

A New Linearization Method of Unbalanced Electrical Distribution Networks

Guodong Liu, Yan Xu
Power & Energy Systems Group
Oak Ridge National Laboratory
Oak Ridge, USA
Email: liug@ornl.gov, xuy3@ornl.gov

Oguzhan Ceylan, Kevin Tomsovic
Dept. of Electrical Engineering and Computer Science
The University of Tennessee
Knoxville, USA
Email: oceylan@utk.edu, tomsovic@utk.edu

Abstract—With increasing penetration of distributed generation in the distribution networks (DN), the secure and optimal operation of DN has become an important concern. As DN control and operation strategies are mostly based on the linearized sensitivity coefficients between controlled variables (e.g., node voltages, line currents, power loss) and control variables (e.g., power injections, transformer tap positions), efficient and precise calculation of these sensitivity coefficients, i.e. linearization of DN, is of fundamental importance. In this paper, the derivation of the node voltages and power loss as functions of the nodal power injections and transformers' tap-changers positions is presented, and then solved by a Gauss-Seidel method. Compared to other approaches presented in the literature, the proposed method takes into account different load characteristics (e.g., constant PQ, constant impedance, constant current and any combination of above) of a generic multi-phase unbalanced DN and improves the accuracy of linearization. Numerical simulations on both IEEE 13 and 34 nodes test feeders show the efficiency and accuracy of the proposed method.

Index Terms—Sensitivity coefficients, linearization, load characteristics, multi-phase unbalanced distribution network.

I. INTRODUCTION

With increasing penetration of distributed energy resources (e.g., wind turbines, PV panels, microturbines, fuel cells, mini-hydro, battery storage, and so on) in the distribution networks (DN), the traditional passive DNs without any sources are gradually transforming into active ones with both dispatchable and undispachable power sources[1]. Correspondingly, the usual “install and forget” principles become technically more difficult and potentially could compromise operating efficiency. Within this context, new control and operation strategies capable of coordinating different types of distributed

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Guodong Liu and Yan Xu are with the Oak Ridge National Laboratory (ORNL), Oak Ridge, TN 37831 USA (email: liug@ornl.gov; xuy3@ornl.gov).

Oguzhan Ceylan and Kevin Tomsovic are with the Department of Electrical Engineering and Computer Science, The University of Tennessee (UT), Knoxville, TN 37996 USA (email: oceylan@utk.edu; tomsovic@utk.edu).

energy resources (DERs) efficiently to achieve operational objectives compatible with sustainability of energy systems are in particular need of development [2].

Management and optimizing the operation of these active DNs requires a full AC-formulation of the power flow equations, and various approaches have been proposed for solving such distribution optimal power flow (DOPF) problem in literature. Generally, these approaches can be classified into two categories. In the first category, the nonlinear optimization problem is directly solved using nonlinear programming methods such as gradient search or interior point methods [3]-[6]. It should be noted that the solution time and convergence characteristic of nonlinear programming in solving the DOPF problem may not satisfy the stringent time constraints required by real-time controls. More commonly, the nonlinear optimization problem is addressed by iteratively solving the linearized problem as in the second category [7]-[12]. These linear programming based methods are generally more efficient in term of solution time. Nevertheless, the linearized problem is formulated based on the calculated sensitivity coefficients, which need to be continuously updated as the system operation condition changes. For this matter, efficient and precise linearization of unbalanced DNs, particularly, calculating the values of sensitivities of node voltages, line currents and power losses to power injections and transformer tap positions is critical.

Currently, there are three common ways to calculate the sensitivity coefficients of interest. The first method involves a series of load flow calculations each performed for a small variation of a single control variable (e.g., power injections or tap positions). By this numerical method, the sensitivity coefficients are directly calculated as the result of a series of load flow studies. The second method uses the Newton-Raphson formulation of the load flow calculation to directly infer the voltage sensitivity coefficients by inverting the Jacobian matrix [13], [14]. It should be noted that this method cannot compute the sensitivities against the transformers' tap positions. In addition, the Jacobian matrix need to be updated and inverted whenever the system operation condition changes, which compromises the computational efficiency of this method. In third method, the derivation of the node

voltages, line currents and power loss as functions of the nodal power injections and transformers' tap-changers positions are deducted based on the "Z" or "Y" matrix of the unbalanced DNs [15], [16]. This method is more efficient than the previous methods in computational time. However, the power injections are all assumed to be constant PQ injections and independent of the voltage. This assumption influences the accuracy of the computed coefficients in generic DNs, which contain various type of power injections (e.g., constant PQ, constant impedance, constant current and any combination of above).

