CS580 Homework 1

Fall 2024

August 21, 2024

(Due 4:10pm, August 28, 2024)

Email homework assignments to ldojcsak@vols.utk.edu by the beginning of class time.

1. Prove by induction that $6^k - 1$ is divisible by 5 for all $k \geq 1$.

2. Prove by contradiction that if $n^3 + 5$ is odd, then n is even.

3. Let R be a relation on the real numbers such that aRb iff (if and only if) $|a-b| < 0.01$. Recall that a relation is a set of pairs, and aRb indicates that the pair (a, b) is in the set R. Prove or disprove that R is an equivalence relation.

4. Find the transitive closure, the reflexive and transitive closure, and the symmetric closure of the relation $\{(a, b), (c, d), (d, e), (e, d)\}$.

5. Prove that the set of positive rational numbers $\mathbb{Q}^+$ is countably infinite.
(If you use a diagram to show the mapping to prove this, you must write an accompanying explanation. Note: This is not the only way to prove this.)

6. Prove or Disprove: The union of a countably infinite collection of countably infinite sets is countably infinite.

7. Prove or Disprove: The Cartesian product of a countably infinite collection of countably infinite sets is countably infinite.