Risk Assessment Using Boundary Load Flow Solutions

Aleksandar Dimitrovski and Kevin Tomsovic

***Abstract* — Risk is the central issue in planning under uncertainty and the load flow is central tool in power system planning. The uncertainty considered here is of non-statistical nature and best modeled using fuzzy set theory. Two methods are generally used for obtaining solutions from load flow when this type of uncertainty is involved. The first one, recently developed, finds the boundary load flow solutions and produces accurate results. The second one is the linearized fuzzy load flow, which gives only approximate results. The fuzzy results obtained are aggregated in a form of system inadequacy indices that are further used to express risk. A comparison between the two methods is made.**

***Index Terms* — Exposure, fuzzy sets, load flow analysis, power system planning, risk assessment, robustness, uncertainty.**

I. Introduction

Uncertainty is one of the most important issues in power system planning when decisions are made regarding the future system expansion and operation. Naturally, if decisions involve uncertainty there is always a financial risk. This fact has become of even greater importance recently within the restructured environment, which brings yet new uncertainties and risks.

In power system planning, the load flow program is the most fundamental and most heavily used analytical tool. In trying to include uncertainty into the solution process, analysts have tried different approaches. Most frequently, planners repeat the analysis under varying system conditions. Still, the need for a different approach has been long recognized. A better solution would be to provide solutions over the range of the uncertainties included, i.e. solutions that are sets of values instead of single points. To date, two families of such load flow algorithms have evolved.

The first, is the so-called probabilistic load flow (PLF) (for example, [1], [2]). PLF considers load and generation as random variables with appropriate probability distributions. The results of the load flow, i.e., voltages, power flows, and so on, are also random variables with the resultant probability distributions obtained using probabilistic techniques. Because of the complexity introduced by using random variables, PLF solutions are obtained using a linearized model and the results are rough approximations.

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The second family of load flow algorithms incorporating uncertainty that has been developed more recently utilizes fuzzy sets for modeling, (for example, [3], [4]). This is a qualitatively different way of expressing uncertainty. It represents imprecise, or vague, knowledge, rather than uncertainty related to a frequency of occurrence. One inherent advantage of this approach is the ability to easily incorporate expert knowledge about the system under study. With this approach input variables are represented as fuzzy numbers (FNs), which are special types of fuzzy sets. When the uncertainty of input variables is of a non-statistical nature (which the authors believe is the most usual case), FNs best represent the inputs.

FNs are defined by membership functions, also known as ‘possibility distributions’. Usually, for the sake of simplicity, trapezoidal membership functions like the one shown in Fig. 1 are assumed. A FN may also be considered as nested intervals with an α−degree of possibility, $0 \le \alpha \le 1$. Each α corresponds to some ordinary interval number. From this viewpoint, interval numbers and interval mathematics are simply a special case of fuzzy numbers and fuzzy mathematics.

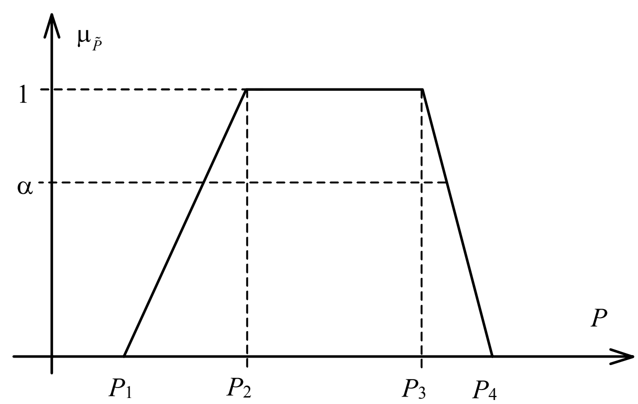

Fig. 1. Trapezoidal membership function of a fuzzy load $\tilde{P} = (P_1, P_2, P_3, P_4)$ expressing the possibility that load may occur between P_1 and P_4, but more typically between P_2 and P_3.