The main contribution of this paper is to extend the analytical method of [15], [16] to take into account various types of power injections (i.e. loads and DG outputs). Specifically, constant PQ, constant impedance and constant current loads are considered separately. In addition, the connection types (e.g., wye-connected or delta-connected) are taken into account. Compared to [15], [16], the proposed method improved the accuracy of the calculated sensitivity coefficients by removing the assumption of constant PQ injections.

The rest of this paper is organized as follows. In Section II, node voltage and loss sensitivity coefficients with respected to real, reactive power injections and transformer tap positions are derived. In Section III, the proposed model is validated on both IEEE 13 and 34 nodes test feeders. Section IV summarizes the paper and presents conclusions.

II. ANALYTICAL DERIVATIONS OF SENSITIVITY COEFFICIENTS

This section contains the main analytical development of this paper related to the derivation of the voltage and loss sensitivity coefficients with respected to real, reactive power injections and transformer tap positions.

A. Load Modeling

The active and reactive power of distribution loads can be changing due to variations in node voltages. When a small power is injected into the distribution system, the node voltage are changes, which results in changing of power injections at probably all nodes, until a balance is reached. For this reason, it is necessary to take into account different load types in calculating node voltage sensitivity coefficients. To this end, there are three well established models for loads representation in power system studies [17]¹:

- Constant power (PQ): in which the active and reactive powers are assumed to be constant irrespective of load node voltage.

$$S = S_0 \quad (1)$$

- Constant current (I): in which the active and reactive powers are assumed to be proportional to the load node voltage.

$$S = \left| \frac{V}{V_0} \right| S_0 = \sqrt{\frac{VV}{V_0V_0}} S_0 \quad (2)$$

- Constant impedance (Z): in which the active and reactive consumed powers are assumed to be proportional to the square of load node voltage.

$$S = \left(\left| \frac{V}{V_0} \right| \right)^2 S_0 = \frac{VV}{V_0V_0} S_0 \quad (3)$$

Taking the 3-phase wye-connected load at node i for example, by using the per-unit value of node voltages, the power injection can be expressed as in (4)

For delta-connected load, the power between phases need to be transformed into phase power entering the delta-connected load first by equation (5). By substituting equation (4) into (5), we can get the phase power entering the delta-connected load represented by node voltages and nominal power between phases.

$$\begin{bmatrix} S_a^i \\ S_b^i \\ S_c^i \end{bmatrix} = \begin{bmatrix} V_a^i & & \\ & V_b^i & \\ & & V_c^i \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} S_{ab}^i \\ S_{bc}^i \\ S_{ca}^i \end{bmatrix} \quad (5)$$

B. Voltage Sensitivity Coefficients With Respect to Power Injection

The 3-phase unbalanced network nodal equations linking the voltages of each phase of the buses to the corresponding injected current is as shown in (6).

$$\begin{bmatrix} I_{abc}^N \\ I_{abc}^S \end{bmatrix} = \begin{bmatrix} Y_{abc}^{NN} & Y_{abc}^{NS} \\ Y_{abc}^{SN} & Y_{abc}^{SS} \end{bmatrix} \begin{bmatrix} V_{abc}^N \\ V_{abc}^S \end{bmatrix} \quad (6)$$

where $I_{abc}^N = [I_a^1, I_b^1, I_c^1 \dots, I_a^N, I_b^N, I_c^N]$, $I_{abc}^S = [I_a^S, I_b^S, I_c^S]$, $V_{abc}^N = [V_a^1, V_b^1, V_c^1 \dots, V_a^N, V_b^N, V_c^N]$, and $V_{abc}^S = [V_a^S, V_b^S, V_c^S]$. S is the slack bus and N is the number of buses except slack buses. a, b and c stands for the 3 phases. $Y_{abc}^{NN}, Y_{abc}^{NS}, Y_{abc}^{SN}$ and Y_{abc}^{SS} are formulated using the compound admittance matrix [18]. Then, the voltages of non-slack buses can be expressed as in (7) with $Z_{abc}^N = (Y_{abc}^{NN})^{-1}$.

