Nonlinear Modal Decoupling: a new approach to real-time power system stability analysis and control

Kai Sun
University of Tennessee

kaisun@utk.edu

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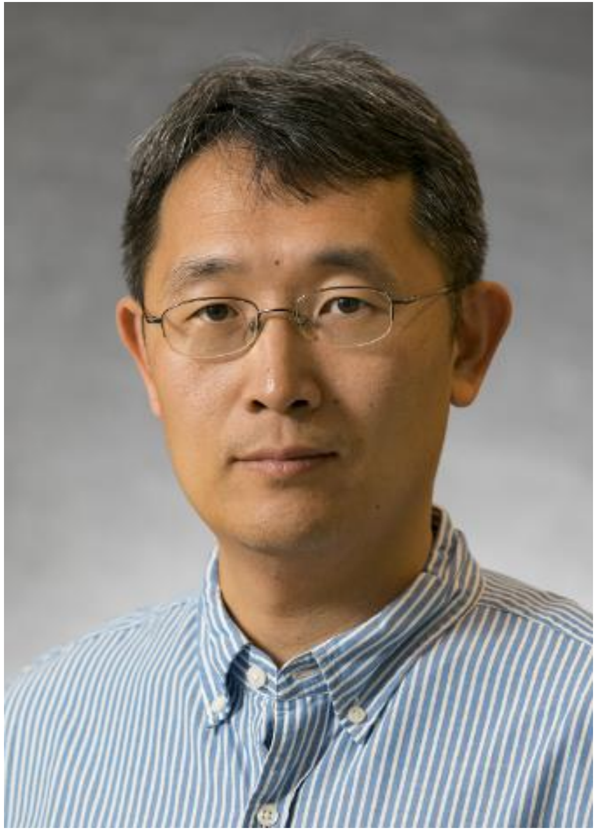
Presenter Bio

Kai Sun

Professor
Department of Electrical Engineering & Computer Science
University of Tennessee, Knoxville



Prof. Sun is received his BE in Automation and PhD in Control Science and Engineering from Tsinghua University, Beijing in 1999 and 2004. He joined the University of Tennessee in 2012. From 2007 to 2012, he was with EPRI, Palo Alto, CA as a project manager for R&D programs in grid operations, planning and renewable integration. Earlier, he was with Arizona State University, Tempe as a research associate. Dr. Sun has served in the editorial boards of *IEEE Transactions on Power Systems*, *IEEE Transactions on Smart Grid* and *IEEE Open Access Journal of Power and Energy.* He received EPRI Chauncey Award in 2009, NSF CAREER Award in 2016, CRSTT Most Valuable Players Award by NASPI in 2016.

Learning Objectives

- Understand nonlinearity in power system oscillation under a large disturbance.
- Learn how to decompose a power grid model regarding its oscillation modes by a Nonlinear Modal Decoupling method.
- Discuss use cases of the method for real-time power system stability monitoring and control.

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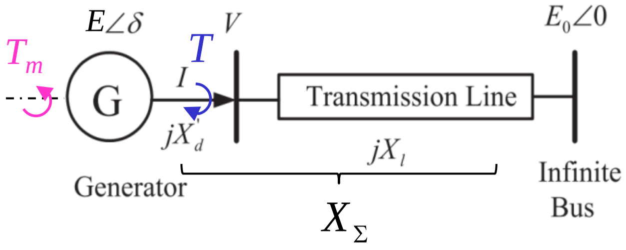
- Characterization of a nonlinear oscillation by its F-A curve
- Introduction of the Nonlinear Modal Decoupling (NMD) method for power grids and other multi-oscillator systems
- Use cases:
    1. Transient stability analysis and monitoring
    2. Real-time damping estimation on nonlinear oscillations
    3. Direct damping feedback control for grid stabilization
- Conclusions and takeaways

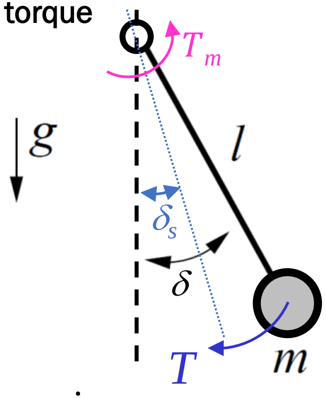
Two 1-DOF nonlinear oscillators

SMIB system



$$\frac{2H}{\omega_0}\cdot\ddot{\delta}+D\cdot\dot{\delta}+T_{\max}\cdot\sin\delta-T_m=0$$

Pendulum subject to a torque



$$ml^2\cdot\ddot{\delta}+D\cdot\dot{\delta}+mgl\cdot\sin\delta-T_m=0$$

$I=2H/\omega_0$

$T_{\max}=P_{\max}=EE_0/X_\Sigma$

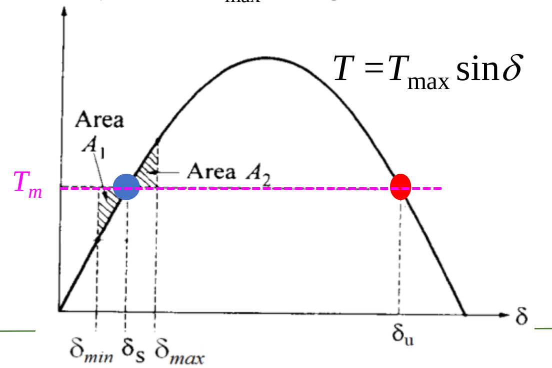
$$\ddot{\delta}+\frac{D}{I}\cdot\dot{\delta}+\frac{T_{\max}}{I}\cdot\sin\delta-\frac{T_m}{I}=0$$

$I=ml^2$

$T_{\max}=mgl$

Equilibria of δ in 360º:

- Stable equilibrium point (SEP): δ_s=arcsin(T_m/$T_{\max}$)
- Unstable equilibrium point (UEP): δ_u=180º-δ_s



Oscillations under small and large disturbances

$$\ddot{\delta}+\frac{D}{I}\cdot\dot{\delta}+\frac{T_{\max}}{I}\cdot\sin\delta-\frac{T_m}{I}=0$$

Let $\Delta\delta=\delta-\delta_s$

$$\Delta\ddot{\delta}+\frac{D}{I}\cdot\Delta\dot{\delta}+\frac{T_{\max}}{I}\cdot\operatorname{sinc}\left(\frac{\Delta\delta}{2}\right)\cos\left(\delta_s+\frac{\Delta\delta}{2}\right)\cdot\Delta\delta=0$$

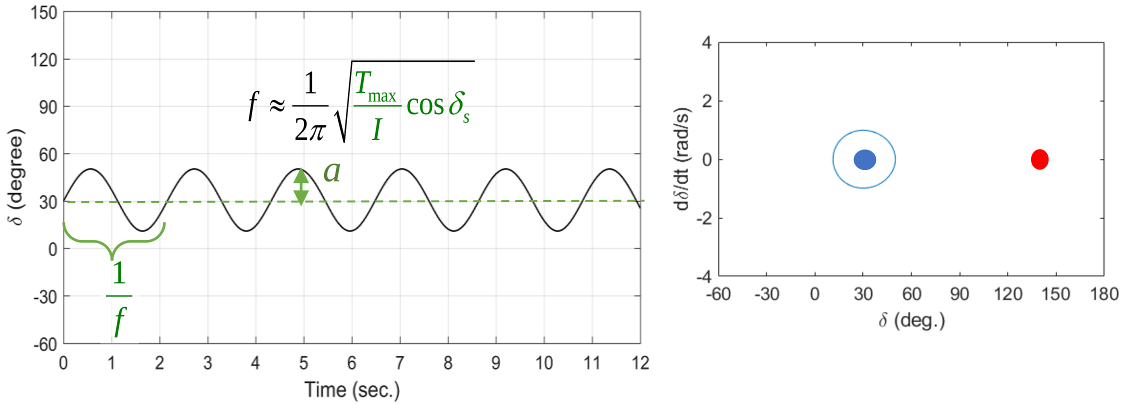
For $\Delta\delta\approx 0$ (small disturbance)

$$\Delta\ddot{\delta}+\frac{D}{I}\cdot\Delta\dot{\delta}+\frac{T_{\max}}{I}\cos\delta_s\cdot\Delta\delta\approx 0$$

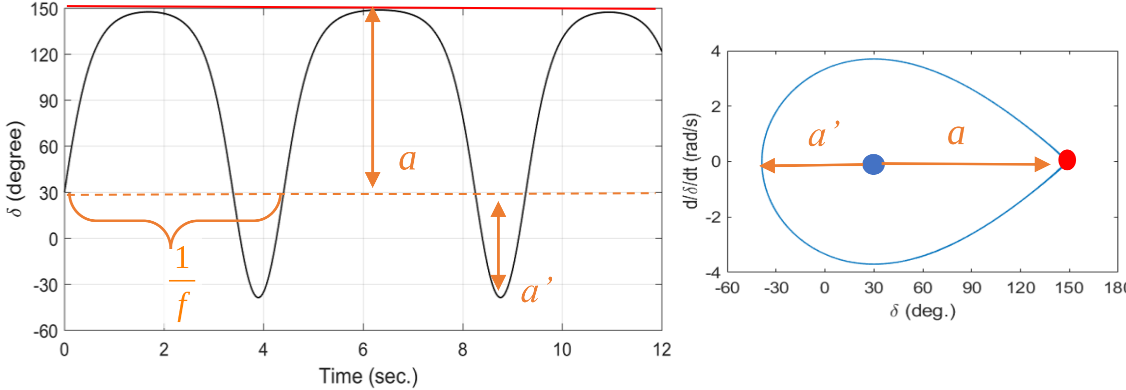
Example:

I=1, D=0, $T_{\max}$=10, T_m=5 (δ_s=30°)

1. Small disturbance: approximately harmonic



2. Large disturbance: marginally stable



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Frequency-Amplitude (F-A) Curve