Although the calculations in fuzzy load flow analysis are somewhat simpler than that in a probabilistic case (convolution is not needed), it is still far too complex to be applied directly to the full system model. Therefore, again a linearized model of the system has been used and the results obtained are approximate. Very recently, however, the authors have developed a methodology where an accurate solution for a non-statistical interval load flow is obtainable [5]. The concept is based on finding the boundary values, which are equivalent to an optimization procedure for implicitly defined vector functions. A simple algorithm was developed that is easy to implement and robust. Finding the accurate boundary values enables us to obtain accurate solutions from a fuzzy/ interval load flow. The results presented in [5] show that a linearized fuzzy/interval load flow can also produce good results given a small degree of uncertainty very efficiently.

This paper addresses the problem of calculating system risks that arise from including uncertain nodal powers (loads and generation) in the analysis. In doing so, it considers both approaches for solving the fuzzy load flow problem, boundary load flow solutions and linearized fuzzy load flow. It compares the results obtained and investigates the feasibility of the later to produce useful information for planners.

II. Boundary Load Flow Solutions

The concept of boundary load flow (BLF) was presented for the first time in [2] within the context of PLF. In this paper, an approximate solution for ranges of values for state and output variables, given the ranges of values of input variables from their probability distributions, was presented. The results for the ranges of variables were then used to determine multiple points of linearization for the load flow equations in order to improve the accuracy of the PLF solutions for the tail regions of probability distributions.

Motivated by this concept, the authors have developed a methodology where an accurate solution for a non-statistical interval load flow is obtainable. In the following, a brief explanation of this methodology is given.

The load flow problem is defined by the following two sets of nonlinear equations:

$$\mathbf{Y} = \mathbf{g}(\mathbf{X}) \tag{1}$$

and

$$\mathbf{Z} = \mathbf{h}(\mathbf{X}), \tag{2}$$

where:

- **X** is the vector of unknown state variables (voltage magnitudes and angles at PQ buses; and voltage angles and reactive power outputs at PV buses),
- **Y** is the vector of predefined input variables (real and reactive injected nodal powers at PQ buses; and voltage magnitudes and real power outputs at PV buses),
- **Z** is the vector of unknown output variables (real and reactive power flows in the network elements), and
- **g**, **h** are the load flow vector functions.

Finding the boundary values in a load flow problem is a process of locating the constrained extrema of implicitly defined vector functions of vector arguments. In our notation, we want to find the extreme values for the elements of **X** and **Z** implicitly expressed in (1) and (2), in terms of the elements of **Y** which, in turn, are constrained.

Although the elements of **X** and **Z** cannot be explicitly expressed in terms of the elements of **Y**, their partial derivatives can be found during the solution of the ordinary load flow.

As is well known, because **X** cannot be explicitly expressed in terms of **Y**, the solution of the system of equations (1) is found by an iterative process. Given an initial trial solution, **X'**, the error is calculated as:

$$\Delta\mathbf{Y} = \mathbf{Y} - \mathbf{Y'} = \mathbf{Y} - \mathbf{g}(\mathbf{X'}). \tag{3}$$

If a Newton-Raphson (N-R) based scheme is used, (1) is linearized around **X'** and an update for the new solution is found as:

$$\Delta\mathbf{X} = \mathbf{K}\cdot\Delta\mathbf{Y}, \tag{4}$$

where **K** is the inverse of the Jacobian of **g** evaluated at **X'**. The element K_{ij} of this matrix is actually the partial derivative of X_i with respect to Y_j.

Similarly, if we linearize (2) and substitute for $\Delta\mathbf{X}$ from (4) we will obtain:

$$\Delta\mathbf{Z} = \mathbf{S}\cdot\Delta\mathbf{X} = \mathbf{L}\cdot\Delta\mathbf{Y}, \tag{5}$$

where **S** is the Jacobian of **h** at the given point of linearization. The matrix $\mathbf{L} = \mathbf{S}\cdot\mathbf{K}$ is the sensitivity coefficient matrix and the element L_{ij} is the partial derivative of Z_i with respect to Y_j.

Each row of **K** and **L** represents the gradient vector of the corresponding state and output variable X_i and Z_i, respectively. Similar to derivative based optimization procedures, by iteratively following the direction of the gradient, extreme points (possibly local) of the state or output variable can be found.