$$V_{abc}^N = Z_{abc}^N I_{abc}^{NN} - Z_{abc}^N Y_{abc}^{NS} V_{abc}^S \quad (7)$$

Now, we can find the derivative of bus voltages with respect to a constant PQ active power injection P_ϕ^j at phase ϕ of bus j as in (8). Note that the voltages at slack bus are assumed to be constant.

$$\frac{\partial V_{abc}^N}{\partial P_\phi^j} = Z_{abc}^N \frac{\partial I_{abc}^N}{\partial P_\phi^j} - Z_{abc}^N Y_{abc}^{NS} \frac{\partial V_{abc}^S}{\partial P_\phi^j} = Z_{abc}^N \frac{\partial I_{abc}^N}{\partial P_\phi^j} \quad (8)$$

In order to calculate the voltage sensitivity coefficients, we need to get the injected current sensitivity first. From $I = S/V$ and (4), the injected current sensitivity for a wye-connected constant PQ load at bus i can be calculated as in (9), where $\frac{\partial S_{0\phi}^i}{\partial P_\phi^j} = 1$ if $i = j$ and $\phi = \varphi$, and $\frac{\partial S_{0\phi}^i}{\partial P_\phi^j} = 0$ otherwise.

¹In the rest of the paper, complex conjugates are denoted with a bar below (e.g., \bar{V}).

$$\begin{bmatrix} S_{0a}^i \\ S_{0b}^i \\ S_{0c}^i \end{bmatrix} = \begin{cases} \begin{bmatrix} S_{0a}^i \\ S_{0b}^i \\ S_{0c}^i \end{bmatrix} & \text{for constant PQ} \\ \begin{bmatrix} \sqrt{V_a^i V_a^i} & 0 & 0 \\ 0 & \sqrt{V_b^i V_b^i} & 0 \\ 0 & 0 & \sqrt{V_c^i V_c^i} \end{bmatrix} \begin{bmatrix} S_{0a}^i \\ S_{0b}^i \\ S_{0c}^i \end{bmatrix} & \text{for constant I} \\ \begin{bmatrix} V_a^i V_a^i & 0 & 0 \\ 0 & V_b^i V_b^i & 0 \\ 0 & 0 & V_c^i V_c^i \end{bmatrix} \begin{bmatrix} S_{0a}^i \\ S_{0b}^i \\ S_{0c}^i \end{bmatrix} & \text{for constant Z} \end{cases} \quad (4)$$

$$\begin{bmatrix} \frac{\partial I_a^i}{\partial P_\phi^j} \\ \frac{\partial I_b^i}{\partial P_\phi^j} \\ \frac{\partial I_c^i}{\partial P_\phi^j} \end{bmatrix} = - \begin{bmatrix} \frac{S_{0a}^i}{(V_a^i)^2} \\ \frac{S_{0b}^i}{(V_b^i)^2} \\ \frac{S_{0c}^i}{(V_c^i)^2} \end{bmatrix} \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} + \begin{bmatrix} \frac{1}{V_a^i} & & \\ & \frac{1}{V_b^i} & \\ & & \frac{1}{V_c^i} \end{bmatrix} \begin{bmatrix} \frac{\partial S_{0a}^i}{\partial P_\phi^j} \\ \frac{\partial S_{0b}^i}{\partial P_\phi^j} \\ \frac{\partial S_{0c}^i}{\partial P_\phi^j} \end{bmatrix} \quad (9)$$

Similarly, we can find the injected current sensitivity for a wye-connected constant Z load and constant I load at bus i as shown in (10) and (11), respectively.