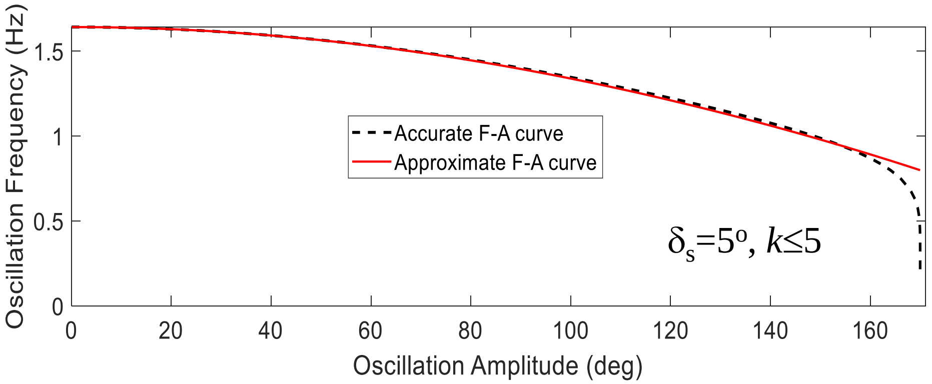
$$f = \frac{1}{t(0,a)+t(-a',0)}$$

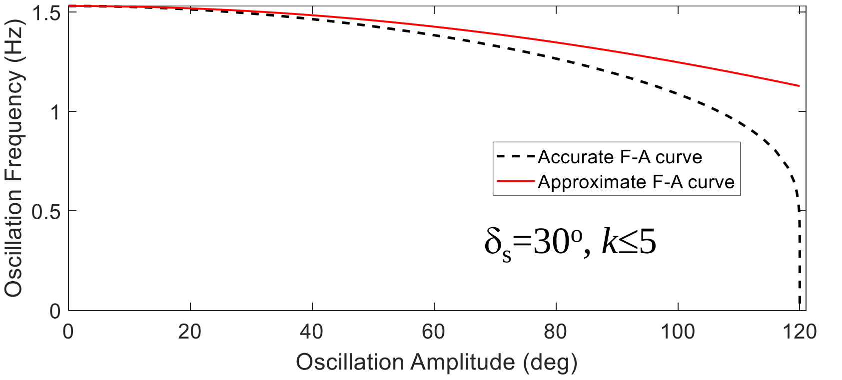
where $t(x,y) = \sqrt{\frac{2I}{T_{max}}} \cdot \int_x^y \frac{d\Delta\delta}{\sqrt{\cos(\delta_s+\Delta\delta)-\cos(\delta_s+x+y)+(\Delta\delta-x-y)\sin\delta_s}}$

Elliptic integral of the first kind

$$f \approx \frac{1}{2\pi}\sqrt{\frac{T_{max}}{I}\cos\delta_s} \cdot J(a)$$

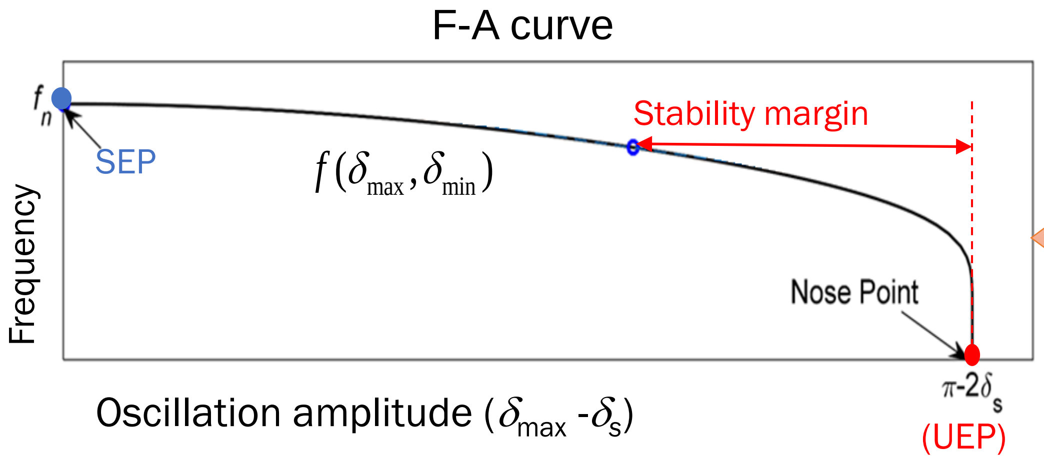
where Bessel function $J(a) = \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k}}{2^{2k} k!(k+1)!} = 1 - \frac{a^2}{8} + \frac{a^4}{192} - \frac{x^6}{9{,}216} + \frac{a^8}{737{,}280} - \frac{a^{10}}{8{,}847{,}360} + \cdots$

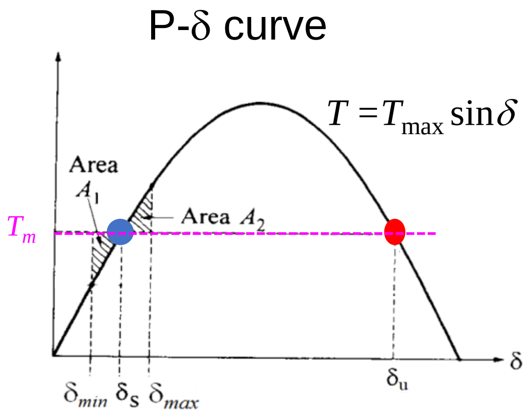




[1] B. Wang, K. Sun, "Formulation and Characterization of Power System Electromechanical Oscillations," IEEE Trans. Power Systems, 2016.
[2] M. Xiong, X. Xu, K. Sun, B. Wang, “Approximation of the Frequency-Amplitude Curve Using the Homotopy Analysis Method,” IEEE PESGM’21.

Frequency-Amplitude (F-A) Curve





- The F-A curve characterizes the **non-linear oscillation mode** of a 1-DOF oscillator while its **linear natural mode** defined by a pair of eigenvalues only corresponds to single point, i.e. SEP, on the curve.
- **Hypothesis**: For a multi-generator system, an F-A curve describes one of nonlinear oscillation modes in which generators keep oscillatory coherency before loss of synchronism,
- In the rest of the presentation, “a mode of X Hz” refers to a non-linear mode whose frequency at the SEP equals X Hz.

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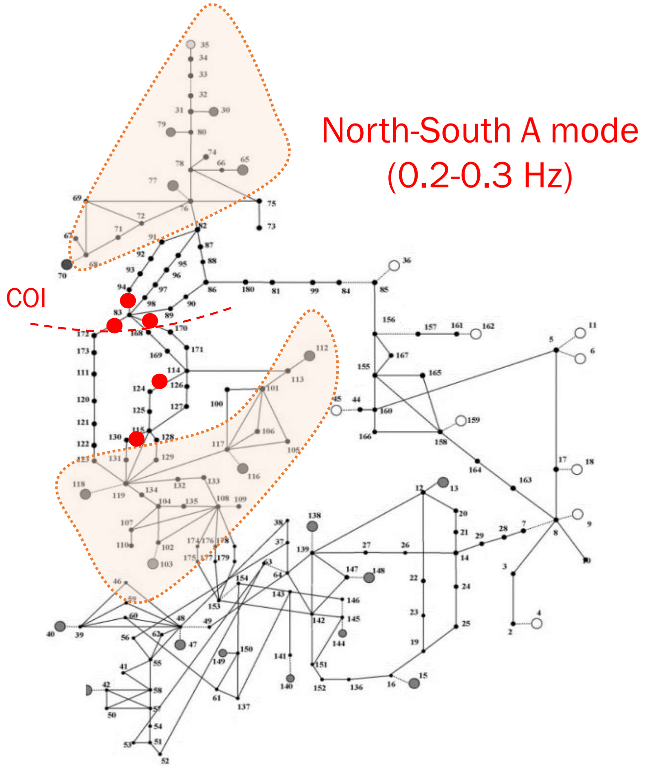
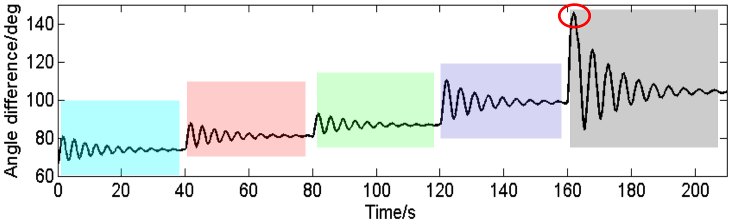
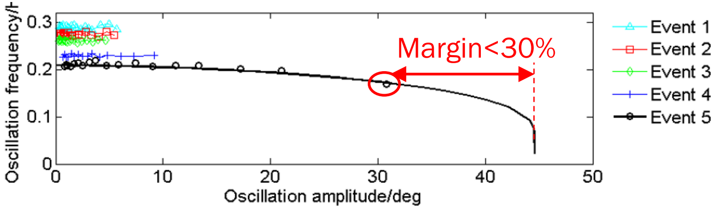
[3] B. Wang, X. Su, K. Sun, “Properties of the Frequency-Amplitude Curve,” IEEE Trans. Power Systems, 2017

F-A curves on WECC 29-machine 179-bus system

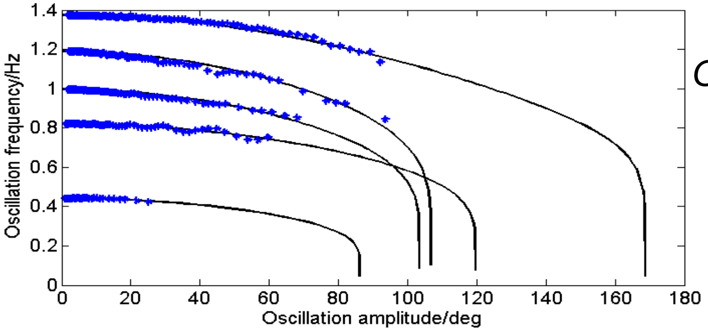
- Consider cascading line outages near the California-Oregon Intertie (COI).







Approximate F-A curves on other modes



Can we decouple these nonlinear modes for stability analysis and control?

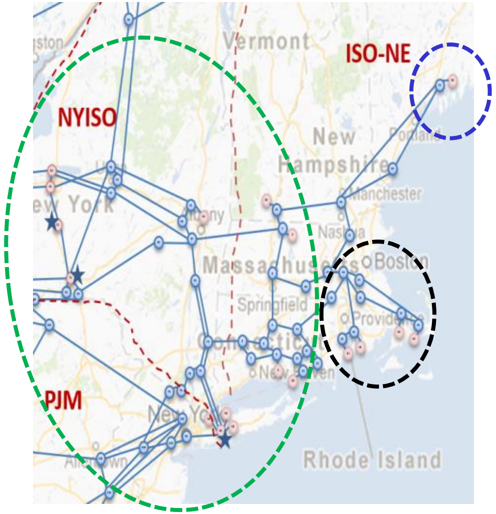
Modal decoupling: from linear to nonlinear oscillations

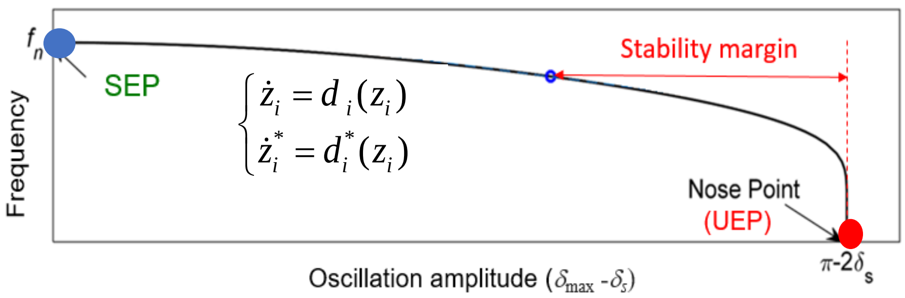
- Under a small disturbance, linear modes are decoupled by diagonalizing Jacobian **A** to analyze small-signal stability for each mode.

