}, x_2^{(k+1)}, x_3^{(k+1)}, \ldots, x_{n-1}^{(k+1)}, x_n^{(k)}\right)
\end{aligned}
$$

- Using an acceleration factor when updating $x_i^{(k)}$

$$x_i^{(k+1)} = x_i^{(k)} + \alpha(x_{i\ cal}^{(k+1)} - x_i^{(k)}) \qquad x_{i\ cal}^{(k+1)} = g_i(x_1^{(k)}, \cdots, x_n^{(k)})$$

G-S Power Flow Solution

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1, j\neq i}^{n} y_{ij} V_j$$

$$Y_{ij} = -y_{ij} \quad j \neq i \qquad Y_{ii} = \sum_{j=0}^{n} y_{ij}$$

$$= V_i Y_{ii} + \sum_{j=1, j\neq i}^{n} Y_{ij} V_j$$

$$= \sum_{j=1}^{n} Y_{ij} V_j$$

$$V_i^{(k+1)} = \frac{\dfrac{P_i^{(k)} - jQ_i^{(k)}}{V_i^{*(k)}} - \displaystyle\sum_{j=1, j\neq i}^{n} Y_{ij} V_j^{(k)}}{Y_{ii}}$$

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

$$Q_i^{(k+1)} = -\mathrm{Im}[V_i^{*(k)} \sum_{j=1}^{n} Y_{ij} V_j^{(k)}]$$

$$P_i^{(k+1)} = \mathrm{Re}[V_i^{*(k)} \sum_{j=1}^{n} Y_{ij} V_j^{(k)}]$$

PQ Buses

- $|V_i|$ and δ_i are unknown
- P_i and Q_i are scheduled (generation or load), denoted by P_i^{sch} and Q_i^{sch}

$$x^{(k+1)}=g(x^{(k)}) \quad \Leftrightarrow \quad V_i^{(k+1)} = \frac{\dfrac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{j=1,\, j\neq i}^{n} Y_{ij} V_j^{(k)}}{Y_{ii}}$$

- Under normal operating conditions:
  - Slack bus: $|V_0|\angle\delta_0$ (typically $1\angle 0^o$)
  - Other buses: $|V_i|$ is close to 1pu or $|V_0|$. For most of cases, there are:
    - Generator buses: $|V_i|>|V_0|$, $\delta_i > \delta_0$,
    - Load buses: $|V_i|<|V_0|$, $\delta_i < \delta_0$

- Initial guess could be $V_i^{(0)} = 1\angle 0^o$ without a better estimation

PV Buses

- P_i=P_i^{sch} and $|V_i|$ are specified
- Starting from an initial estimate of $\delta_i^{(0)} \rightarrow V_i^{(0)}$=$|V_i|\angle\, \delta_i^{(0)}$

$$Q_i^{(k+1)} = -\mathrm{Im}[V_i^{*(k)}\sum_{j=1}^{n} Y_{ij}V_j^{(k)}] \qquad V_{ci}^{(k+1)} = \frac{\dfrac{P_i^{sch} - jQ_i^{(k+1)}}{V_i^{*(k)}} - \sum_{j=1,\, j\neq i}^{n} Y_{ij}V_j^{(k)}}{Y_{ii}}$$

- Since $|V_i|$ is specified, only $V_{I,i}^{(k+1)}$=$\mathrm{Im}[V_{ci}^{(k+1)}]$ is retained

$$V_{R,i}^{(k+1)} = \sqrt{|V_i|^2 - (V_{I,i}^{(k+1)})^2}$$

Update $V_i^{(k+1)}$=$V_{R,i}^{(k+1)}$+j$\cdot$ $V_{I,i}^{(k+1)}$

- Continue the iterations until $|V_{R,i}^{(k+1)} - V_{R,i}^{(k)}| \le \varepsilon \quad |V_{I,i}^{(k+1)} - V_{I,i}^{(k)}| \le \varepsilon$

  or, the power mismatch, i.e. the largest element in $\boldsymbol{\Delta P}$ and $\boldsymbol{\Delta Q}$ $<\varepsilon$

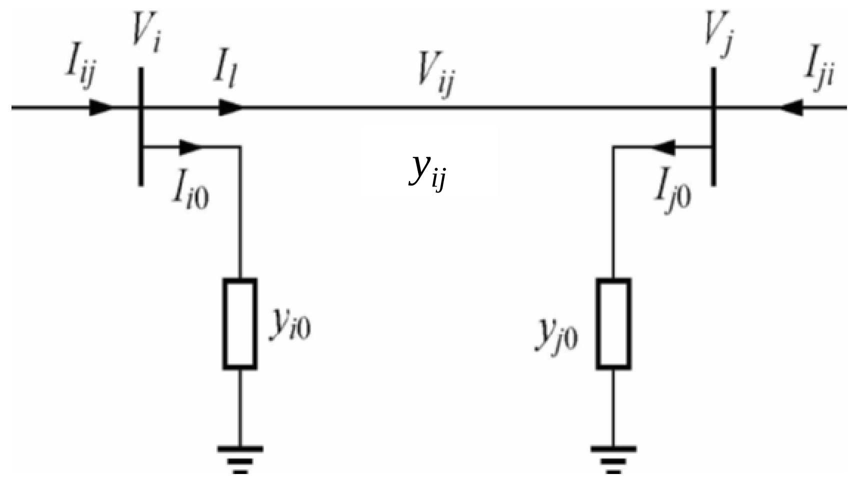
- Using acceleration factor α=1.3~1.7

$$V_{ic}^{(k+1)} = V_i^{(k)} + \alpha\left(\frac{\dfrac{P_i^{sch} - jQ_i^{(k+1)}}{V_i^{*(k)}} - \sum_{j=1,\, j\neq i}^{n} Y_{ij}V_j^{(k)}}{Y_{ii}} - V_i^{(k)}\right)$$

Slack Bus

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

Line Flows and Losses



- At bus *i*:

$$I_{ij} = I_l + I_{i0} = y_{ij}(V_i - V_j) + y_{i0} V_i$$

$$S_{ij} = V_i I_{ij}^*$$

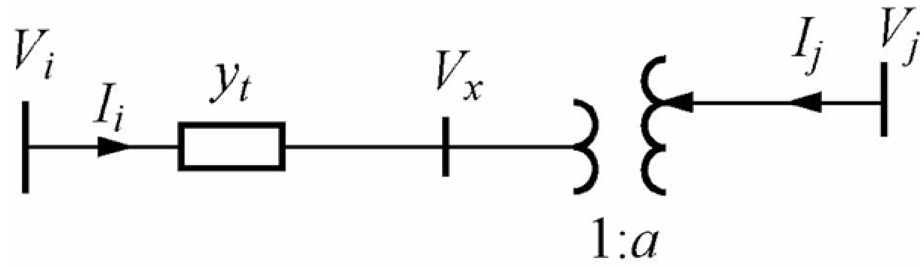
- At bus *j*:

$$I_{ji} = -I_l + I_{j0} = y_{ij}(V_j - V_i) + y_{j0} V_j$$

$$S_{ji} = V_j I_{ji}^*$$

- Power loss in line *i – j*: $S_{Lij} = S_{ij} + S_{ji}$

Tap Changing Transformers



- a is the per unit off-nominal tap position (usually, $|a|$ = 0.9~1.1)
  - Complex number for phase shifting transformers

$$S_T = V_x I_i^* = -V_j I_j^* \qquad V_x = \frac{1}{a} V_j \quad \Rightarrow \quad I_i = -a^* \cdot I_j$$