$$\begin{bmatrix} \frac{\partial I_a^i}{\partial P_\phi^j} \\ \frac{\partial I_b^i}{\partial P_\phi^j} \\ \frac{\partial I_c^i}{\partial P_\phi^j} \end{bmatrix} = \begin{bmatrix} S_{0a}^i & & \\ & S_{0b}^i & \\ & & S_{0c}^i \end{bmatrix} \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} \frac{\partial I_a^i}{\partial P_\phi^j} \\ \frac{\partial I_b^i}{\partial P_\phi^j} \\ \frac{\partial I_c^i}{\partial P_\phi^j} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{S_{0a}^i}{\sqrt{V_a^i V_a^i}} \\ \frac{S_{0b}^i}{\sqrt{V_b^i V_b^i}} \\ \frac{S_{0c}^i}{\sqrt{V_c^i V_c^i}} \end{bmatrix} \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} - \frac{1}{2} \begin{bmatrix} \frac{S_{0a}^i}{V_a^i} \sqrt{\frac{V_a^i}{V_a^i}} \\ \frac{S_{0b}^i}{V_b^i} \sqrt{\frac{V_b^i}{V_b^i}} \\ \frac{S_{0c}^i}{V_c^i} \sqrt{\frac{V_c^i}{V_c^i}} \end{bmatrix} \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} \quad (11)$$

For a delta-connected load, we need to calculate the phase power entering the delta-connected load first by equation (5) first. Then, the phase current can be calculated by $I = S/V$. The injected current sensitivity for a wye-connected constant PQ load at bus i with respect to a constant PQ active power

injection P_ϕ^j at phase ϕ of bus j can be calculated as in (12),

$$\text{where } s = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \text{ and } v = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} \frac{\partial I_a^i}{\partial P_\phi^j} \\ \frac{\partial I_b^i}{\partial P_\phi^j} \\ \frac{\partial I_c^i}{\partial P_\phi^j} \end{bmatrix} = -s \begin{bmatrix} \frac{S_{0ab}^i}{(V_{ab}^i)^2} \\ \frac{S_{0bc}^i}{(V_{bc}^i)^2} \\ \frac{S_{0ca}^i}{(V_{ca}^i)^2} \end{bmatrix} v \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} \quad (12)$$

Similarly, we can deduct the injected current sensitivity for a delta-connected constant Z load and constant I load at bus i as shown in (10) and (11), respectively.

$$\begin{bmatrix} \frac{\partial I_a^i}{\partial P_\phi^j} \\ \frac{\partial I_b^i}{\partial P_\phi^j} \\ \frac{\partial I_c^i}{\partial P_\phi^j} \end{bmatrix} = \frac{1}{3} s \begin{bmatrix} S_{0ab}^i & & \\ & S_{0bc}^i & \\ & & S_{0ca}^i \end{bmatrix} v \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} \quad (13)$$

Back to equation (8), the voltage sensitivity is a linear function of injected current sensitivity. By (9)-(14), we have get injected current sensitivity as linear functions of voltage sensitivity and conjugate voltage sensitivity with known bus voltages and nominal load power. In this paper, a Gauss-Seidel method is used to solve the voltage sensitivity due to its robustness to initial operating points. The algorithm flow chart is shown in Fig. 1.

Similarly, the voltage sensitivity coefficients with respect to a constant PQ reactive power injection Q_ϕ^j at phase ϕ of bus j can also be calculated since equation (8)-(14) also hold for voltage sensitivity with respect to reactive power. Although we only found the voltage sensitivity with respect to constant PQ injection here, the proposed method can easily be adapted to calculate the sensitivities with respect to constant current and constant impedance injections. This is very useful in demand response management.

Once the derivatives of voltage with respect to power injections are calculated, the derivatives of voltage magnitude with respect to power injections can be got by (15).

$$\begin{bmatrix} \frac{\partial I_a^i}{\partial P_\phi^j} \\ \frac{\partial I_b^i}{\partial P_\phi^j} \\ \frac{\partial I_c^i}{\partial P_\phi^j} \end{bmatrix} = \frac{s}{2\sqrt{3}} \left(\begin{bmatrix} \frac{S_{0ab}^i}{\sqrt{V_{ab}^i V_{ab}^i}} & & \\ & \frac{S_{0bc}^i}{\sqrt{V_{bc}^i V_{bc}^i}} & \\ & & \frac{S_{0ca}^i}{\sqrt{V_{ca}^i V_{ca}^i}} \end{bmatrix} v \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} - \begin{bmatrix} \frac{S_{0ab}^i}{V_{ab}^i} \sqrt{\frac{V_{ab}^i}{V_{ab}^i}} & & \\ & \frac{S_{0bc}^i}{V_{bc}^i} \sqrt{\frac{V_{bc}^i}{V_{bc}^i}} & \\ & & \frac{S_{0ca}^i}{V_{ca}^i} \sqrt{\frac{V_{ca}^i}{V_{ca}^i}} \end{bmatrix} v \begin{bmatrix} \frac{\partial V_a^i}{\partial P_\phi^j} \\ \frac{\partial V_b^i}{\partial P_\phi^j} \\ \frac{\partial V_c^i}{\partial P_\phi^j} \end{bmatrix} \right) \quad (14)$$