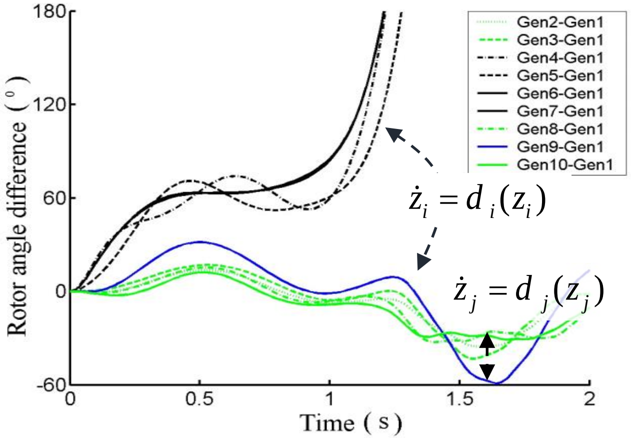
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad \overset{\mathbf{x}=\boldsymbol{\Phi}\mathbf{z}}{\Leftrightarrow} \quad \dot{\mathbf{z}} = \boldsymbol{\Lambda}\mathbf{z} \quad \text{with decoupled } \dot{z}_i = \lambda_i z_i \quad (z_i \in \mathbb{C})$$

- Under a large disturbance, can we decouple **non-linear** modes?

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad \overset{\mathbf{x}=\mathbf{H}(\mathbf{z})}{\Leftrightarrow} \quad \dot{\mathbf{z}} = \mathbf{d}(\mathbf{z}) \text{ with deoupled } \dot{z}_i = d_i(z_i) \quad (z_i \in \mathbb{C})$$







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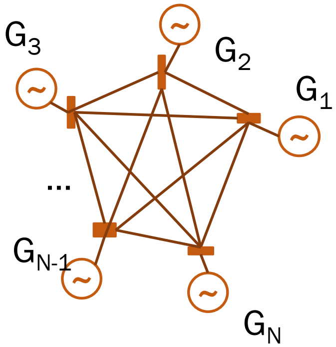
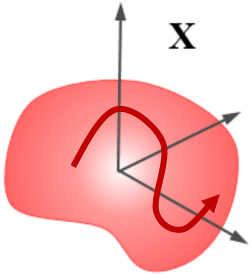
Nonlinear Modal Decoupling of a Multi-Oscillator System

N-oscillator system

Decoupled 1-DOF oscillators

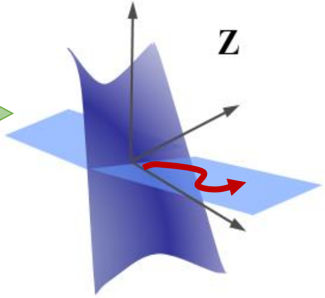


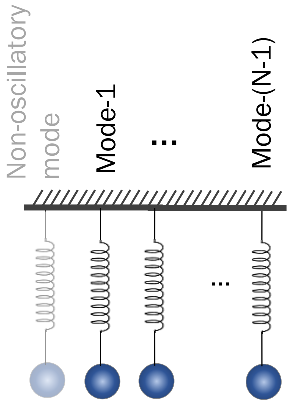
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) = \mathbf{\Lambda x} + \mathbf{f}^{(2)}(\mathbf{x}) + \ldots$$



$$\mathbf{x} = \mathbf{H}(\mathbf{z}) = \mathbf{z} + \mathbf{h}^{(2)}(\mathbf{z}) + \ldots$$

$$\dot{\mathbf{z}} = \mathbf{d}(\mathbf{z}) = \mathbf{\Lambda z} + \mathbf{d}^{(2)}(\mathbf{z}) + ..$$





- By a coordinate transformation **x=H(z)**, the goal is to decompose the system's high-dimensional vector field into low-dimensional vector fields on as many decoupled nonlinear 1-DOF oscillators as oscillation modes.

[4] B. Wang, K. Sun, W. Kang, "Nonlinear Modal Decoupling of Multi-Oscillator Systems with Applications to Power Systems," IEEE Access, 2018

Poincaré's Normal Form Theorem: formal linearization

Consider a smooth, nonlinear system having an equilibrium at $\mathbf{x}$=0. If eigenvalues of its Jacobian $\mathbf{A}$ are *non-resonant*, then the system can be transformed into a linear system by changing coordinates:

$$\mathbf{x} = \mathbf{z} + \mathbf{h}(\mathbf{z}) = \mathbf{z} + \mathbf{h}^{(2)}(\mathbf{z}) + \mathbf{h}^{(3)}(\mathbf{z}) + \ldots$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{f}^{(2)}(\mathbf{x}) + \mathbf{f}^{(3)}(\mathbf{x}) + \cdots \quad \longrightarrow \quad \dot{\mathbf{z}} = \mathbf{A}\mathbf{z}$$

$\mathbf{h}(\mathbf{z})$ can be obtained by solving a **homological equation**:

$$[\mathbf{A}\mathbf{z}, \mathbf{h}(\mathbf{z})] = \frac{\partial \mathbf{h}}{\partial \mathbf{z}}\mathbf{A}\mathbf{z} - \mathbf{A}\mathbf{h}(\mathbf{z}) = \mathbf{f}^{(2)}(\mathbf{z} + \mathbf{h}(\mathbf{z})) + \mathbf{f}^{(3)}(\mathbf{z} + \mathbf{h}(\mathbf{z})) + \ldots$$

If $\mathbf{f}^{(n)}(\mathbf{z}) \equiv 0$ for $\forall n<k$, there is:

$$[\mathbf{A}\mathbf{z}, \mathbf{h}^{(k)}(\mathbf{z})] = \boxed{\frac{\partial \mathbf{h}^{(k)}}{\partial \mathbf{z}}\mathbf{A}\mathbf{z} - \mathbf{A}\mathbf{h}^{(k)}(\mathbf{z}) = \mathbf{f}^{(k)}(\mathbf{z})}$$

Note: this equation is a **linear equation** about *k*-th order monomials of $\mathbf{z}$. Thus, to formally linearize the system up to the *K*-th order, we only need to solve such a linear equation for (*K*-1) times.

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[5] V.I. Arnold, Geometrical Methods in the Theory of Ordinary Differential Equations (2nd Ed), Springer, 1988

1-D example of formal linearization

$$\dot{y} = \lambda y + c_2 y^2 + c_3 y^3 + O(y^4)$$

Let $y = z + h(z) = z + h_2 z^2 + h_3 z^3 + O(z^4)$

$$\dot{z} + \frac{d}{dt} h(z) = \lambda z + \lambda h(z) + c_2 y^2 + c_3 y^3 + O(y^4)$$

$$\frac{\partial h}{\partial z} \lambda z - \lambda h(z) = c_2 y^2 + c_3 y^3 + O(y^4) \quad \textbf{(homological equation)}$$

$$(1 + 2h_2 z + 3h_3 z^2)\lambda z - \lambda(z + h_2 z^2 + h_3 z^3) = c_2(z^2 + 2h_2 z^3) + c_3 z^3 + O(y^4)$$

$$\dot{z} = \lambda z + O(z^4)$$

$$z^2: \quad 2\lambda h_2 - \lambda h_2 = c_2 \quad \Rightarrow \quad \boxed{h_2 = \frac{c_2}{\lambda}}$$

$$z^3: \quad 3\lambda h_3 - \lambda h_3 - 2c_2 h_2 = c_3 \quad \Rightarrow \quad \boxed{h_3 = \frac{c_3}{2\lambda} + h_2^2}$$

Nonlinear Modal Decoupling (NMD)

Theorem (NMD): If system (1) has no resonance at equilibrium **x**=0, and its eigenvalues belong to the Poincare domain, i.e. 0$\notin$ their convex hull (this is a sufficient condition), then these exists a convergent coordinate transformation to (2), which is decoupled up to the k^{th} order with desired modal nonlinearity.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{f}^{(2)}(\mathbf{x}) + \mathbf{f}^{(3)}(\mathbf{x}) + \cdots \quad (1)$$

x=Φy

$$\dot{\mathbf{y}} = \boldsymbol{\Lambda}\mathbf{y} + \mathbf{c}^{(2)}(\mathbf{y}) + \cdots + \mathbf{c}^{(k)}(\mathbf{y}) + \mathbf{c}^{(k+1)}(\mathbf{y}) + \cdots \quad (1')$$

y=z+h(z)

x=H(z)
=Φz+ Φh(z)

$$\dot{\mathbf{z}} = \mathbf{d}(\mathbf{z}) = \underbrace{\boldsymbol{\Lambda}\cdot\mathbf{z} + \mathbf{d}^{(2)}(\mathbf{z}) + \cdots + \mathbf{d}^{(k)}(\mathbf{z})}_{\text{decoupled } k\text{-jet}} + \mathbf{O}(\|\mathbf{z}\|^{k+1}) \quad (2)$$

$\mathbf{c}^{(k)}$: **coupling**, k^{th} order **inter-modal** terms (to be eliminated)

$\mathbf{d}^{(k)}$: **decoupled**, k^{th} order **intra-modal** terms (carrying designed modal nonlinearity if nonzero)

When transforming $\mathbf{c}^{(k)}$ terms, if $\mathbf{c}^{(n)}(\mathbf{z})\equiv\mathbf{d}^{(n)}(\mathbf{z})$ for $\forall n<k$, there is $\dfrac{\partial\mathbf{h}^{(k)}}{\partial\mathbf{z}}\boldsymbol{\Lambda}\mathbf{z} - \boldsymbol{\Lambda}\mathbf{h}^{(k)}(\mathbf{z}) = \mathbf{c}^{(k)}(\mathbf{z}) - \mathbf{d}^{(k)}(\mathbf{z})$

Solution: $h_{i,s_1\cdots s_k} = \dfrac{c_{i,s_1\cdots s_k} - d_{i,s_1\cdots s_k}}{\lambda_{s_1} + \cdots + \lambda_{s_k} - \lambda_i}$

If $i = s_1 = \cdots = s_k$, there is $h_{i,\underbrace{i\cdots i}_{k}} = \dfrac{c_{i,i\cdots i} - d_{i,i\cdots i}}{(k-1)\lambda_i}$ for any desired $d_{i,\underbrace{i\cdots i}_{k}}$

Otherwise ($\exists s_j \neq i$), all $d_{i,s_1\cdots s_k}$=0, i.e. $h_{i,s_1\cdots s_k} = \dfrac{c_{i,s_1\cdots s_k}}{\lambda_{s_1} + \cdots + \lambda_{s_k} - \lambda_i}$.