$$I_i = y_t (V_i - V_x) = y_t V_i - \frac{y_t}{a} V_j$$

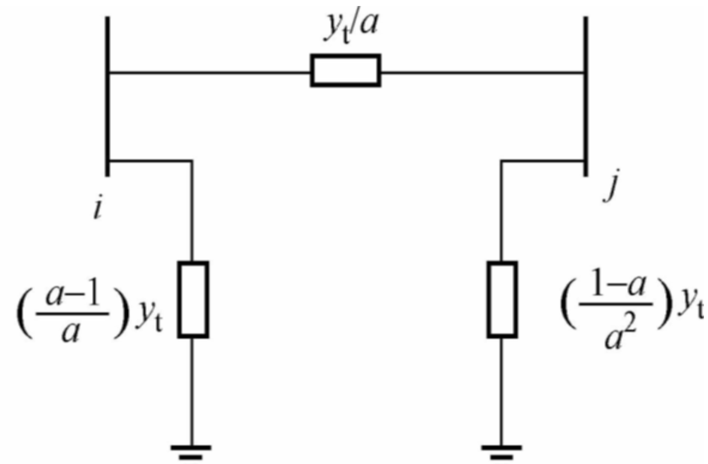
$$I_j = -\frac{1}{a^*} I_i = -\frac{y_t}{a^*} V_i + \frac{y_t}{|a|^2} V_j$$

$$\boxed{\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_t & -\dfrac{y_t}{a} \\ -\dfrac{y_t}{a^*} & \dfrac{y_t}{|a|^2} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}}$$

$\mathbf{Y}_{\mathbf{bus}}$ is not symmetrical with a phase shifting transformer

Equivalent circuit if *a* is real
(ignoring phase shifting)

Non-tap side Tap side



Example 6.7
(V-δ and P-Q buses)

Using the G-S method to find the power flow solution:

(a) Determine the voltage phasors at P-Q buses 2 and 3 accurate to 4 decimal places

(b) Find the slack bus real and reactive power

(c) Determine the line flows and losses. Show line flow directions in a power-flow diagram

(Solve P_1, Q_1, $|V_2|$, δ_2, $|V_3|$, δ_3, S_{ij} and S_{lij})

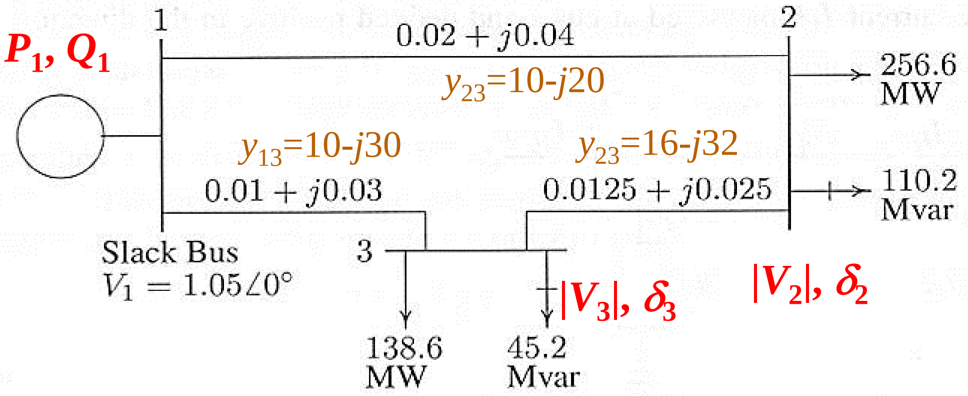


FIGURE 6.9
One-line diagram of Example 6.7 (impedances in pu on 100-MVA base).

Step 1. Check what are given

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$.

At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(256.6 + j110.2)}{100} = -2.566 - j1.102 \quad \text{pu}$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452 \quad \text{pu}$$

Step 2. Set initial estimates and iterate

Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$,

$$V_i^{(k+1)} = \frac{\dfrac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{j=1, j\neq i}^{n} Y_{ij} V_j^{(k)}}{Y_{ii}}$$

$$V_2^{(1)} = \frac{\dfrac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

$$= \frac{\frac{-2.566 + j1.102}{1.0 - j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.0 + j0)}{(26 - j52)}$$

$$= 0.9825 - j0.0310$$

$$V_3^{(1)} = \frac{\dfrac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{-1.386 + j0.452}{1 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9825 - j0.0310)}{(26 - j62)}$$

$$= 1.0011 - j0.0353$$

$V_2^{(2)} = 0.9816 - j0.0520$ $\quad$ $V_3^{(2)} = 1.0008 - j0.0459$

$V_2^{(3)} = 0.9808 - j0.0578$ $\quad$ $V_3^{(3)} = 1.0004 - j0.0488$

$V_2^{(4)} = 0.9803 - j0.0594$ $\quad$ $V_3^{(4)} = 1.0002 - j0.0497$

$V_2^{(5)} = 0.9801 - j0.0598$ $\quad$ $V_3^{(5)} = 1.0001 - j0.0499$

$V_2^{(6)} = 0.9801 - j0.0599$ $\quad$ $V_3^{(6)} = 1.0000 - j0.0500$

$V_2^{(7)} = 0.9800 - j0.0600$ $\quad$ $V_3^{(7)} = 1.0000 - j0.0500$

The final solution is

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle{-3.5035^\circ} \quad \text{pu}$$
$$V_3 = 1.0000 - j0.0500 = 1.00125\angle{-2.8624^\circ} \quad \text{pu}$$

Step 3. Calculate P and Q of the slack bus

(b) With the knowledge of all bus voltages, the slack bus power is

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.05[1.05(20 - j50) - (10 - j20)(0.98 - j.06) - \\ &\quad (10 - j30)(1.0 - j0.05)] \\ &= 4.095 - j1.890 \end{aligned}$$

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

the slack bus real and reactive powers are $P_1 = 4.095$ pu $= 409.5$ MW and $Q_1 = 1.890$ pu $= 189$ Mvar.

Step 4. Calculate line flows and losses

(c) To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1.05 + j0) - (0.98 - j0.06)] = 1.9 - j0.8$$
$$I_{21} = -I_{12} = -1.9 + j0.8$$
$$I_{13} = y_{13}(V_1 - V_3) = (10 - j30)[(1.05 + j0) - (1.0 - j0.05)] = 2.0 - j1.0$$
$$I_{31} = -I_{13} = -2.0 + j1.0$$
$$I_{23} = y_{23}(V_2 - V_3) = (16 - j32)[(0.98 - j0.06) - (1 - j0.05)] = -.64 + j.48$$
$$I_{32} = -I_{23} = 0.64 - j0.48$$

The line flows are

$$S_{12} = V_1 I_{12}^* = (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84 \text{ pu} \quad = 199.5 \text{ MW} + j84.0 \text{ Mvar}$$
$$S_{21} = V_2 I_{21}^* = (0.98 - j0.06)(-1.9 - j0.8) = -1.91 - j0.67 \text{ pu} \quad = -191.0 \text{ MW} - j67.0 \text{ Mvar}$$
$$S_{13} = V_1 I_{13}^* = (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05 \text{ pu} \quad = 210.0 \text{ MW} + j105.0 \text{ Mvar}$$
$$S_{31} = V_3 I_{31}^* = (1.0 - j0.05)(-2.0 - j1.0) = -2.05 - j0.90 \text{ pu} \quad = -205.0 \text{ MW} - j90.0 \text{ Mvar}$$
$$S_{23} = V_2 I_{23}^* = (0.98 - j0.06)(-0.656 + j0.48) = -0.656 - j0.432 \text{ pu} = -65.6 \text{ MW} - j43.2 \text{ Mvar}$$
$$S_{32} = V_3 I_{32}^* = (1.0 - j0.05)(0.64 + j0.48) = 0.664 + j0.448 \text{ pu} \quad = 66.4 \text{ MW} + j44.8 \text{ Mvar}$$

and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 8.5 \text{ MW} + j17.0 \text{ Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 5.0 \text{ MW} + j15.0 \text{ Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.8 \text{ MW} + j1.60 \text{ Mvar}$$