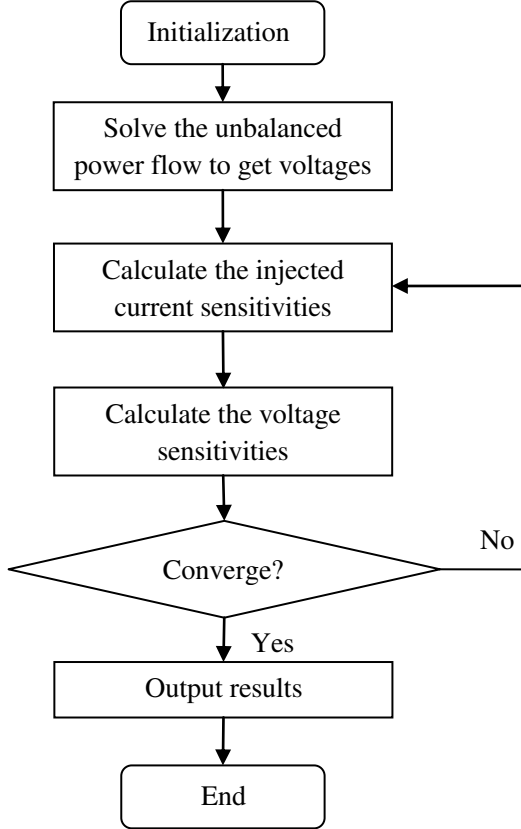


Fig. 1. Gauss-Seidel method to solve voltage sensitivity

$$\frac{\partial |V_\phi^i|}{\partial P_\phi^j} = \frac{1}{|V_\phi^i|} \operatorname{Re} \left(V_\phi^i \frac{\partial V_\phi^i}{\partial P_\phi^j} \right) \quad (15)$$

C. Voltage Sensitivity Coefficients With Respect to Tap Positions of Transformers

Similar to [16], we also assume the transformers' tap-changers are located at the slack bus, which in general is the beginning of the distribution feeder at the substation. As a result, the voltage sensitivity with respect to tap positions of transformers is equivalent to voltage sensitivity with respect to slack bus voltage. Thus, we can find the derivative of bus voltages with respect to the voltage V_ϕ^S at phase ϕ of slack bus S as in (16).

$$\frac{\partial V_{abc}^N}{\partial |V_\phi^S|} = Z_{abc}^N \frac{\partial I_{abc}^N}{\partial |V_\phi^S|} - Z_{abc}^N Y_{abc}^{NS} \frac{\partial V_{abc}^S}{\partial |V_\phi^S|} \quad (16)$$

For the first part, we can write $\frac{\partial I_{abc}^N}{\partial |V_\phi^S|}$ as a function of $\frac{\partial V_{abc}^N}{\partial |V_\phi^S|}$ and $\frac{\partial V_{abc}^S}{\partial |V_\phi^S|}$ similar to equation (9)-(14). For the second part, we write $V_{abc}^S = [|V_a^S| e^{j\theta_a}, |V_b^S| e^{j\theta_b}, |V_c^S| e^{j\theta_c}]$ with $\theta_a = 0^\circ$, $\theta_b = -120^\circ$, and $\theta_c = 120^\circ$, then $\frac{\partial V_\phi^S}{\partial |V_\phi^S|} = e^{j\theta_\phi}$ when $\phi = \phi$, and 0 otherwise. By Gauss-Seidel method, we can calculate the voltage sensitivity with respect to slack bus voltage. The derivatives of voltage magnitude with respect to slack bus voltage can be calculated by (17).

$$\frac{\partial |V_\phi^i|}{\partial |V_\phi^S|} = \frac{1}{|V_\phi^i|} \operatorname{Re} \left(V_\phi^i \frac{\partial V_\phi^i}{\partial |V_\phi^S|} \right) \quad (17)$$