2-D example of nonlinear modal decoupling

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}f_{1,11}x_1^2+f_{1,12}x_1x_2+f_{1,22}x_2^2\\ f_{2,11}x_1^2+f_{2,12}x_1x_2+f_{2,22}x_2^2\end{bmatrix}+\mathbf{O}(\|\mathbf{x}\|^3)$$

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix}=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}c_{1,11}y_1^2+c_{1,12}y_1y_2+c_{1,22}y_2^2\\ c_{2,11}y_1^2+c_{2,12}y_1y_2+c_{2,22}y_2^2\end{bmatrix}+\mathbf{O}(\|\mathbf{y}\|^3)$$

Let $\mathbf{y}=\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\mathbf{z}+\mathbf{h}^{(2)}(\mathbf{z})=\begin{bmatrix}z_1\\ z_2\end{bmatrix}+\begin{bmatrix}h_{1,11}z_1^2+h_{1,12}z_1z_2+h_{1,22}z_2^2\\ h_{2,11}z_1^2+h_{2,12}z_1z_2+h_{2,22}z_2^2\end{bmatrix}$

$$\frac{\partial\mathbf{h}^{(2)}}{\partial\mathbf{z}}\boldsymbol{\Lambda}\mathbf{z}-\boldsymbol{\Lambda}\mathbf{h}^{(2)}(\mathbf{z})=\mathbf{c}^{(2)}(\mathbf{z})-\mathbf{d}^{(2)}(\mathbf{z})$$

$$\begin{bmatrix}2h_{1,11}z_1+h_{1,12}z_2 & h_{1,12}z_1+2h_{1,22}z_2\\ 2h_{2,11}z_1+h_{2,12}z_2 & h_{2,12}z_1+2h_{2,22}z_2\end{bmatrix}\times\begin{bmatrix}\lambda_1z_1\\ \lambda_2z_2\end{bmatrix}-\begin{bmatrix}\lambda_1(h_{1,11}z_1^2+h_{1,12}z_1z_2+h_{1,22}z_2^2)\\ \lambda_2(h_{2,11}z_1^2+h_{2,12}z_1z_2+h_{2,22}z_2^2)\end{bmatrix}=\begin{bmatrix}c_{1,11}z_1^2+c_{1,12}z_1z_2+c_{1,22}z_2^2\\ c_{2,11}z_1^2+c_{2,12}z_1z_2+c_{2,22}z_2^2\end{bmatrix}-\begin{bmatrix}d_{1,22}z_1^2\\ d_{2,22}z_2^2\end{bmatrix}$$

Solve $h_{i,jk}$ to cancel z_1z_2 terms:

$$h_{i,st}=\frac{c_{i,st}-d_{i,st}}{\lambda_s+\lambda_t-\lambda_i}$$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}\lambda_1z_1+d_{1,11}z_1^2\\ \lambda_2z_2+d_{2,22}z_2^2\end{bmatrix}+\mathbf{O}(\|\mathbf{z}\|^3)$$

$$h_{1,11}=\frac{c_{1,11}-d_{1,11}}{\lambda_1},\quad h_{1,12}=\frac{c_{1,12}}{\lambda_2},\quad h_{1,22}=\frac{c_{1,22}}{2\lambda_2-\lambda_1},$$

$$h_{2,11}=\frac{c_{2,11}}{2\lambda_1-\lambda_2},\quad h_{2,12}=\frac{c_{2,12}}{\lambda_1},\quad h_{2,22}=\frac{c_{2,22}-d_{2,22}}{\lambda_2}$$

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Real-valued decoupled k-jet

- How to transform a decoupled, **complex-valued** *k*-jet into an approximately **real-valued** *k*-jet?

1-DOF oscillator near **x**=0: $\mathbf{x}=\begin{bmatrix}\dot{\delta}\\ \delta\end{bmatrix}=\mathbf{\Phi z}=\begin{bmatrix}\lambda & \lambda^*\\ 1 & 1\end{bmatrix}\begin{bmatrix}z_1\\ z_2=z_1^*\end{bmatrix}$

For a multi-oscillator system, define: $\begin{bmatrix}w_i\\ w_{i+1}\end{bmatrix}=\begin{bmatrix}\lambda_i & \lambda_i^*\\ 1 & 1\end{bmatrix}\begin{bmatrix}z_i\\ z_i^*\end{bmatrix}\sim\begin{bmatrix}\dot{\delta}\\ \delta\end{bmatrix}$ speed / angle

$$\begin{cases}\dot{z}_i=\lambda_i z_i+d^{(2)}(z_i)+d^{(3)}(z_i)+\ldots\\ z_{i+1}=z_i^*\end{cases}$$

$$\begin{bmatrix}z_i\\ z_i^*\end{bmatrix}=\begin{bmatrix}\lambda_i & \lambda_i^*\\ 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}w_i\\ w_{i+1}\end{bmatrix}$$

$$\begin{cases}\dot{w}_i=g(w_i,w_{i+1})\\ \dot{w}_{i+1}\approx w_i\end{cases}$$

or

$$\ddot{w}_{i+1}-g(\dot{w}_{i+1},w_{i+1})\approx 0$$

Resonance of eigenvalues

- Distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ are said to be **resonant** if there exists a vector $\mathbf{m}=[m_1, \ldots, m_n]^T$ of **non-negative** integers such that the following equation holds. The **order of resonance** is defined as $\Sigma_i m_i$. If there is "≈" instead of "=", these eigenvalues have **near-resonance**.

$$\lambda_i = m_1\lambda_1 + m_2\lambda_2 + \ldots + m_n\lambda_n \quad \text{where } \Sigma_i m_i \geq 2$$

- **Examples (n=2):**

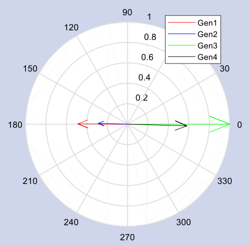
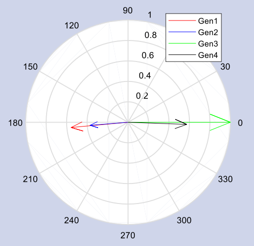
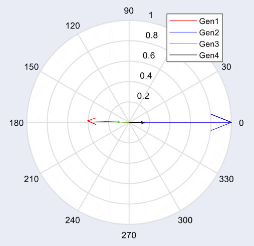
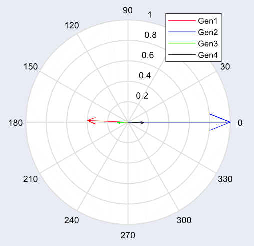
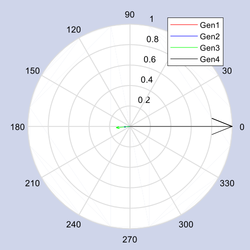
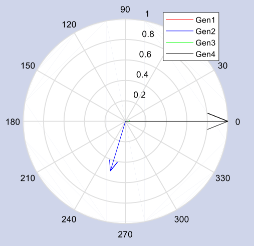
$\lambda_1 = 2\lambda_2$	a resonance of order 2;
$2\lambda_1 = 3\lambda_2$	not a resonance;
$\lambda_1 + \lambda_2 = 0$	a resonance of order 3 since $\lambda_1 = 2\lambda_1 + \lambda_2$.

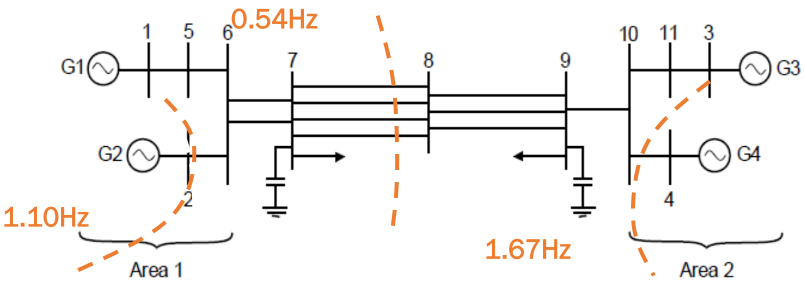
- **A necessary condition of normal form/NMD transformation:** in order to eliminate or transform $\mathbf{c}^{(k)}$ terms, the system cannot have kth order resonance.

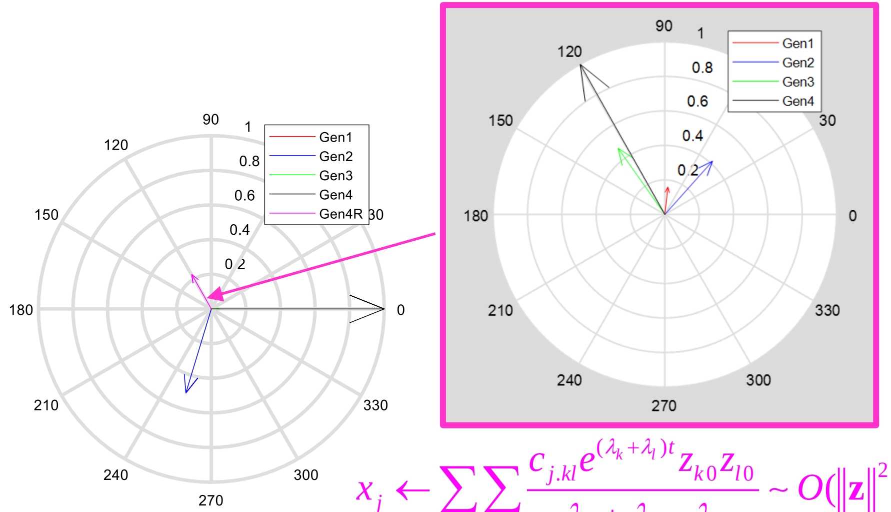
$$h_{i,s_1\cdots s_k} = \frac{c_{i,s_1\cdots s_k} - d_{i,s_1\cdots s_k}}{\lambda_{s_1} + \cdots + \lambda_{s_k} - \lambda_i}$$

[5] V.I. Arnold, Geometrical Methods in the Theory of Ordinary Differential Equations (2nd Ed), Springer, 1988

Mode shape on near-resonance oscillation

Mode (Hz)	From Linear Model	From Measurements
0.54		
1.10		
1.67		





$$x_j \leftarrow \sum_k \sum_l \frac{c_{j.kl} e^{(\lambda_k+\lambda_l)t} z_{k0} z_{l0}}{\lambda_k+\lambda_l-\lambda_j} \sim O(\|\mathbf{z}\|^2)$$

[6] T. Xia, Z. Yu, K. Sun, D. Shi, Z. Wang, "Extended Prony Analysis on Power System Oscillation Under a Near-Resonance Condition," IEEE PESGM, 2020

NMD with designed modal nonlinearity

How to design new $\mathbf{d}^{(k)}$ terms introduced by $\mathbf{h}$ for desired modal nonlinearity (e.g. the stability limit)?