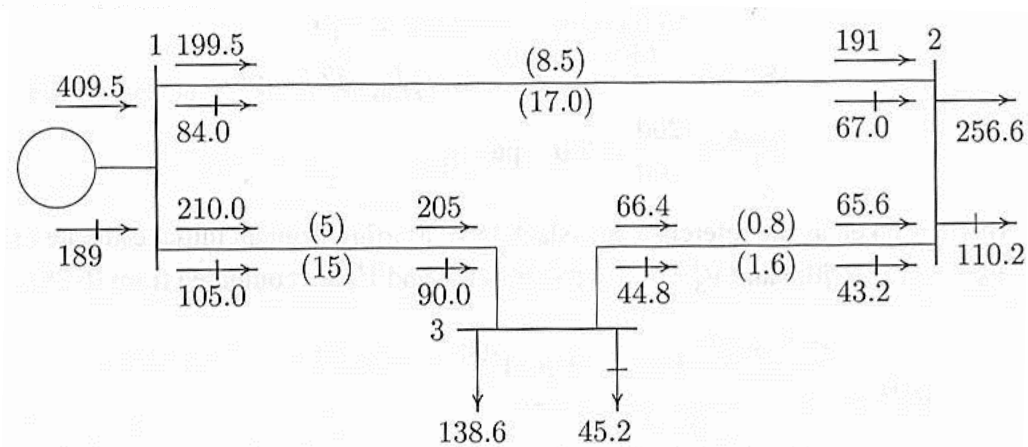


FIGURE 6.11
Power flow diagram of Example 6.7 (powers in MW and Mvar).

Example 6.8
(V-δ, P-Q and P-V buses)

Line charging susceptances are neglected. Obtain the power flow solution by the G-S method including line flows and line losses

(Solve P_1, Q_1, $|V_2|$, δ_2, Q_3, δ_3, S_{ij} and S_{lij})

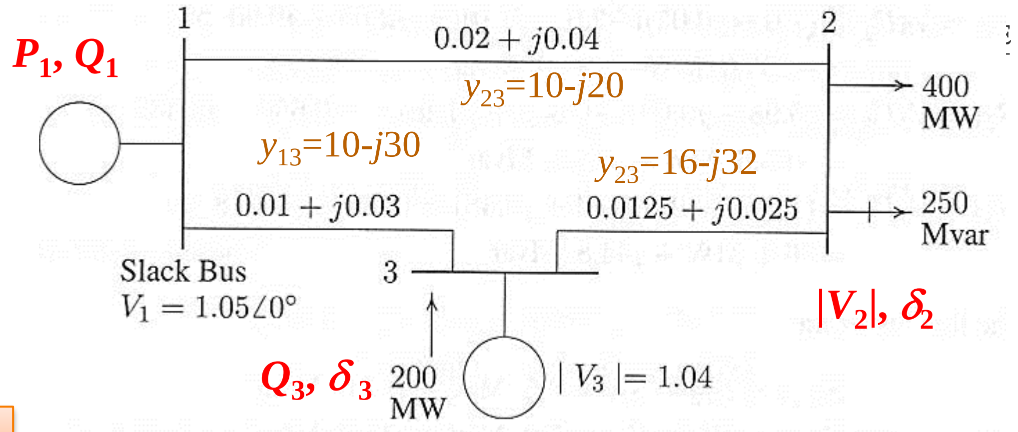


FIGURE 6.12
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

Step 1. Check what are given

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

Step 2. Set initial estimates and iterate

Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.04 + j0.0$,

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

$$= \frac{\frac{-4.0+j2.5}{1.0-j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.04 + j0)}{(26 - j52)}$$

$$= 0.97462 - j0.042307$$

$$Q_3^{(1)} = -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\}$$

$$= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - (16 - j32)(0.97462 - j0.042307)]\}$$

$$= 1.16$$

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{(1)}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{2.0-j1.16}{1.04-j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.97462 - j0.042307)}{(26 - j62)}$$

$$= 1.03783 - j0.005170$$

Note: $|V_{c3}^{(1)}| = 1.0378 \neq 1.04 = |V_3|$

Since $|V_3|$ is held constant at 1.04 pu,

only the imaginary part of $V_{c3}^{(1)}$ is retained,

and its real part is obtained from

$$V_{R,3}^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$$

$$V_3^{(1)} = 1.039987 - j0.005170$$

Bus 2 (P-Q): Solve $\lvert V_2 \rvert, \delta_2$	Bus 3 (P-V): Solve Q_3, δ_3

$$V_i^{(k+1)} = \frac{\dfrac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{j=1, j\neq i}^{n} Y_{ij} V_j^{(k)}}{Y_{ii}}$$

$$Q_i^{(k+1)} = -\mathrm{Im}[V_i^{*(k)} \sum_{j=1}^{n} Y_{ij} V_j^{(k)}]$$

$$V_{ci}^{(k+1)} = \frac{\dfrac{P_i^{sch} - jQ_i^{(k+1)}}{V_i^{*(k)}} - \sum_{j=1, j\neq i}^{n} Y_{ij} V_j^{(k)}}{Y_{ii}}$$

$$\mathrm{Re}[V_3^{(k+1)}] = \sqrt{1.04^2 - \{\mathrm{Im}[V_{c3}^{(k+1)}]\}^2}$$

Bus 2	Q_3	V_{c3}	V_3
$V_2^{(2)} = 0.971057 - j0.043432$	$Q_3^{(2)} = 1.38796$	$V_{c3}^{(2)} = 1.03908 - j0.00730$	$V_3^{(2)} = 1.039974 - j0.00730$
$V_2^{(3)} = 0.97073 - j0.04479$	$Q_3^{(3)} = 1.42904$	$V_{c3}^{(3)} = 1.03954 - j0.00833$	$V_3^{(3)} = 1.03996 - j0.00833$
$V_2^{(4)} = 0.97065 - j0.04533$	$Q_3^{(4)} = 1.44833$	$V_{c3}^{(4)} = 1.03978 - j0.00873$	$V_3^{(4)} = 1.03996 - j0.00873$
$V_2^{(5)} = 0.97062 - j0.04555$	$Q_3^{(5)} = 1.45621$	$V_{c3}^{(5)} = 1.03989 - j0.00893$	$V_3^{(5)} = 1.03996 - j0.00893$
$V_2^{(6)} = 0.97061 - j0.04565$	$Q_3^{(6)} = 1.45947$	$V_{c3}^{(6)} = 1.03993 - j0.00900$	$V_3^{(6)} = 1.03996 - j0.00900$
$V_2^{(7)} = 0.97061 - j0.04569$	$Q_3^{(7)} = 1.46082$	$V_{c3}^{(7)} = 1.03995 - j0.00903$	$V_3^{(7)} = 1.03996 - j0.00903$

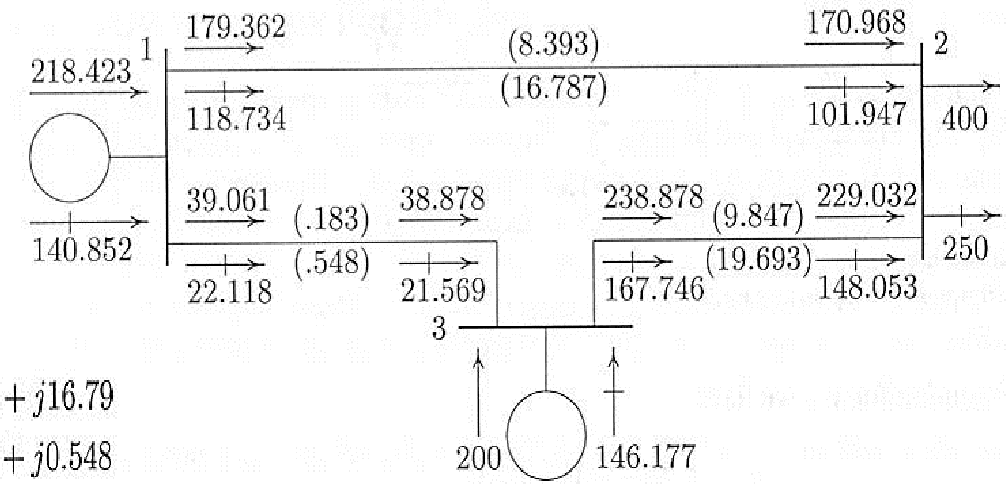
The final solution is

$$V_2 = 0.97168\angle -2.6948^\circ \quad \text{pu}$$

$$S_3 = 2.0 + j1.4617 \quad \text{pu}$$

$$V_3 = 1.04\angle -.498^\circ \quad \text{pu}$$

$$S_1 = 2.1842 + j1.4085 \quad \text{pu}$$



$S_{12} = 179.36 + j118.734$ $\quad S_{21} = -170.97 - j101.947$ $\quad S_{L12} = 8.39 + j16.79$