Here, only the signs of the partial derivatives that comprise the gradient are used. Experience has shown that the values of these partials are not useful for efficiently determining the step length. Further, procedure is needed to maintain feasibility of the solution, i.e., ensure the input variables are within constrains for all iterations.

Specifically, suppose that the *minimum* value of X_i is sought. If K_{ij} is positive (negative), then decrease (increase) the value of Y_j. After repeating for all Y_j we obtain a new point of **Y** from which a new **X** from (1) can be found. From this new point, the above steps are repeated until one of the following is true for <u>each</u> input variable:

- the partial derivative is positive and the associated variable is at a minimum;
- the partial derivative is negative and the associated variable is at a maximum;
- the partial derivative is zero.

If the final condition does not hold for any of the variables, then the solution is a vertex of the X_i's domain and clearly a point of constrained minimum. Because of the nonlinearity of the function, this point may not be the only minimum, i.e., there may be other vertices that are also points of local constrained minima. Still, our experience has shown that the physical nature of the load flow problem dictates either a unique solution or a solution, which is dominated by a few input variables in a unique manner.

When one or more of the partial derivates are zero, the solution point lies somewhere on the boundary surface. Such a point is either a local constrained extremum (either minimum or maximum) or a saddle point. Though it is highly unlikely that by preceding in a downhill direction one will end up trapped in a local maximum or a saddle point, theoretically such a possibility exists. Thus, additional conditions are imposed. Previous values of X_i are recorded and compared with the newly obtained one. If X_i fails to decrease, then different length steps are to be employed.

Finally, in the special case when all the partial derivatives are zero, a solution cannot be obtained due to the singularity of the Jacobian. Such a point typically indicates infeasibility of the load flow and a loading limit for the system considered. Further, a singularity of the Jacobian may occur even if not all of the partial derivatives are zero. In such cases, the ranges of values of the input variables are too great and one must repeat the calculations with reduced variations for some or all of the variables.

It should be clear that the procedure described here must be repeated for each state and output variable considered. Therefore, it is computationally intensive. This is the cost for finding an accurate solution for a fuzzy/interval load flow. The next section describes a linearized fuzzy/ interval load flow.

III. Linearized Fuzzy / Interval Load Flow

The fuzzy approach to the load flow problem presented in [4] uses linearized equations (1) and (2). The points of linearization ($\mathbf{X}_0$, $\mathbf{Y}_0$) and ($\mathbf{X}_0$, $\mathbf{Z}_0$) are obtained from a deterministic load flow (DLF) with input variables set at their midpoints with the highest degree of possibility (i.e., α = 1). These equations now have the form:

$$\tilde{\mathbf{X}} = \mathbf{X}_0 + \mathbf{K} \cdot (\tilde{\mathbf{Y}} - \mathbf{Y}_0) \tag{6}$$

and

$$\tilde{\mathbf{Z}} = \mathbf{Z}_0 + \mathbf{L} \cdot (\tilde{\mathbf{Y}} - \mathbf{Y}_0)\,, \tag{7}$$

where $\tilde{\mathbf{Y}}$ denotes the vector with fuzzy input variables, and $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Z}}$ denote the resultant vectors with fuzzy state and output variables, respectively.

Using the rules of fuzzy arithmetic [6], which are based on interval operations for a given α, the approximate resultant fuzzy variables are readily available in a few single non-iterative steps. Typically, computations are performed only for two values of α (α = 0 and α = 1), and then assuming trapezoidal membership functions for the output variables, results for intermediary values of α are found by interpolation.

IV. System Adequacy Indices

In this paper we are confined to the steady state operation of the system and its adequacy in terms of bus voltage and branch current magnitudes. If uncertainty is allowed in the input variables (nodal powers), the resultant state variables (voltages) and output variables (currents) will become uncertain also. Given the predefined operational constraints for these variables, we may express the *adequacy* or, conversely, *inadequacy* of the system to accommodate such uncertainty [7].

A. Voltage Inadequacy

Let us assume, for example, that the possibility distribution of voltage magnitude at some particular bus in the system is described with the following trapezoidal FN: (0.93, 0.99, 1.02, 1.03) p.u., as shown in Fig. 2. If the criterion for the minimum acceptable voltage is 0.95 p.u. (vertical dashed line), then there is a possibility of 0.33 that the voltage in this bus will be bellow this constraint.