- **Strategy 1:** Normal form, i.e. a **linear *k*-jet system** with $\mathbf{d}^{(2)}=\ldots=\mathbf{d}^{(k)}=0$.
  – Unable to estimate the stability limit from each decoupled linear system.
- **Strategy 2:** Let $h_{i,ii\ldots i}=0$, i.e. $d_{i,i\ldots i}=c_{i,i\ldots i}$ for $\forall n<k$ to avoid adding any new $\mathbf{d}^{(k)}$ terms.
  – The F-A curve on each decoupled oscillator only depends on remaining $\mathbf{c}^{(k)}$ terms.

$$\begin{bmatrix}\dot y_1\\ \dot y_2\end{bmatrix}=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}c_{1,11}y_1^2+c_{1,12}y_1y_2+c_{1,22}y_2^2\\ c_{2,11}y_1^2+c_{2,12}y_1y_2+c_{2,22}y_2^2\end{bmatrix}+\ldots+\mathbf{O}(\|\mathbf{y}\|^3)$$

$$h_{1,11}=\frac{c_{1,11}-d_{1,11}}{\lambda_1}=0,\quad h_{1,12}=\frac{c_{1,12}}{\lambda_2},\quad h_{1,22}=\frac{c_{1,22}}{2\lambda_2-\lambda_1},$$
$$h_{2,11}=\frac{c_{2,11}}{2\lambda_1-\lambda_2},\quad h_{2,12}=\frac{c_{2,12}}{\lambda_1},\quad h_{2,22}=\frac{c_{2,22}-d_{2,22}}{\lambda_2}=0$$

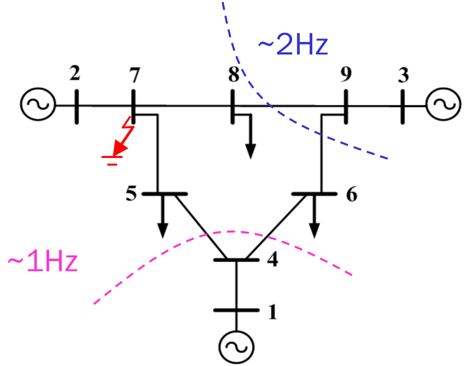
$$\begin{bmatrix}\dot z_1\\ \dot z_2\end{bmatrix}=\begin{bmatrix}\lambda_1 z_1+c_{1,11}z_1^2\\ \lambda_2 z_2+c_{1,11}z_2^2\end{bmatrix}+\mathbf{O}(\|\mathbf{z}\|^3)\quad\Rightarrow\quad \begin{aligned}\dot w_1&=g(w_1,w_2)\\ \dot w_2&\approx w_1\end{aligned}$$

- **Strategy 3:** each decoupled system is a *k*-jet **SMIB system**.

$$\begin{cases}\dot z_i=\lambda_i z_i+d^{(2)}(z_i)+\ldots+d^{(k)}(z_i)\\ z_{i+1}=z_i^*\end{cases}\quad\Rightarrow\quad\begin{cases}\dot w_i=-\dfrac{D}{I}\cdot w_i-\dfrac{T_{\max}}{I}\sin^{(k)}(w_{i+1})+\dfrac{T_m}{I}\\ \dot w_{i+1}\approx w_i\end{cases}$$

  – New $\mathbf{d}^{(k)}$ terms change the shape of the F-A curve on each mode.
  – The stability of each decoupled system can be analyzed and controlled as an SMIB system.

Example: NMD on IEEE 9-bus system



- Consider a 3-phase fault (CCT=0.17s)
- Decouple its 3-jet system (CCT=0.16s) by 3 strategies:

Real valued 3-jet of the post-fault system:

$$\begin{cases} \dot{x}_1 = x_2 + 3.12 \\ \dot{x}_2 = -14.1x_1 - 0.5x_2 + 5.42x_3 + 8.85x_5 - 2.99x_1^2 - 1.65x_3^2 - 1.34x_5^2 + 3.31x_1x_3 + 2.68x_1x_5 \\ \qquad + 2.38x_1^3 - 0.904x_3^3 - 1.48x_5^3 - 2.71x_1^2x_3 + 2.71x_1x_3^2 - 4.43x_1^2x_5 + 4.43x_1x_5^2 \\ \dot{x}_3 = x_4 + 3.12 \\ \dot{x}_4 = 14.4x_1 - 49.5x_3 - 0.5x_4 + 35.1x_5 + 9.26x_1^2 + 17.1x_3^2 + 7.89x_5^2 - 18.5x_1x_3 - 15.8x_3x_5 \\ \qquad - 2.40x_1^3 + 8.25x_3^3 - 5.84x_5^3 + 7.21x_1^2x_3 - 7.21x_1x_3^2 - 17.5x_3^2x_5 + 17.5x_3x_5^2 \\ \dot{x}_5 = x_6 + 3.12 \\ \dot{x}_6 = 58.5x_1 + 81.3x_3 - 140.0x_5 - 0.5x_6 + 21.5x_1^2 - 4.37x_3^2 + 17.1x_5^2 - 43.0x_1x_5 + 8.73x_3x_5 \\ \qquad - 9.76x_1^3 - 13.5x_3^3 + 23.3x_5^3 + 29.3x_1^2x_5 - 29.3x_1x_5^2 + 40.6x_3^2x_5 - 40.6x_3x_5^2 \end{cases}$$

Strategy 1 (NF)

$$\begin{cases} \dot{z}_1 = (-0.25 + j12.9)z_1 \\ z_3 = z_1^* \end{cases} \quad \sim 2\text{Hz}$$

$$\begin{cases} \dot{z}_2 = (-0.25 + j6.08)z_2 \\ z_4 = z_2^* \end{cases} \quad \sim 1\text{Hz}$$

Strategy 2 (ND-ST)

$$\begin{cases} \dot{z}_1 = (-0.25 + j12.9)z_1 - (0.0019 + j0.0975)z_1^2 + (0.0038 - j0.195)z_1z_3 \\ \qquad + (0.0057 - j0.097)z_3^2 - (0.0101 + j0.262)z_1^3 + (0.00013 - j1.01)z_1^2z_3 \\ \qquad + (0.0389 - j1.01)z_1z_3^2 + (0.02 - j0.262)z_3^3 \\ z_3 = z_1^* \end{cases}$$

$$\begin{cases} \dot{z}_2 = (-0.25 + j6.08)z_2 - (0.023 + j0.57)z_2^2 + (0.047 - j1.14)z_2z_4 \\ \qquad + (0.07 - j0.566)z_4^2 - (0.009 + j0.093)z_2^3 - (0.002 + j0.29)z_2^2z_4 \\ \qquad + (0.025 - j0.289)z_2z_4^2 + (0.017 - j0.092)z_4^3 \\ z_4 = z_3^* \end{cases}$$

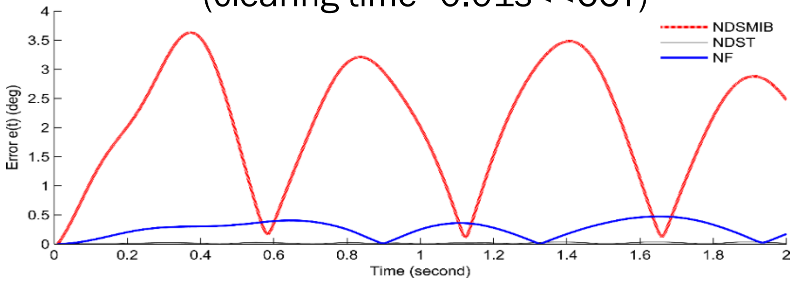
Strategy 3 (ND-SMIB)

$$\begin{cases} \dot{z}_1 = (-0.25 + j12.9)z_1 - j1.42z_1^2 - j2.83z_1z_3 - j1.42z_3^2 \\ \qquad - j1.08z_1^3 - j3.23z_1^2z_3 - j3.23z_1z_3^2 - j1.08z_3^3 \\ z_3 = z_1^* \end{cases}$$

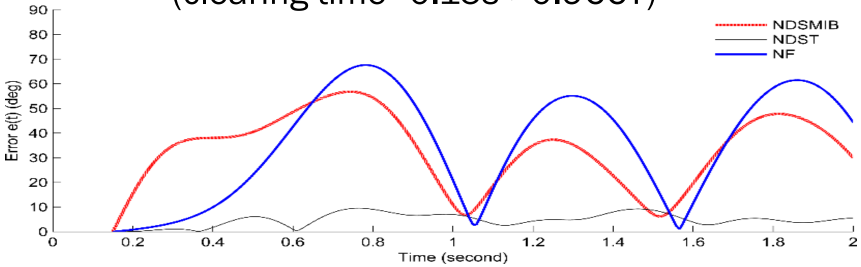
$$\begin{cases} \dot{z}_2 = (-0.25 + j6.08)z_2 - j0.86z_2^2 - j1.72z_2z_4 - j0.86z_4^2 \\ \qquad - j0.51z_2^3 - j1.52z_2z_4^2 - j1.52z_2^2z_4 - j0.51z_4^3 \\ z_4 = z_2^* \end{cases}$$

Accuracy on transient stability

Small disturbance
(clearing time=0.01s<<CCT)

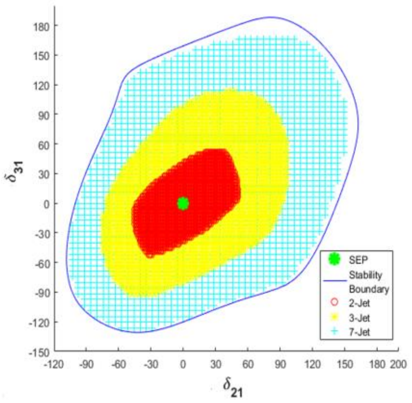


Large disturbance
(clearing time=0.15s >0.9CCT)

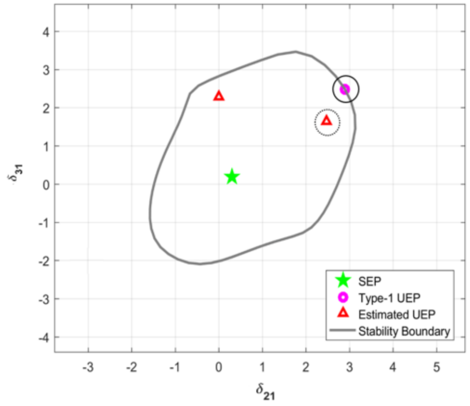


- For small disturbances, all strategies give accurate decoupled 3-jets.
- For a large disturbance (90% of CCT), strategy 2 is much more accurate than Strategy 3 (decoupled into SMIBs).

Error<5 deg.