$S_{13} = 39.06 + j22.118$ $\quad S_{31} = -38.88 - j21.569$ $\quad S_{L13} = 0.18 + j0.548$

$S_{23} = -229.03 - j148.05$ $\quad S_{32} = 238.88 + j167.746$ $\quad S_{L23} = 9.85 + j19.69$

Newton-Raphson Method

- Based on Taylor's series expansion at an initial estimate of the solution

$$f(x) = c$$

$$\Leftrightarrow \quad f(x^{(0)} + \Delta x^{(0)}) = c$$

$$\Leftrightarrow \quad f(x^{(0)}) + (\frac{df}{dx})^{(0)} \Delta x^{(0)} \cancel{+ \frac{1}{2!}(\frac{d^2 f}{dx^2})^{(0)} (\Delta x^{(0)})^2 + \cdots} = c$$

- Ignore all terms with orders ≥2

Comparison: G-S method ignores all differential terms (orders ≥1)

$$(\frac{df}{dx})^{(0)} \Delta x^{(0)} \simeq c - f(x^{(0)}) \stackrel{\Delta}{=} \Delta c^{(0)}$$

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{(\frac{df}{dx})^{(0)}} \qquad x^{(1)} = x^{(0)} + \Delta x^{(0)}$$

- Iteration 1: (0)→(1)

$$x^{(1)} \stackrel{\Delta}{=} x^{(0)} + \Delta x^{(0)} = x^{(0)} + \frac{\Delta c^{(0)}}{(\frac{df}{dx})^{(0)}} = x^{(0)} + \frac{c - f(x^{(0)})}{(\frac{df}{dx})^{(0)}}$$

- Iteration k+1: (k) →(k+1)

$$\boxed{x^{(k+1)} \stackrel{\Delta}{=} x^{(k)} + \Delta x^{(k)} = x^{(k)} + \frac{\Delta c^{(k)}}{(\frac{df}{dx})^{(k)}} = x^{(k)} + \frac{c - f(x^{(k)})}{(\frac{df}{dx})^{(k)}}}$$

$$\Delta x^{(k)} = \frac{\Delta c^{(k)}}{(\frac{df}{dx})^{(k)}}$$

- Until $\quad | x^{(k+1)} - x^{(k)} | \le \varepsilon$

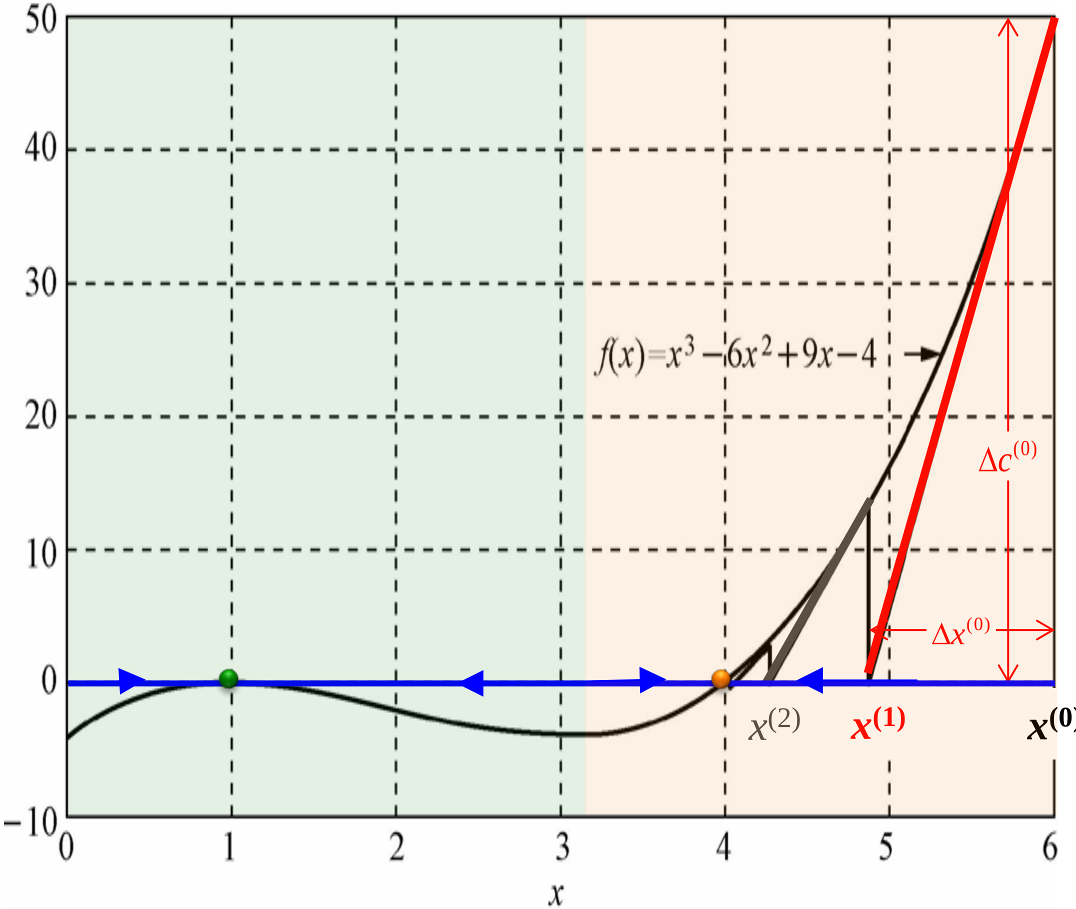
- $c=f(x)$ is actually approximated by the tangent line on the curve at $x^{(k)}$.

$$x = x^{(k)} + \frac{c - f(x^{(k)})}{(\frac{df}{dx})^{(k)}}$$

It is straight line function
$c=kx+b$

Example 6.4

$$\boxed{x^{(k+1)} = x^{(k)} + \frac{c - f(x^{(k)})}{\left(\frac{df}{dx}\right)^{(k)}}}$$



Let $x^{(0)}=6$

$$\frac{df(x)}{dx} = 3x^2 - 12x + 9$$

$$\left(\frac{df}{dx}\right)^{(0)} = 3(6)^2 - 12(6) + 9 = 45$$

$$\Delta c^{(0)} = c - f(x^{(0)}) = 0 - [(6)^3 - 6(6)^2 + 9(6) - 4] = -50$$

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}} = \frac{-50}{45} = -1.1111$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = 6 - 1.1111 = 4.8889$$