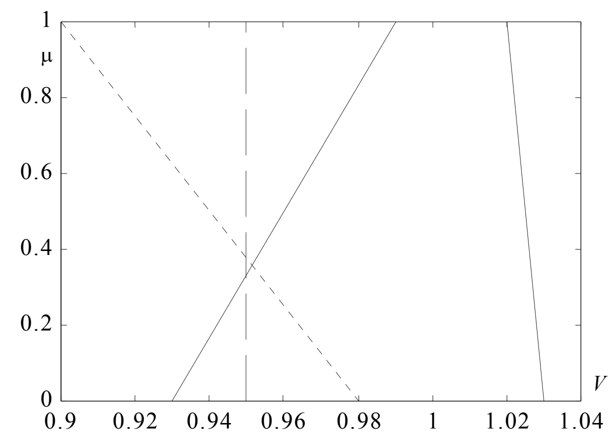

Fig. 2. Possibility distribution of a bus voltage expressed by a trapezoidal membership function (solid line); Hard constraint for low voltage (dashed line); Soft constraint for low voltage (dotted line).

The low voltage inadequacy index for this bus could be defined by the triangular FN: ((0.93,0), (0.95,0.33), (0.95,0)), obtained from the left tail of the possibility distribution beyond the voltage level 0.95 p.u. Such definition of inadequacy may be useful when a single bus is considered. Practically, there are numerous buses and one wants to compare voltages and, possibly, aggregate the results into a system wide index. Therefore, it is much more convenient to define the voltage inadequacy index in terms of voltage drop from the minimum acceptable voltage. Using fuzzy arithmetic, the voltage inadequacy index of bus i can be expressed as:

$$\tilde{V}_{\mathrm{IN}i} = \max\{\tilde{V}_{\min i} - \tilde{V}_i, 0\}, \quad \text{for each } \alpha \in [0,1] \tag{8}$$

where $\tilde{V}_{\min i}$ is the minimum acceptable voltage constraint, which can also be a fuzzy number, and $\tilde{V}_i$ is the fuzzy voltage magnitude at bus i.

Using this definition, the voltage inadequacy from the previous example is defined with the following triangular FN: ((0,0), (0,0.33), (0.02,0)), shown in Fig. 3 with dashed line. This number can be obtained geometrically by flipping the previously obtained FN horizontally and shifting it to the right of the x-axis origin.

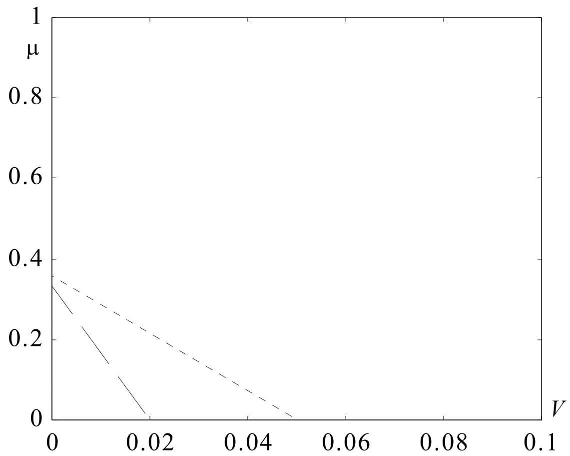

Fig. 3. Voltage inadequacies for the fuzzy bus voltage in Fig. 2 when the low voltage constraint is 'hard' (dashed line) and 'soft' (dotted line).

As noted in the definition of the voltage inadequacy index, the operational constraints may also be defined in a 'soft', fuzzy, manner. For example, we may define a 'good voltage' with the following trapezoidal FN: (0.9, 0.98, 1.02, 1.1) p.u. From here, the definition of 'bad voltage' is obtained by applying the negation operator over the fuzzy set 'good voltage', i.e. subtracting its membership function (possibility distribution) from 1. The left part of this new membership function is the fuzzy 'low voltage' criterion and it defines the minimum acceptable voltage as a fuzzy constraint, given by the line ((0.9,1), (0.98,0)). This line is shown in Fig. 2 as a dotted line. In this case, by applying (8), the voltage inadequacy index is given with the following FN: ((0,0), (0,0.357), (0.05,0)), also shown in Fig. 3 (dotted line).