D. Loss Sensitivity Coefficients With Respect to Power Injection and Tap Positions of Transformers

Since loss can be expressed as a function of bus voltages [19], the loss sensitivity with respect to power injection and tap positions of transformer (i.e., slack bus voltage) can be deducted as functions of voltage sensitivities as in (18) and (19),

$$\frac{\partial P_{loss}}{\partial P_\phi^j} = 2 \operatorname{Re} \left(\underline{V}^T G \frac{\partial V}{\partial P_\phi^j} \right) \quad (18)$$

$$\frac{\partial P_{loss}}{\partial |V_\phi^S|} = 2 \operatorname{Re} \left(\underline{V}^T G \frac{\partial V}{\partial |V_\phi^S|} \right) \quad (19)$$

where $V = [V_a^1, V_b^1, V_c^1, \dots, V_a^N, V_b^N, V_c^N, V_a^S, V_b^S, V_c^S]$ and G is the real part of the admittance matrix including the slack bus. So far, we have developed the method of calculating voltage and loss sensitivity coefficients with respect to real, reactive power injections and transformer tap positions by taking into account of various types of loads.

III. CASE STUDIES

In this section, the proposed method for the computation of voltage/loss sensitivities is validated on both IEEE 13 and 34 nodes test feeders [20]. First of all, the voltage and loss sensitivity coefficients are calculated by using the numerical approach where a series of load flow problems are solved by applying small injection perturbations into the tested

TABLE I
ERRORS OF TWO METHODS ON IEEE 13 NODE FEEDER.

Sensitivities	Method in [16]		Proposed Method	
	Min Error	Max Error	Min Error	Max Error
V to P	-0.1637	0.261	-3e-7	3e-5
V to Q	-0.247	1.556	-4e-6	5e-6
V to V_s	-18.2947	580.9146	-5e-4	0.003
P_{loss} to P	-0.018	0.049	-2e-7	3e-5
P_{loss} to Q	-152.5625	0.55	-1e-5	0.003
P_{loss} to V_s	-10.740	7.628	1e-4	1e-4

network. The sensitivities calculated by this numerical method are the most accurate ones because it directly utilizes the unbalanced 3-phase test feeders without any assumption. Thus, these sensitivities are taken as the reference value. Then, the voltage and loss sensitivity coefficients are calculated using the method in [16], which assumes all load are constant PQ loads, and the proposed method in this paper, which accounts for different load characteristics. After that, the calculated sensitivity coefficients are compared with the ones calculated by the numerical approach. The relative errors are calculated by equation (20), where reference values are the sensitivity coefficients calculated by the numerical approach.

$$\text{Error} = \frac{\text{Reference Value} - \text{Calculated Value}}{\text{Reference Value}} \quad (20)$$

In order to validate the accuracy of the proposed method, all the voltage and loss sensitivities with respect to real power injection, reactive power injection and slack bus voltage are calculated. The errors of the method assuming all constant PQ loads and the proposed method on IEEE 13 node feeder are compared in Table I. For the sake of brevity, only maximum and minimum values of errors are listed here. As can be seen, the assumption of all constant PQ load propagates large error, especially in voltage sensitivity with respect to reactive power injection. The extremely large error is due to the reactive power injection changes the node voltage, which have a large effect on the consumed power of constant I and constant Z loads. By the proposed method, the calculated sensitivities are actually much closer to their real values.

The relative errors of voltage sensitivity with respect to real power injection on the 13 node feeder by method in [16] are shown in Fig. 2. As can be seen, the errors have both positive and negative values. Since the linearized DOPF problem is formulated based on the calculated sensitivity coefficients, these errors can probably reduce the speed of convergence and mislead to a non-optimal solution by giving wrong searching directions. In some scenarios, the linearized DOPF might not be able to converge to a solution. The relative errors of voltage sensitivity with respect to reactive power injection on the 13 node feeder by method in [16] are shown in Fig. 3. Similarly, the errors have both positive and negative values, which will mislead the searching direction.