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = 4.8889 - \frac{13.4431}{22.037} = 4.2789$$

$$x^{(3)} = x^{(2)} + \Delta x^{(2)} = 4.2789 - \frac{2.9981}{12.5797} = 4.0405$$

$$x^{(4)} = x^{(3)} + \Delta x^{(3)} = 4.0405 - \frac{0.3748}{9.4914} = 4.0011$$

$$x^{(5)} = x^{(4)} + \Delta x^{(4)} = 4.0011 - \frac{0.0095}{9.0126} = 4.0000$$

N-dimensional System

$$f(x) = c$$

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)} = x^{(k)} + \left[\left(\frac{df}{dx}\right)^{(k)} \right]^{-1} \Delta c^{(k)}$$

$$\Delta c^{(k)} = c - f(x^{(k)})$$

$$f_1(x_1, x_2, \cdots, x_n) = c_1$$
$$f_2(x_1, x_2, \cdots, x_n) = c_2$$
$$\cdots$$
$$f_n(x_1, x_2, \cdots, x_n) = c_n$$

$$X^{(k+1)} = X^{(k)} + \Delta X^{(k+1)} = X^{(k)} + \left[J^{(k)} \right]^{-1} \Delta C^{(k)}$$

Jacobian Matrix:

$$J^{(k)} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(k)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(k)} \end{bmatrix}$$

$$\Delta X^{(k)} = \begin{bmatrix} \Delta x_1^{(k)} \\ \Delta x_2^{(k)} \\ \vdots \\ \Delta x_n^{(k)} \end{bmatrix}$$

$$\Delta C^{(k)} = \begin{bmatrix} c_1 - (f_1)^{(k)} \\ c_2 - (f_2)^{(k)} \\ \vdots \\ c_n - (f_n)^{(k)} \end{bmatrix}$$

Example 6.5

- Use the N-R method to find the intersections of the curves

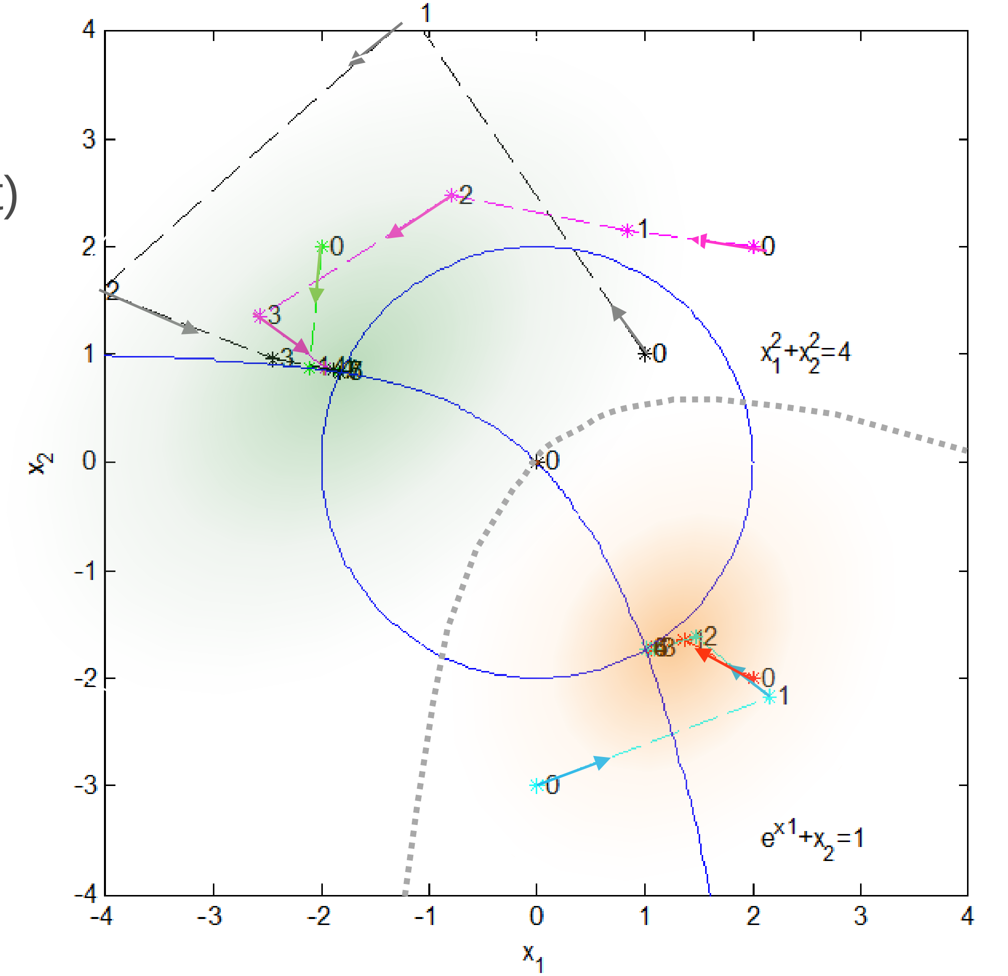
$$x_1^2 + x_2^2 = 4 \qquad J = \begin{pmatrix} 2x_1 & 2x_2 \\ e^{x_1} & 1 \end{pmatrix}$$

$$e^{x_1} + x_2 = 1$$

- J tells the fastest direction (gradient) toward a solution

If $x_1^{(0)}=2$, $x_2^{(0)}= -2$:

k	ΔC	J		Δx	x
1	-4.0000	4.0000	-4.0000	-0.6424	1.3576
	-4.3891	7.3891	1.0000	0.3576	-1.6424
2	-0.5406	2.7152	-3.2848	-0.2989	1.0587
	-1.2445	3.8869	1.0000	-0.0825	-1.7249
3	-0.0962	2.1173	-3.4499	-0.0530	1.0056
	-0.1576	2.8825	1.0000	-0.0047	-1.7296
4	-0.0028	2.0112	-3.4592	-0.0014	1.0042
	-0.0040	2.7336	1.0000	-0.0000	-1.7296
5	-0.0000	2.0083	-3.4593	-0.0000	1.0042
	-0.0000	2.7296	1.0000	-0.0000	-1.7296



Compared to the Gauss-Seidel Method

- Since higher-order terms are ignored, the N-R method also needs the initial estimation to be sufficiently close to the actual solution
- The N-R method converges much faster
  - N-R method: quadratic convergence (ignoring the 2nd and higher orders)
  - G-S method: linear convergence (ignoring the 1st and higher orders)
- The N-R method has some computational issues:
  - Needs $[J^{(k)}]^{-1}$ during each iteration, which is computationally intense

Dealing with $[J^{(k)}]^{-1}$

$$\Delta X^{(k+1)} = \left[J^{(k)} \right]^{-1} \Delta C^{(k)}$$

- Try not to update $J^{(k)}$ so often (at least not in every iteration)

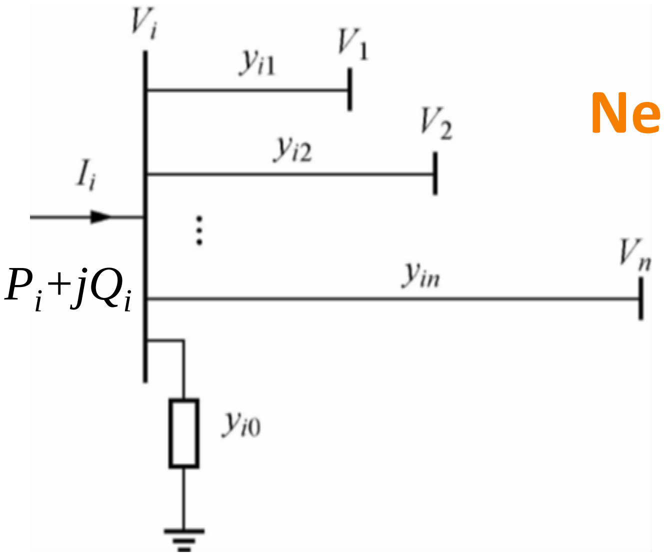
- Apply LU decomposition (triangular factorization)

$$J^{(k)} \Delta X^{(k+1)} = \Delta C^{(k)}$$

$$L^{(k)} \underline{U^{(k)} \underline{\Delta X^{(k+1)}}} = \Delta C^{(k)}$$

In MATLAB, the solution of $J\Delta X = \Delta C$ can be obtained by $\Delta X = J \backslash \Delta C$