Having defined the fuzzy 'low voltage' criterion, the degree of 'low voltage' can be found by applying the fuzzy *and* operator on both membership functions. This operator, while falling within the family of *t-norms*, can be defined in various ways. For the examples here, the *min* operator appears to be adequate. The result is the intersection of the fuzzy voltage and fuzzy constraint sets.

In our case, the result for 'low voltage' is the triangular FN: ((0.93,0), (0951, 0.357), (0.98,0)) (see Fig. 2). From here, the voltage inadequacy index can be obtained geometrically by first flipping horizontally this 'low voltage' FN, aligning its left side with the y-axis, and shifting it to the right of the *x*-axis origin. Then, the system voltage inadequacy index can be defined as the fuzzy sum of voltage inadequacies for all buses in the system:

$$\tilde{V}_{\mathrm{IN}sys} = \sum_i \tilde{V}_{\mathrm{IN}i} \tag{9}$$

At this point, only the low voltage inadequacy has been considered. Although this is usually the main concern in power systems, there may be cases when some of the voltages in the system may reach unacceptably high values. In such cases, one may define the high voltage inadequacy index analogously to the above.

B. Current Inadequacy

Now, similarly to that for voltage inadequacy, current inadequacy indices for each branch in the system can be defined, given the possibility distribution for the current and the maximum loading criterion for the branch.

For example, Fig. 4 shows a possibility distribution of current magnitude in some particular branch in the system, described with the trapezoidal FN: (1.5, 2.0, 2.2, 2.5) p.u. If the maximum current for this branch is given as 'hard' and equals, for example, 2.3 p.u. (dashed line), then there is a possibility of 0.666 that the current in this branch will be beyond this constraint. Again, if the maximum current for this branch is given as 'soft', for example, as line ((2.2,0), (2.4,1)) (dotted line), then the possibility of overloading is 0.6.

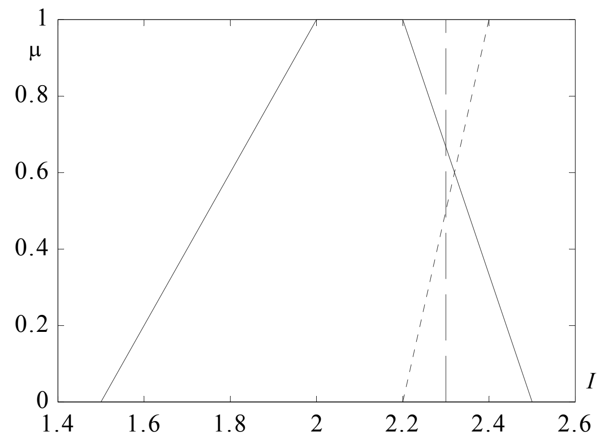

Fig. 4. Possibility distribution of a branch current expressed by a trapezoidal membership function (solid line); Hard constraint for maximum current (dashed line); Soft constraint for maximum current (dotted line).

The current inadequacy indices can be defined as:

$$\tilde{I}_{INi} = \max\{\tilde{I}_i - \tilde{I}_{\max i}, 0\}, \quad \text{for each } \alpha \in [0,1] \tag{10}$$

where $\tilde{I}_{\max i}$ is the maximum current constraint (in general, fuzzy number), and $\tilde{I}_i$ is the fuzzy current magnitude in branch *i*.

Using this definition, the current inadequacy indices similar to those for voltage inadequacies shown in Fig. 3 can be found.

The system current inadequacy index can be defined as the fuzzy sum of current inadequacies for all branches in the system:

$$\tilde{I}_{\mathrm{IN}sys} = \sum_i \tilde{I}_{\mathrm{IN}i} \tag{11}$$

V. Risk Indices

As uncertainty has increased with deregulation, the concept of risk has become central issue in power system planning [8]. Risk can be defined as the hazard to which we are exposed because of uncertainty [9]. It is associated with some set of decisions and it has the following two dimensions:

- The likelihood of making a regrettable decision;
- The amount by which the decision is regrettable.