The errors of the method assuming all constant PQ loads and the proposed method on IEEE 13 node feeder are compared in Table II. As can be seen, the errors of proposed method

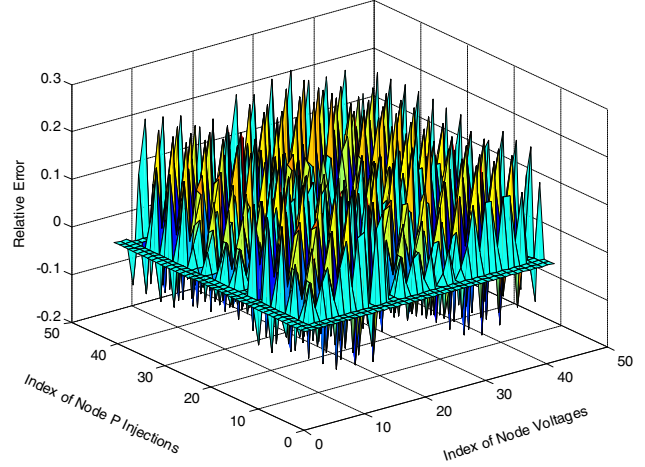


Fig. 2. Relative errors of voltage sensitivity with respect to P on the 13 node feeder by method in [16].

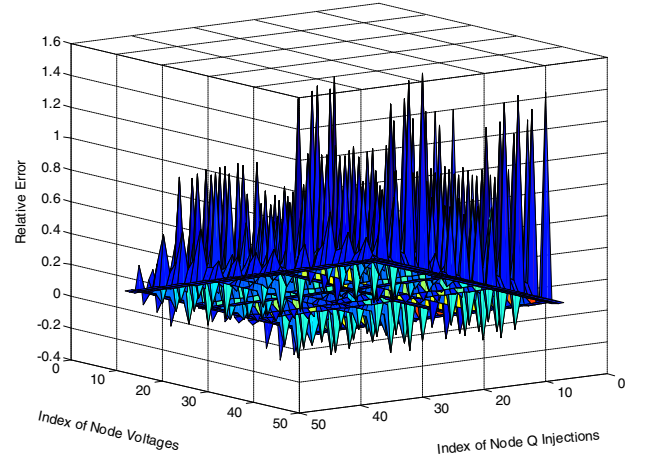


Fig. 3. Relative errors of voltage sensitivity with respect to Q on the 13 node feeder by method in [16].

are much smaller than that of the method in [16]. Therefore, the accuracy of the proposed method is higher by considering different load characteristics for the cases studied.

All methods are coded in MATLAB environment and run on a Windows-based PC with 2.3 GHz and 4 G bytes of RAM. The solution time of proposed method are compared with that

TABLE II
ERRORS OF TWO METHODS ON IEEE 34 NODE FEEDER.

Sensitivities	Method in [16]		Proposed Method	
	Min Error	Max Error	Min Error	Max Error
V to P	-0.274	1.355	-1e-5	2e-5
V to Q	-852.788	270.	-0.005	0.011
P_{loss} to P	0	0.378	0	4e-5
P_{loss} to Q	-14.167	29.548	-0.002	0.003

TABLE III
SOLUTION TIME OF CALCULATING VOLTAGE AND LOSS SENSITIVITIES
WITH RESPECT TO POWER INJECTIONS AT ALL PHASES OF ALL NODES.

System	Numerical Approach	Proposed Method
13 Node Feeder	1.71s	0.22s
34 Node Feeder	25.08s	3.83s

of numerical approach using both IEEE 13 and 34 nodes test feeders as shown in Table III. It is obvious that the proposed method is much better than the numerical approach in terms of solution time.

IV. CONCLUSIONS

In this paper, a new linearization method of unbalanced electrical distribution networks is proposed. The node voltage and power loss sensitivities with respect to real and reactive power injections as well as transformers' tap-changers positions are found, and then solved by Gauss-Seidel method. Compared to other approaches presented in the literature, the proposed method takes into account of the various load characteristics. Specifically, constant PQ, constant impedance and constant current loads are considered separately. Also, the connection types (e.g., wye-connected or delta-connected) are also taken into account. Simulation results on both IEEE 13 and 34 nodes test feeders show that the proposed method improved the accuracy of the calculated sensitivity coefficients by removing the assumption of constant PQ injections. In addition, the proposed method is much better than the numerical approach in terms of solution time.

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