Newton-Raphson Power Flow Solution



G-S method

$X=G(X)$

$$V_i^{(k+1)}=\frac{\dfrac{P_i^{(k)}-jQ_i^{(k)}}{V_i^{*(k)}}-\displaystyle\sum_{j=1,\,j\neq i}^{n}Y_{ij}V_j^{(k)}}{Y_{ii}}$$

$$P_i^{(k+1)}-jQ_i^{(k+1)}=V_i^{*(k)}\sum_{j=1}^{n}Y_{ij}V_j^{(k)}$$

$$I_i=\frac{P_i-jQ_i}{V_i^*}=V_iY_{ii}+\sum_{j=1,\,j\neq i}^{n}Y_{ij}V_j$$

$$=\sum_{j=1}^{n}Y_{ij}V_j$$

N-R method

$F(X)=C$

$\mathbf{J}\times\Delta X=\Delta C$

$$V_i^*\sum_{j=1}^{n}Y_{ij}V_j=P_i-jQ_i$$

$$\begin{bmatrix}\mathbf{J}_1 & \mathbf{J}_2\\ \mathbf{J}_3 & \mathbf{J}_4\end{bmatrix}\begin{bmatrix}\boldsymbol{\Delta\delta}\\ \boldsymbol{\Delta|V|}\end{bmatrix}=\begin{bmatrix}\mathbf{\Delta P}\\ \mathbf{\Delta Q}\end{bmatrix}$$

$$P_i-jQ_i=V_i^*\sum_{j=1}^{n}Y_{ij}V_j$$

Use polar forms:

$V_i=|V_i|\angle\delta_i \quad Y_{ij}=|Y_{ij}|\angle\theta_{ij}$

$$=|V_i|\angle-\delta_i\sum_{j=1}^{n}|Y_{ij}||V_j|\angle\theta_{ij}+\delta_j$$

$$P_i = f_i(\Delta|\mathbf{V}|, \Delta\boldsymbol{\delta}) \qquad \text{eq.(1)}$$

$$= \sum_{j=1}^{n} |V_j||V_i||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = g_i(\Delta|\mathbf{V}|, \Delta\boldsymbol{\delta}) \qquad \text{eq.(2)}$$

$$= -\sum_{j=1}^{n} |V_j||V_i||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta|\mathbf{V}| \end{bmatrix}$$

Algorithm:

- For the slack bus (say bus 1) :
  - No need to include bus 1 in **J**
  - calculate P_1 and Q_1 by (1) and (2) at the end
- For m voltage-controlled (P-V) buses:
  - solve δ_i by (1)
  - calculate Q_i by (2)
- For the n-1-m load (P-Q) buses left:
  - solve $|V_i|$ and δ_i by (1) and (2)

There are 2n-2-m independent (1)'s and (2)'s

P-V bus

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \frac{\partial P_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \frac{\partial P_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_n}{\partial |V_n|}^{(k)} \\ \hline \frac{\partial Q_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \frac{\partial Q_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \frac{\partial Q_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_n}{\partial |V_n|}^{(k)} \end{array}\right] \begin{bmatrix} \Delta\delta_2^{(k)} \\ \vdots \\ \Delta\delta_n^{(k)} \\ \hline \Delta|V_2^{(k)}| \\ \vdots \\ \Delta|V_n^{(k)}| \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{J}_{1,(n-1)\times(n-1)} & \mathbf{J}_{2,(n-1)\times(n-1-m)} \\ \mathbf{J}_{3,(n-1-m)\times(n-1)} & \mathbf{J}_{4,(n-1-m)\times(n-1-m)} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta|\mathbf{V}| \end{bmatrix}$$

J: (2n-2-m)×(2n-2-m)

Jacobian Matrix

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \Delta |\mathbf{V}| \end{bmatrix}$$

- Diagonal and off-diagonal elements of $\mathbf{J}_1$~$\mathbf{J}_2$ ($i \neq$ slack bus)

$\mathbf{J}_1$ (n-1)×(n-1) $i \neq$ slack bus

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

$\mathbf{J}_2$ (n-1)×(n-1-m) $i \notin$ PV and slack buses

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{j \neq i} |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

$\mathbf{J}_3$ (n-1-m)×(n-1) $i \notin$ PV and slack buses

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

$\mathbf{J}_4$ (n-1-m)×(n-1-m) $i \notin$ PV and slack buses

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j \neq i} |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

Procedure for Power Flow Solution by N-R Method

1. Initial values
   - **Load buses:** P_i^{sch} and Q_i^{sch} specified, $|V_i^{(0)}|\angle\delta_i^{(0)}=1\angle 0$ or equal to the slack bus
   - **Voltage-regulated buses**: $|V_i|$ and P_i^{sch} specified, $\delta_i^{(0)}=0$ or the slack bus angle
2. Calculate $P_i^{(k)}$, $\Delta P_i^{(k)}$, $Q_i^{(k)}$ and $\Delta Q_i^{(k)}$
   - **Load buses:** calculate $P_i^{(k)}$ and $Q_i^{(k)}$ by eq. (1) and (2) and then

$$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \qquad \Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)}$$

   - **Voltage-controlled buses:** calculate $P_i^{(k)}$ by eq. (1) and $\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)}$
3. Calculate $\mathbf{J_1}$, $\mathbf{J_2}$, $\mathbf{J_3}$ and $\mathbf{J_4}$ and solve $\Delta|V_i^{(k)}|$ and $\Delta\delta_i^{(k)}$ from

$$\begin{bmatrix} \mathbf{\Delta P} \\ \mathbf{\Delta Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \mathbf{\Delta \delta} \\ \mathbf{\Delta |V|} \end{bmatrix}$$

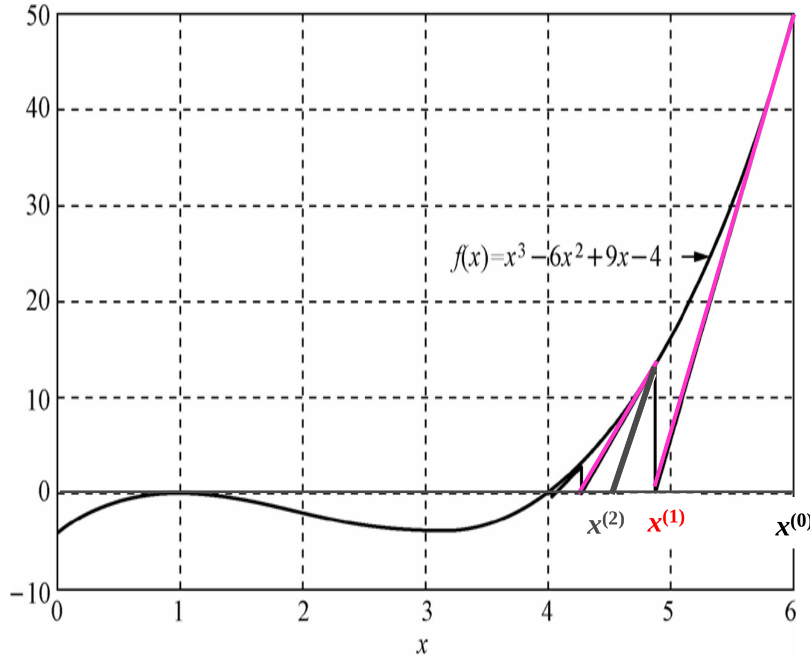
(applying triangular factorization and Gaussian elimination)

4. Calculate $|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta|V_i^{(k)}| \qquad \delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$
5. Iterate Steps 2~5 until $|\Delta P_i^{(k)}| \le \varepsilon \qquad |\Delta Q_i^{(k)}| \le \varepsilon$

How to accelerate the N-R method?