UEPs by Strategy 2 (ND-ST)



[7] X. Xu, B. Wang, K. Sun, "Approximation of Closest Unstable Equilibrium Points via Nonlinear Modal Decoupling," IEEE PESGM, 2019.

[8] X. Xu, B. Wang, K. Sun, "Initial Study of the Power System Stability Boundary Estimated from Nonlinear Modal Decoupling," IEEE PES Powertech'19, Milano.

Transient stability analysis using decoupled oscillators

Two decoupled real-valued 3-jets:

$$\begin{cases}\dot{w}_1 = -166.0w_2 + 5.0w_2^2 + 32.7w_2^3 \\ \dot{w}_2 = w_1\end{cases} \qquad \begin{cases}\dot{w}_3 = -37.1w_4 + 13.8w_4^2 + 4.62w_4^3 \\ \dot{w}_4 = w_3\end{cases}$$

w_1, w_3 ~ speed ω

w_2, w_4 ~ angle δ

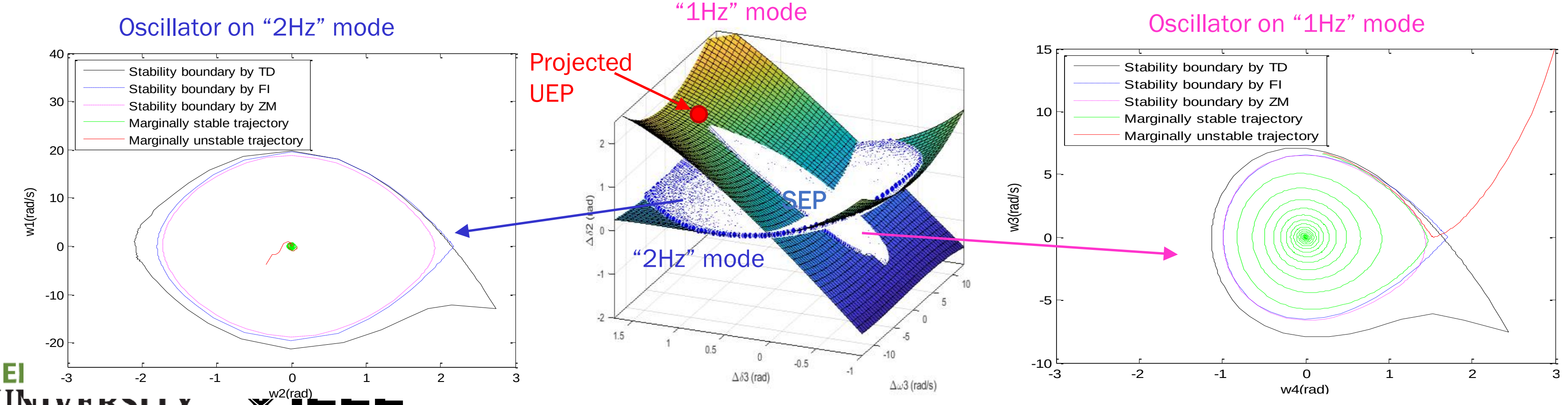
First-Integral based Lyapunov functions:

$$V_1(w_2, \dot{w}_2) = \frac{\dot{w}_2^2}{2} - 83w_2^2 + 1.6667w_2^3 + 8.175w_2^4 \qquad V_2(w_4, \dot{w}_4) = \frac{\dot{w}_4^2}{2} - 18.55w_4^2 + 4.6w_4^3 + 1.155w_4^4$$

Closest UEPs: $w_{2,\mathrm{UEP}} = 2.181$, $w_{4,\mathrm{UEP}} = 1.711$.

Critical energies: $V_{cr1}(0, w_{2,\mathrm{UEP}}) = 193.671$, $V_{cr2}(0, w_{4,\mathrm{UEP}}) = \mathbf{21.402}$.

Estimated CCT = 0.16 s (when V_{cr2} is reached) ≈ 0.17s (true CCT)



NMD-based transient stability analysis for a large power system

- **Step 1:** Derive modal space representation for the classical system model:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \qquad \dot{\mathbf{y}} = \mathbf{\Phi}^{-1}\mathbf{f}(\mathbf{\Phi}\mathbf{y}) \triangleq \tilde{\mathbf{f}}(\mathbf{y}), \quad \text{where } \mathbf{y} = \mathbf{\Phi}^{-1}\mathbf{x}$$

- **Step 2:** Obtain decoupled *k*-jet (*k*≥3) systems for selected *m* critical modes

$$\begin{pmatrix} \dot{\mathbf{y}}_{cr} \\ \dot{\mathbf{y}}_{other} \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{f}}_{cr}(\mathbf{y}_{cr}, \mathbf{y}_{other}) \\ \tilde{\mathbf{f}}_{other}(\mathbf{y}_{cr}, \mathbf{y}_{other}) \end{pmatrix} \Rightarrow \dot{\mathbf{y}}_{cr} = \tilde{\mathbf{f}}_{cr}(\mathbf{y}_{cr}, \mathbf{0}) \qquad \Rightarrow \dot{\mathbf{z}} = \mathbf{\Lambda}\cdot\mathbf{z} + \mathbf{d}^{(2)}(\mathbf{z}) + \mathbf{d}^{(3)}(\mathbf{z}) + \cdots$$

- **Step 3:** Find the stability boundary for each decoupled system by a direct method. Considering reduction and truncation errors, adjust the boundaries for conservative stability assessment.

- **Step 4:** Compare the post-disturbance system trajectory in **z** or **w**-coordinates with the stability boundaries regarding *m* critical modes

Remarks: If the system has near-resonance modal interaction, elimination of some inter-modal terms $\mathbf{c}^{(k)}$ becomes difficult due to large *h*-coefficients. However, NMD can still decouple non-resonant modes.

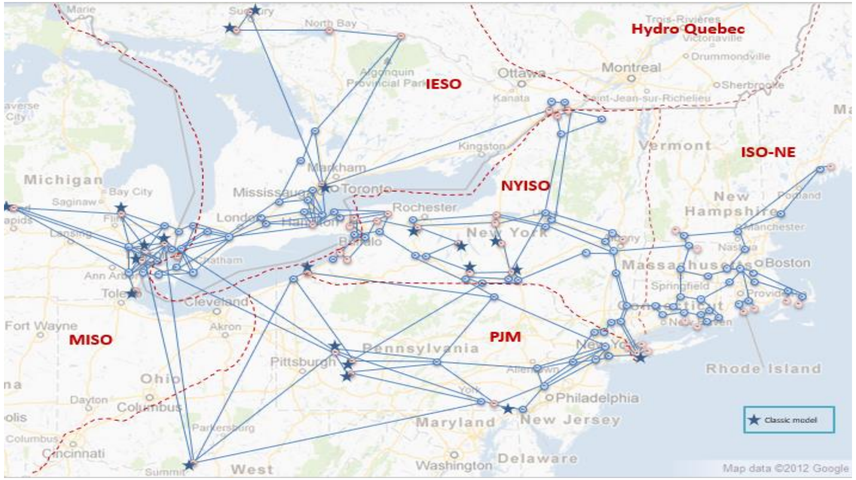
IEEE PES UNIVERSITY IEEE

[9] B. Wang, K. Sun, X. Xu, "Nonlinear Modal Decoupling Based Power System Transient Stability Analysis," IEEE Trans. Power Systems, 2019.

Use Cases of the NMD Method

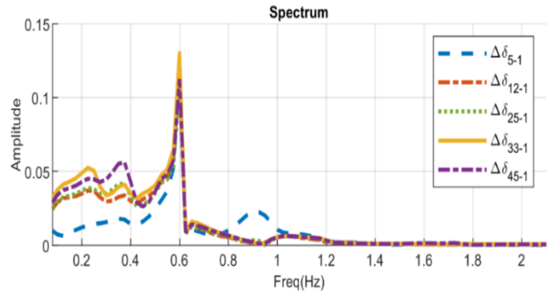
Demonstrations on the NPCC 48-machine 140-bus testbed system

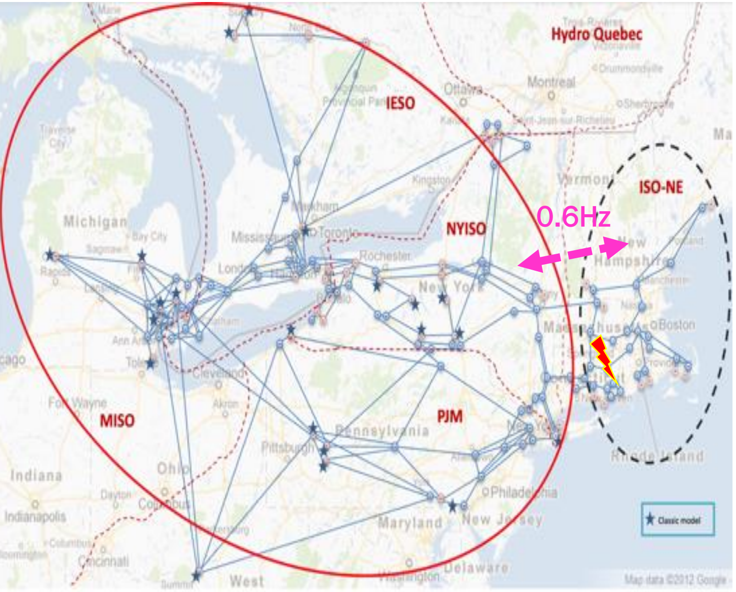
1. Transient stability analysis and monitoring
2. Real-time oscillation damping estimation under small/large disturbances
3. Direct damping feedback control for grid stabilization

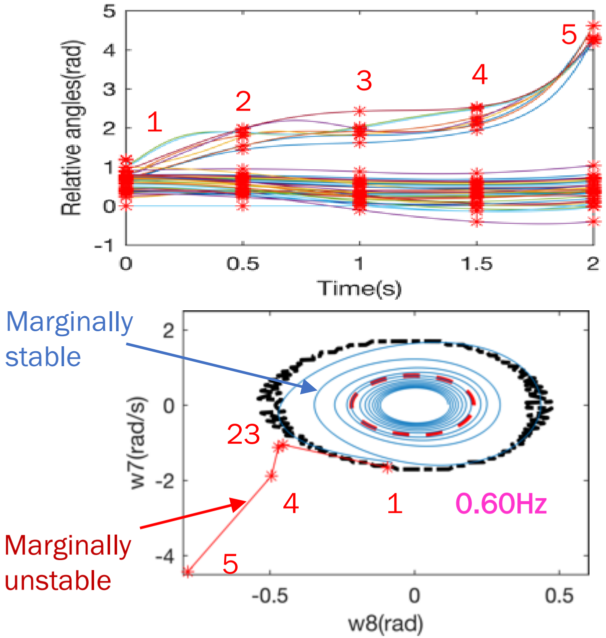


Case 1 - Transient stability monitoring

- Consider a large fault between NYISO and ISO-NE regions.
- **Offline:** For the post-fault system, find decoupled 3-jet oscillators on top-5 modes (>99% total oscillatory energy) and their stability boundaries.
- **Real-time:** Monitor 5 oscillators for transient stability. In the test, a MATLAB program takes 0.3s to locate 5 consecutive post-fault states of decoupled oscillators and only <0.1s to predict instability on 0.6Hz mode.