The decision in power system planning is the particular system configuration.

A. Robustness

Robustness is the likelihood of making a regrettable decision. It is the fundamental measure of risk. In the possibilistic framework used here, it is the possibility for which the system still accommodates uncertainty without any inadequacy. Therefore, in connection with inadequacy indices defined in the previous section, robustness can be expressed as:

$$\text{Robustness} = 1 - \alpha_{IN} \quad (12)$$

where α_{IN} is the highest possibility for the given inadequacy, i.e., the possibility for zero voltage or zero current inadequacy. In the previous example, given the current uncertainty, the robustness of the branch with the soft constraint for maximum current is 0.4.

B. Exposure

Exposure is the amount by which the decision is regrettable. It usually represents the loss that will occur for an adverse materialization of uncertainties. Unfortunately in power system planning, it is difficult to directly express the loss and instead we deal with inadequacy [7]. Thus, exposure is the possibility for which an inadequacy in the system occurs. It follows that it simply takes the value of α_{IN}. In the previous example, the exposure of the branch to the current uncertainty, with the soft constraint for maximum current, is 0.6.

VI. Case Study

Now we find system inadequacy indices for the IEEE 14-bus system shown in Fig. 5. The system data and the base case description can be found elsewhere (for example, [10]). Both solution methods are analyzed here. The boundary load flow will give us exact solutions for the ranges of voltage and current magnitudes, given the ranges of values for input nodal powers. The linearized fuzzy load flow will give us approximate solutions. The point of linearization for the later is the base case solution.

The system current inadequacy, assuming that all the input variables are trapezoidal FNs with (80%, 90%, 110%, 120%) of the base case values, are shown in Fig. 6. The same indices are shown again in Fig. 7, for the case when the 0-support set is changed significantly such that the trapezoidal FNs are (50%, 90%, 110%, 150%) of the base case values.

These figures, together with the fact that the results from both approaches are equal when there is no uncertainty in the system, reveal an interesting phenomenon. Namely, as the uncertainty increases, the difference in the exact and approximate robustness (as well as, exposure) increases up to a certain level of uncertainty, and then starts to decrease.

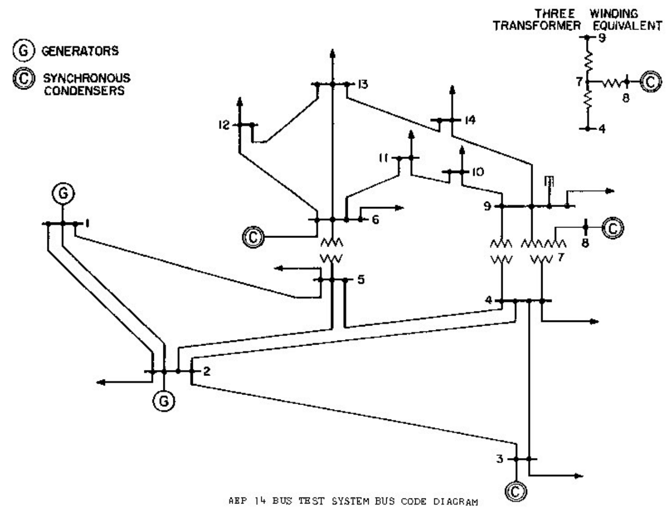

Fig. 5. IEEE/AEP 14-bus test system.

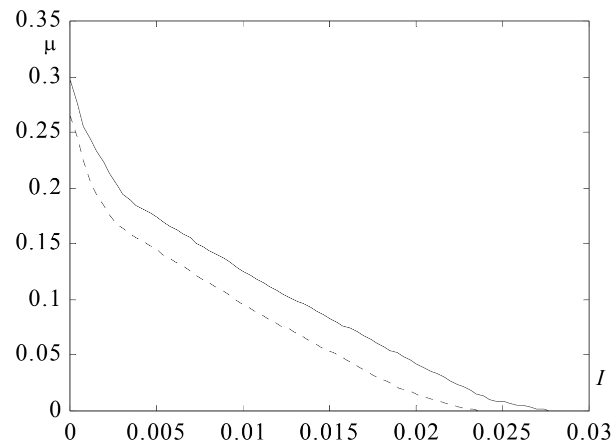

Fig. 6. System current inadequacies for the IEEE 14-bus system when input nodal powers are trapezoidal FNs with (80%, 90%, 110%, 120%) of the base case values. Solid line - exact solution from boundary load flow; Dotted line – approximate solution from the linearized fuzzy load flow.