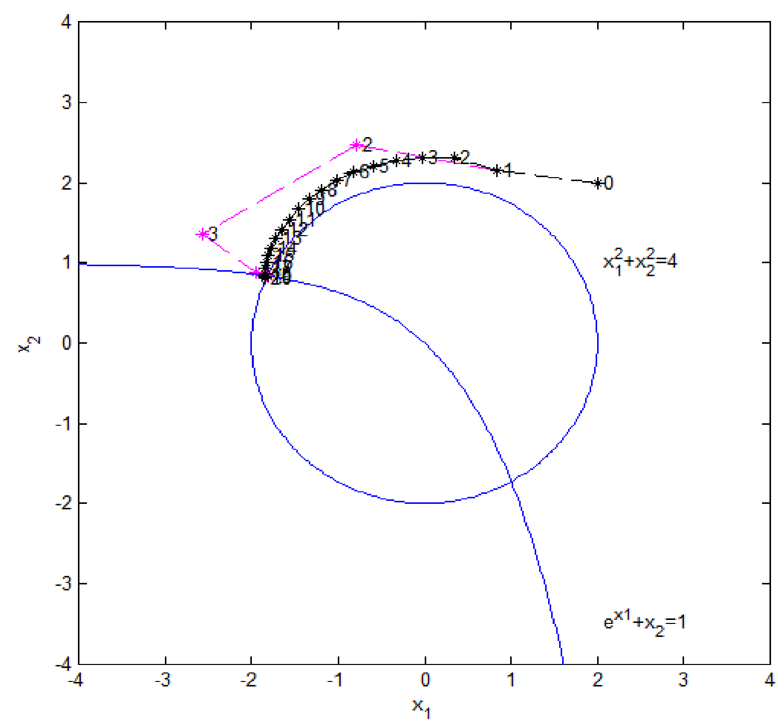
Original N-R method

- $J^{(k)}$ is computed at each iteration
- Computing $[J^{(k)}]^{-1}$ is expensive



Ideas: approximate **J**

- Constant matrix
- Block-diagonal matrix (ignoring small off-diagonal elements)



Fast Decoupled Power Flow Solution

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \Delta |\mathbf{V}| \end{bmatrix}$$

- Some elements of **J** may be close to 0
- Transmission lines usually have a high X/R ratio (close to lossless lines),

$$P_{ij} \approx \frac{|V_i||V_j|}{X_{ij}} \sin(\delta_i - \delta_j)$$

$$Q_{ij} \approx \frac{|V_i|}{X_{ij}} [|V_i| - |V_j| \cos(\delta_i - \delta_j)]$$

  - $\Delta \mathbf{P}$ is less sensitive to $\Delta|\mathbf{V}|$ than it is to $\Delta\delta$ $\rightarrow \mathbf{J}_2 = \partial \mathbf{P}/\partial|\mathbf{V}| \approx \mathbf{0}$
  - $\Delta \mathbf{Q}$ is less sensitive to $\Delta\delta$ than it is to $\Delta|\mathbf{V}|$ $\rightarrow \mathbf{J}_3 = \partial \mathbf{Q}/\partial\delta \approx \mathbf{0}$

- **F-D method:**

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} \approx \begin{bmatrix} \mathbf{J}_1 & 0 \\ 0 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \Delta |\mathbf{V}| \end{bmatrix}$$

$$\Delta \mathbf{P} = \mathbf{J}_1 \Delta \boldsymbol{\delta} = [\frac{\partial \mathbf{P}}{\partial \boldsymbol{\delta}}] \Delta \boldsymbol{\delta}$$

$$\Delta \mathbf{Q} = \mathbf{J}_4 \Delta |\mathbf{V}| = \left[\frac{\partial \mathbf{Q}}{\partial |\mathbf{V}|} \right] \Delta |\mathbf{V}|$$

Jacobian Matrix

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & 0 \\ 0 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \Delta |\mathbf{V}| \end{bmatrix}$$

$V_i = |V_i| \angle \delta_i \quad Y_{ij} = |Y_{ij}| \angle \theta_{ij}$

$\mathbf{J}_1$ (n-1)×(n-1) i ≠ slack bus

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$= \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2 |Y_{ii}| \sin\theta_{ii}$$

$$= -Q_i - |V_i|^2 \; B_{ii}$$

$$\approx -|V_i| B_{ii} \qquad (Q_i << B_{ii}, \; |V_i|^2 \approx |V_i| \approx 1)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

$$\approx -|V_i||V_j||Y_{ij}| \sin\theta_{ij}$$

$$\approx -|V_i| B_{ij}$$

$\mathbf{J}_4$ (n-1-m)×(n-1-m) i ∉ PV and slack buses

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}| \sin\theta_{ii} - \sum_{j \neq i} |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\approx -2|V_i| B_{ii} + |V_i| B_{ii}$$

$$= -|V_i| B_{ii}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

$$\approx -|V_i| B_{ij}$$

$$\frac{\Delta \mathbf{P}}{|\mathbf{V}|} = -\mathbf{B}' \Delta \boldsymbol{\delta}$$

B' ~(n-1)×(n-1) about P-Q & P-V buses

$$\frac{\Delta \mathbf{Q}}{|\mathbf{V}|} = -\mathbf{B}'' \Delta |\mathbf{V}|$$

B"~ (n-1-m)×(n-1-m) about P-Q buses

$$\Delta \boldsymbol{\delta} = -[\mathbf{B}']^{-1} \frac{\Delta \mathbf{P}}{|\mathbf{V}|}$$

Dot division (./)

$$\Delta |\mathbf{V}| = -[\mathbf{B}'']^{-1} \frac{\Delta \mathbf{Q}}{|\mathbf{V}|}$$

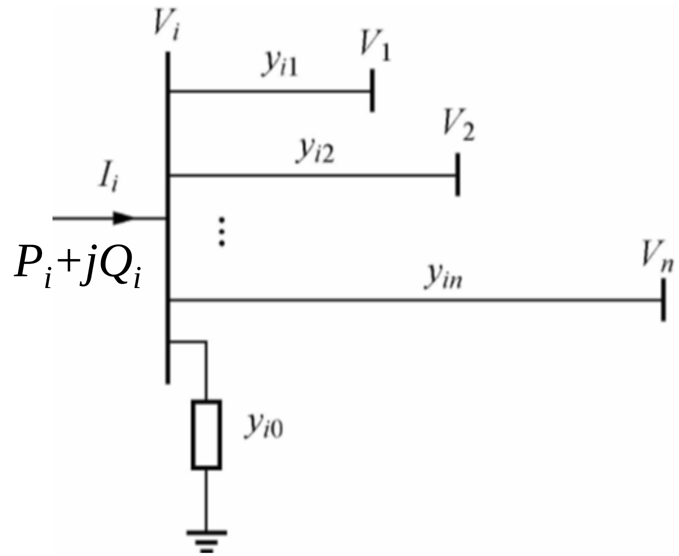
Proof: $Q_i << B_{ii}$

$$Q_{ij} \approx \frac{|V_i|}{X_{ij}}[|V_i| - |V_j|\cos(\delta_i - \delta_j)]$$

$$Q_i \approx |V_i|\sum_j y_{ij}[|V_i| - |V_j|\cos(\delta_i - \delta_j)]$$

$$\approx \sum_j y_{ij} \cdot [1 - \cos(\delta_i - \delta_j)]$$

$$<< \sum_j y_{ij} \cdot 1 \approx B_{ii}$$



In Example 6.7, $Q_i \approx 0.02 B_{ii}$

Compared to the N-R Method

$$\begin{bmatrix} \Delta\delta \\ \Delta|\mathbf{V}| \end{bmatrix} = -\begin{bmatrix} \mathbf{B}'^{-1} & 0 \\ 0 & \mathbf{B}''^{-1} \end{bmatrix} \begin{bmatrix} \dfrac{\Delta\mathbf{P}}{|\mathbf{V}|} \\ \dfrac{\Delta\mathbf{Q}}{|\mathbf{V}|} \end{bmatrix}$$