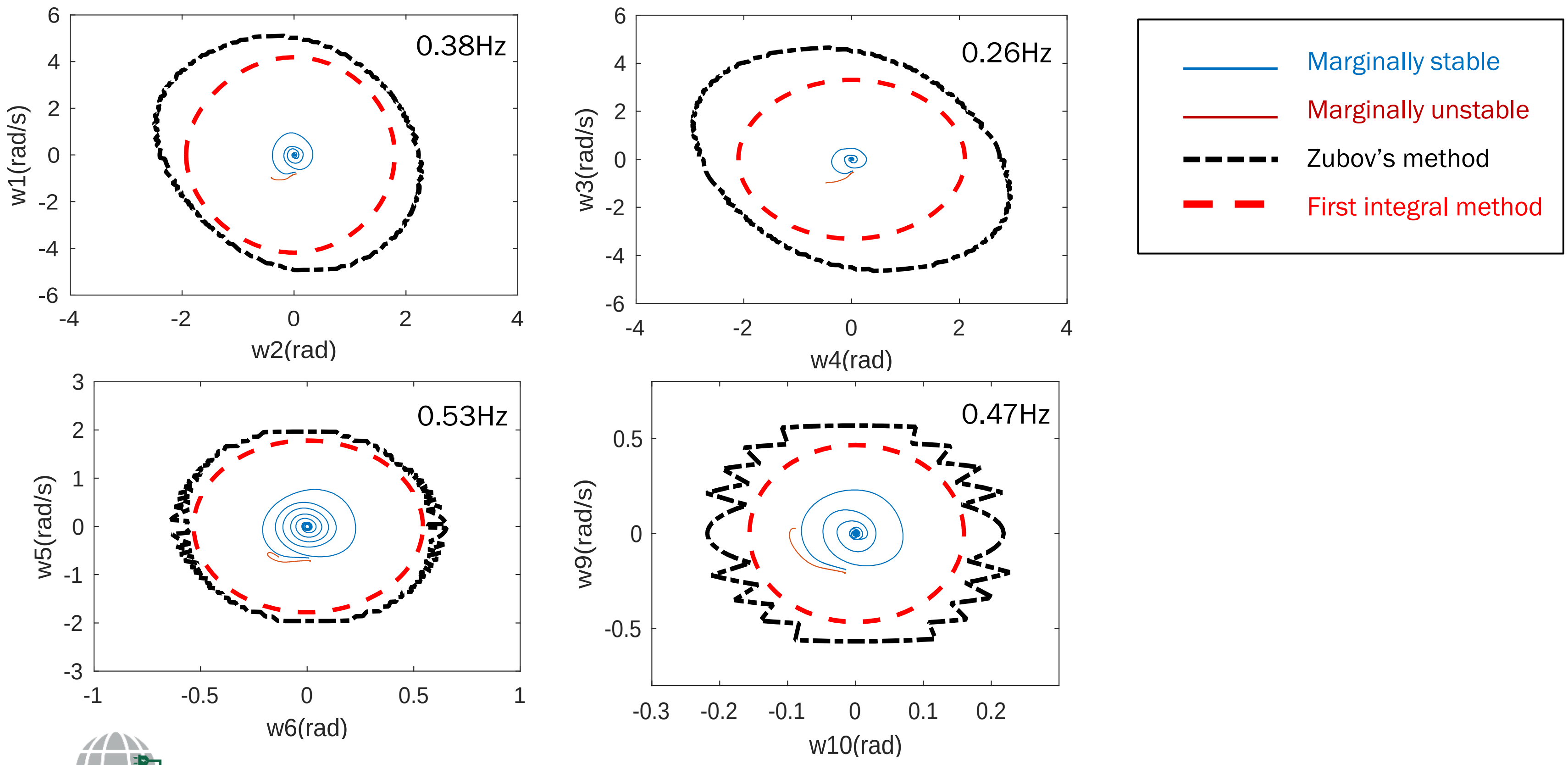




Mode No.	Freq. (Hz)	Modal Energy
1	0.38	54.6%
2	0.26	27.9%
3	0.53	9.3%
4	0.60	6.6%
5	0.47	1.1%
…	…	<1%

[9] B. Wang, K. Sun, X. Xu, "Nonlinear Modal Decoupling Based Power System Transient Stability Analysis," IEEE Trans. Power Systems, 2019.

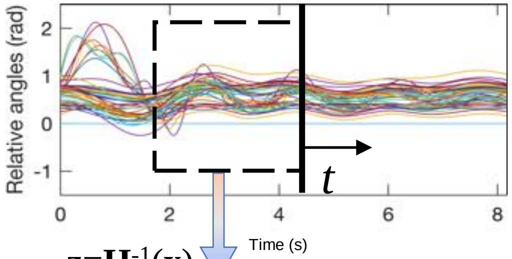
Other four oscillators are all stable



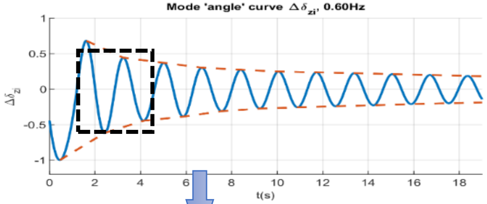
Case 2 - Real-Time Damping Estimation for Nonlinear Oscillations

Steps:

1. Apply inverse NMD transformation $\mathbf{z}=\mathbf{H}^{-1}(\mathbf{x})$ to measurement data of $\mathbf{x}$ over a sliding window.
2. Find a nonlinear oscillator of the best fit for data of the targeted mode, and calculate its damping ratio.



$\mathbf{z}=\mathbf{H}^{-1}(\mathbf{x})$



Optimal fitting at time t:

$$\ddot{\delta}+2\sigma\cdot\dot{\delta}+g_1\delta+g_2\delta^2...+g_k\delta^k=0$$

Damping ratio

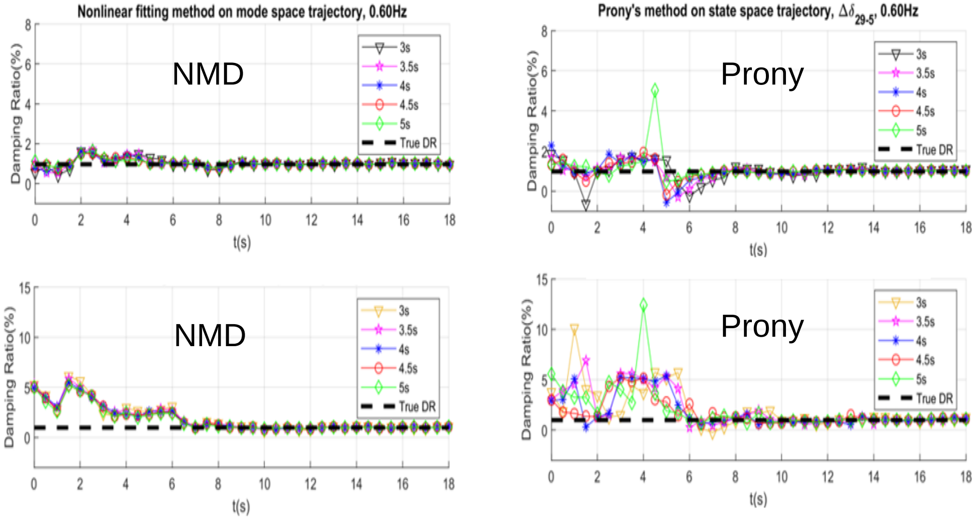
$$\varsigma(t)=\frac{\sigma}{\sqrt{g_1}}$$

Comparing damping ratios from short sliding windows (3-5 s)

NPCC 0.6Hz inter-area mode under small/large disturbances

Fault duration: CCT/4

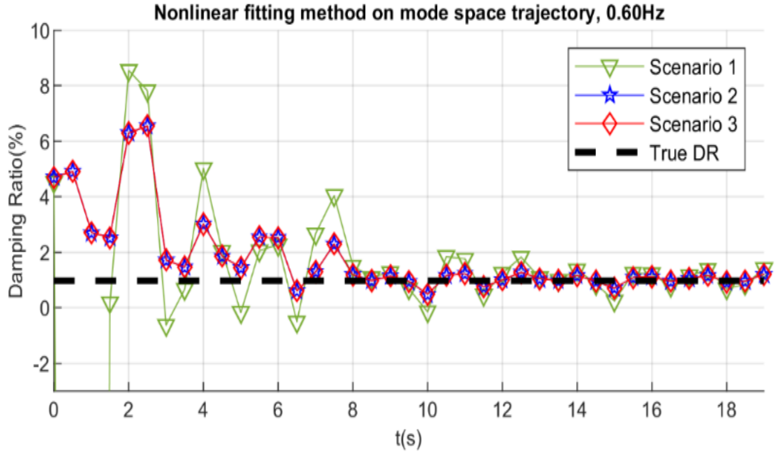
CCT



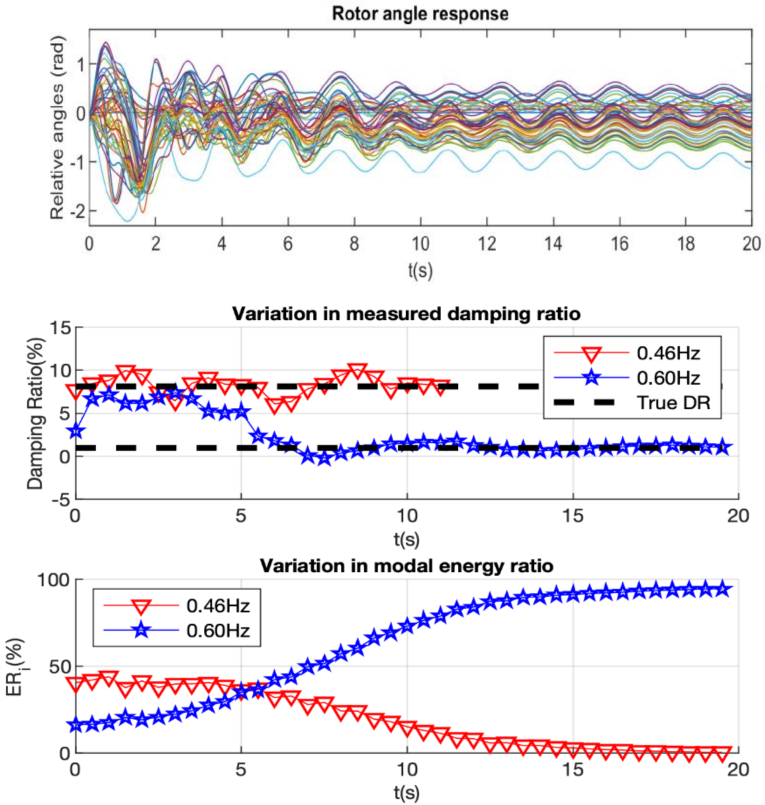
[10] X. Xu, W. Ju, B. Wang, K. Sun, "Real-time Damping Estimation on Nonlinear Electromechanical Oscillation," IEEE Trans. on Power Systems, 2021.