This follows since the inadequacy of the system is a property that has saturation. Once a certain threshold is reached it can no longer increase significantly, but slowly converges to 1. When this happens, the exact and the approximate method will give similar results, because the error is beyond the critical point. At intermediary values, there may be a significant difference. For example, if we let the loads vary with (0%, 90%, 110%, 200%) of the base case value, the system voltage inadequacy shown in Fig. 8 arises. Heavy load is needed for this system in order to drive the voltages down because it has many buses with voltage support. The important point here is two fold: first, that care needs to be taken if using approximate load flow solutions; and second, that the approximation must be evaluated in terms of the decision variable, i.e., adequacy, not simply the load flow solution.

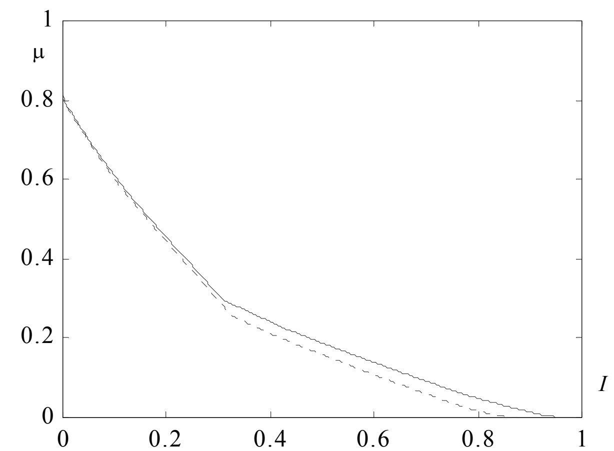

Fig. 7. System current inadequacies for the IEEE 14-bus system when input nodal powers are trapezoidal FNs with (50%, 90%, 110%, 150%) of the base case values. Solid line - exact solution from boundary load flow; Dotted line – approximate solution from the linearized fuzzy load flow.

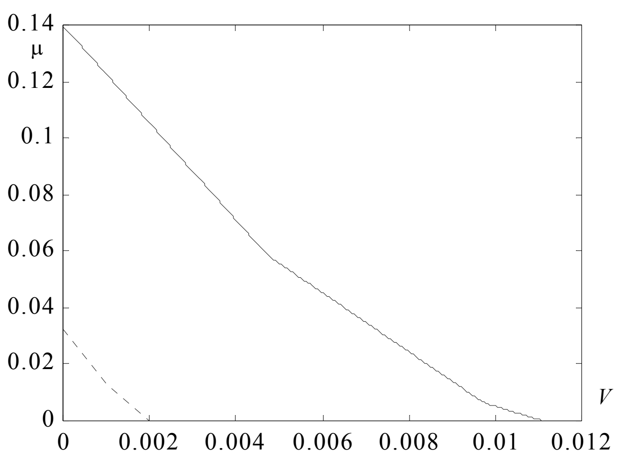

Fig. 8 System voltage inadequacies for the IEEE 14-bus system when input nodal powers are trapezoidal FNs with (0%, 90%, 110%, 200%) of the base case values. Solid line - exact solution from boundary load flow; Dotted line – approximate solution from the linearized fuzzy load flow.

VII. Discussion and Conclusions

This paper presents a risk assessment application of a newly developed concept for finding accurate boundary values of load flow solutions. To approximately obtain the same results one may also apply a linearized fuzzy/interval load flow. While this may produce some useful results quickly, the problem is how to know when the solutions are valid. There is no direct answer to this problem and it depends on the system itself and the decision process. The tool developed by the authors can not only produce exact results, but check the validity of the approximate ones.

A final point to emphasize here is that while the risk measures employed here are fairly simplistic, they show the types of assessments the authors believe should be employed in the planning process. In a more sophisticated, i.e., practical tool, errors of the type exposed here could be exacerbated.

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IX. Biographies

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