B' ~(n-1)×(n-1) about P-Q & P-V buses

B"~ (n-1-m)×(n-1-m) about P-Q buses

- F-D method deals with constant Jacobian matrix: no need to update in every iteration
- It requires more iterations than the N-R method, but each iteration requires considerably less time
- Overall, the F-D method is much faster
- The F-D method is very useful in fast contingency screening

Examples 6.10 & 6.12

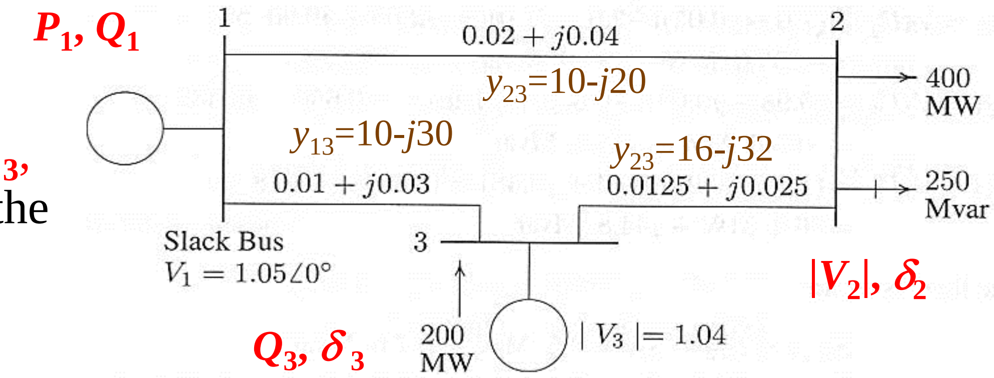


FIGURE 6.12
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

- Obtain the powerflow solution (**V_2, δ_2, δ_3, P_1, Q_1 and Q_3**) by the N-R method and the F-D method

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \text{ pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \text{ pu}$$

$$\begin{array}{cccc} & \text{Slack} & \text{P-Q} & \text{P-V} \\ Y_{bus} = & \left[\begin{array}{c} 20 - j50 \\ -10 + j20 \\ -10 + j30 \end{array}\right. & \begin{array}{c} -10 + j20 \\ 26 - j52 \\ -16 + j32 \end{array} & \left.\begin{array}{c} -10 + j30 \\ -16 + j32 \\ 26 - j62 \end{array}\right] \end{array} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle -1.1737 \end{bmatrix}$$

For F-D Method:

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix} \qquad B'' = [-52]$$

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

N-R Method

- 2n-2-m=3 independent equations:

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos(\theta_{33})$$

Eq (1)

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{33}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Eq (2)

- J has 3x3 elements

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_3} & \dfrac{\partial P_2}{\partial |V_2|} \\ \dfrac{\partial P_3}{\partial \delta_2} & \dfrac{\partial P_3}{\partial \delta_3} & \dfrac{\partial P_3}{\partial |V_2|} \\ \dfrac{\partial Q_2}{\partial \delta_2} & \dfrac{\partial Q_2}{\partial \delta_3} & \dfrac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_2 \\ \Delta |V_2| \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{23}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Procedure of the N-R Method

1. Initial values:

$$V_1 = 1.05\angle 0 \text{ pu} \qquad |V_3| = 1.04 \text{ pu} \qquad |V_2^{(0)}| = 1.0 \qquad \delta_2^{(0)} = 0.0 \qquad \delta_3^{(0)} = 0.0$$

2. Calculate $P_2^{(k)}$, $P_3^{(k)}$ and $Q_2^{(k)}$ by eq. (1) and (2) and then $\Delta P_2^{(k)}$, $\Delta P_3^{(k)}$, and $\Delta Q_2^{(k)}$, e.g.:

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

3. Calculate **J,** solve $\Delta|V_2^{(k)}|$, $\Delta\delta_2^{(k)}$ and $\Delta\delta_3^{(k)}$ and then $|V_2^{(k+1)}|$, $\delta_2^{(k+1)}$ and $\delta_3^{(k+1)}$, e.g.:

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.1400 & 16.64000 & 49.72000 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

$$\Delta\delta_2^{(0)} = -0.045263 \qquad \delta_2^{(1)} = (0) + (-0.045263) = -0.0452653$$
$$\Delta\delta_3^{(0)} = -0.007718 \qquad \delta_3^{(1)} = (0) + (-0.007718) = -0.007718$$
$$\Delta|V_2^{(0)}| = -0.026548 \qquad |V_2^{(1)}| = 1 + (-0.026548) = 0.97345$$

$$\begin{bmatrix} -0.099218 \\ 0.021715 \\ -0.050914 \end{bmatrix} = \begin{bmatrix} 51.724675 & -31.765618 & 21.302567 \\ -32.981642 & 65.656383 & -15.379086 \\ -28.538577 & 17.402838 & 48.103589 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

$$\Delta\delta_2^{(1)} = -0.001795 \qquad \delta_2^{(2)} = -0.045263 + (-0.001795) = -0.04706$$
$$\Delta\delta_3^{(1)} = -0.000985 \qquad \delta_3^{(2)} = 0.007718 + (-0.000985) = -0.00870$$
$$\Delta|V_2^{(1)}| = -0.001767 \qquad |V_2^{(2)}| = 0.973451 + (-0.001767) = 0.971684$$

4. Stop after **3** iterations: $V_2 = 0.97168\angle -2.696^\circ \qquad V_3 = 1.04\angle -0.4988^\circ \qquad \varepsilon = 2.5\times 10^{-4}$

5. Calculate:

$$Q_3 = -|V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2|Y_{33}|\sin\theta_{33} = 1.4617 \text{ pu}$$
$$P_1 = |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3||Y_{13}|\cos(\theta_{13} - \delta_1 + \delta_3) = 2.1842 \text{ pu}$$
$$Q_1 = -|V_1|^2|Y_{11}|\sin\theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3||Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3) = 1.4085 \text{ pu}$$

Procedure of the F-D Method

1. Initial values:

$$V_1 = 1.05\angle 0 \text{ pu} \qquad |V_3| = 1.04 \text{ pu} \qquad |V_2^{(0)}| = 1.0 \qquad \delta_2^{(0)} = 0.0 \qquad \delta_3^{(0)} = 0.0$$

2. Calculate $P_2^{(k)}$, $P_3^{(k)}$ and $Q_2^{(k)}$ by eq. (1) and (2) and then $\Delta P_2^{(k)}$, $\Delta P_3^{(k)}$, and $\Delta Q_2^{(k)}$, e.g.:

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$

$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

3. Use **B'** and **B"** to solve $\Delta|V_2^{(k)}|$, $\Delta\delta_2^{(k)}$ and $\Delta\delta_3^{(k)}$ and then $|V_2^{(k+1)}|$, $\delta_2^{(k+1)}$ and $\delta_3^{(k+1)}$, e.g.:

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix} \begin{bmatrix} \dfrac{-2.8600}{1.0} \\ \dfrac{1.4384}{1.04} \end{bmatrix} = \begin{bmatrix} -0.060483 \\ -0.008909 \end{bmatrix} \Rightarrow$$

$$\delta_2^{(1)} = 0 + (-0.060483) = -0.060483$$

$$\delta_3^{(1)} = 0 + (-0.008989) = -0.008989$$

$$\Delta|V_2^{(0)}| = -\left[\frac{-1}{52}\right]\left[\frac{-0.22}{1.0}\right] = -0.0042308$$

$$|V_2^{(1)}| = 1 + (-0.0042308) = 0.995769$$

4. Stop after **14** iterations: $V_2 = 0.97168\angle -2.696^\circ \qquad V_3 = 1.04\angle -0.4988^\circ$

5. Calculate:

$$Q_3 = 1.4617 \text{ pu}$$

$$P_1 = 2.1842 \text{ pu}$$

$$Q_1 = 1.4085 \text{ pu}$$