More tests on practical scenarios

- Decrease PMU coverage from 100% to 85%:

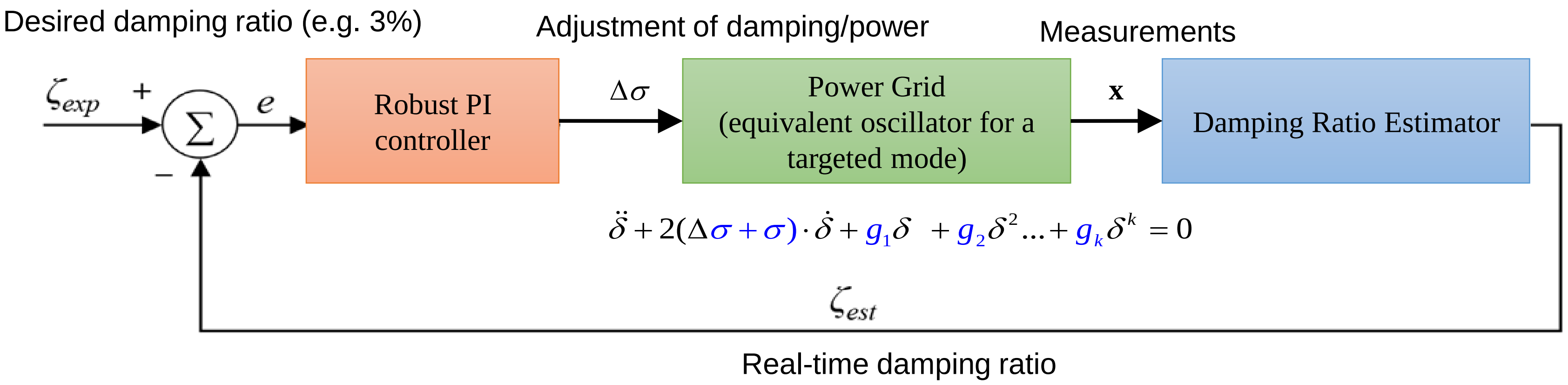


- Co-existence of two dominant modes



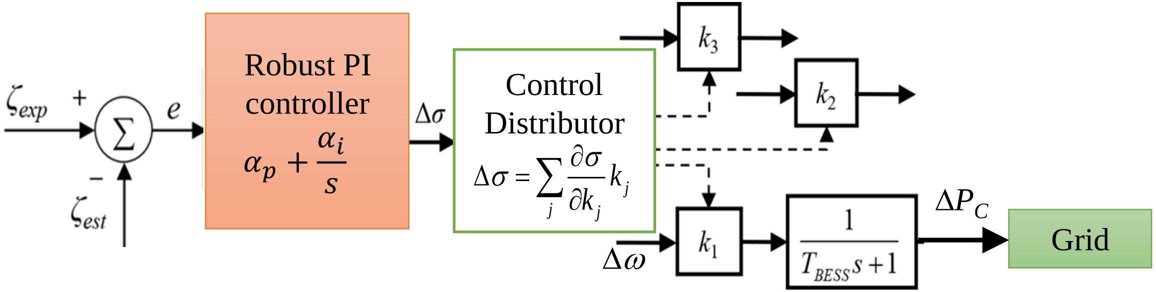
Case 3 - Direct Damping Feedback Control and Grid Stabilization

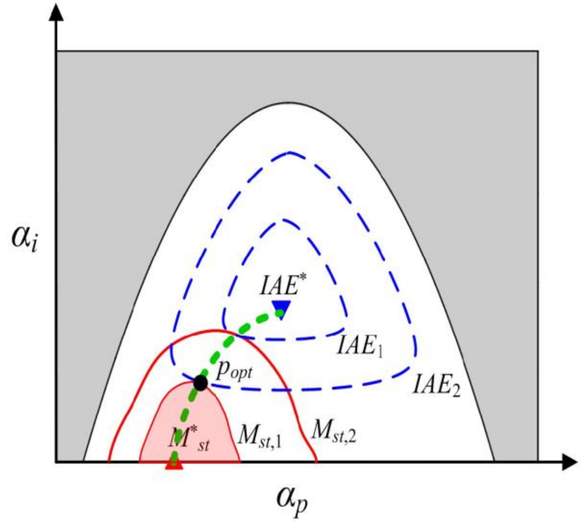
- An oscillating power grid can be stabilized if the damping ratio of each nonlinear mode is accurately controlled at a desired value.
- **Direct Damping Feedback Control System**:
  - NMD-based real-time damping ratio estimator
  - Robust PI controller optimized for each equivalent oscillator of the grid
  - Distributed/decentralized actuators using energy storages and other IBRs (inverter-based resources).



Distributed and Robust Damping Control by IBRs

- **Actuation:** Change power injections from distributed IBRs (e.g. battery energy storage systems) to the grid.





- **Robust Control:** Design PI parameters for the best tradeoff between the integrated absolute control error (IAE) and the sensitivity (M_{st}) of damping estimation to a perturbation in the system condition.

$$M_{st} = \max_{\omega}(|S(j\omega)|, |T(j\omega)|), \qquad IAE = \int_0^{\infty} |e(t)|\, dt$$

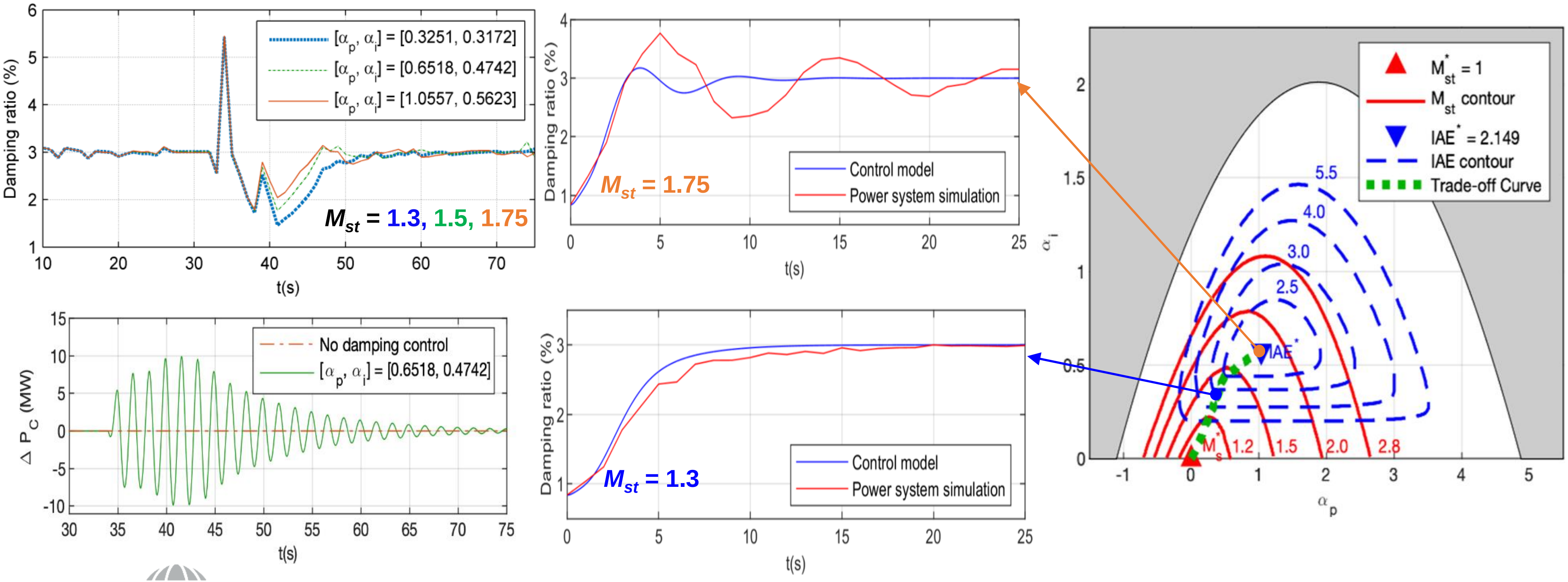
Fig. 5. Trade-off curve and contours of M_{st} and IAE.

[11] X. Xu, K. Sun, "Direct Damping Feedback Control Using Power Electronics-Interfaced Resources," IEEE Trans. On Power Systems, 2022.

[12] Y. Zhu, C. Liu, K. Sun, D. Shi, Z. Wang, "Optimization of Battery Energy Storage to Improve Power System Oscillation Damping," IEEE Trans. Sustainable Energy, 2019

Tests on Stabilization of the NPCC system

- Increase damping ratio from <1% to 3% for 0.6Hz mode after a permanent 3-phase fault lasting for CCT.



Summary

Contributions of the presented work:

- Characterization of nonlinear power system oscillation using a new tool "F-A Curve".
- Establishment of the Nonlinear Modal Decoupling (NMD) method for power grids and other multi-oscillator nonlinear systems.
- Demonstration of three use cases of the NMD method for stability analysis and control.
    1. Transient stability analysis and monitoring;
    2. Real-time oscillation damping estimation under small/large disturbances;
    3. Direct damping feedback control for grid stabilization using IBRs.

Takeaways:

- Small-signal and transient instabilities will be less separate with future oscillation events.
- Future grid controllers need to consider nonlinearities in power system oscillations.
- The NMD method suggests a new approach for decoupling and harnessing various power system oscillations having nonlinearities.

Acknowledgement

- The content of this presentation is based on these PhD dissertations:
  - Bin Wang, “Nonlinear Oscillation Analysis and Modal Decoupling for Power Systems,” University of Tennessee, 2017
  - Yongli Zhu, ”Control and Placement of Battery Energy Storage Systems for Power System Oscillation Damping,” University of Tennessee, 2018
  - Xin Xu, “Stability Analysis and Control of Nonlinear Power System Oscillations, University of Tennessee, 2020

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8. X. Xu, B. Wang, K. Sun, “Initial Study of the Power System Stability Boundary Estimated from Nonlinear Modal Decoupling,” *IEEE PES Powertech*, June 23-27, 2019, Milano, Italy.
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QUESTIONS?