CS580 Homework 11

Fall 2024

November 13, 2024

(Due 5:00pm, **Monday, November 18, 2024**)

Email homework assignments to ldojcsak@vols.utk.edu by the beginning of class time.

1. Given the various scenarios listed below, fill in each row of the table noting the class to which A belongs. Use a "Y" if A belongs to a given class, an "N" if it does not, and a "U" if there's not enough information to decide. Also provide a short justification for each of your answers.

	$A \in P$	$A \in \mathcal{NP}$	A is $\mathcal{NP}$-hard	A is $\mathcal{NP}$-complete
a. $A \propto B$ and $B \in NTIME(n^3)$				
b. $B \propto A$ and $B \in NTIME(n^3)$				
c. $A \propto B$ and $B \in DTIME(n^3)$				
d. $A \propto B$ and the problem of verifying a candidate solution for problem B is in $DTIME(n^5)$				
e. B is $\mathcal{NP}$-complete and $A \propto B$				
f. $SAT \propto B$ and $B \propto A$				
g. $B \propto SAT$ and $A \propto B$				

2. Using the proof of the NP-completeness of the vertex cover problem as a model (Theorem 13.4, pg. 331), construct a polynomial time reduction of the 3-CNF problem below to L_{vc}. Draw the graph and show that a choice of vertices that satisfy F correctly correspond to a vertex cover, similar to Example 13.2.

   $F = (x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (x_2 \vee \overline{x_3} \vee x_5) \wedge (\overline{x_3} \vee x_4 \vee \overline{x_5}) \wedge (x_3 \vee \overline{x_4} \vee \overline{x_5})$

3. (Bonus exercise. Will not be scored.) A dominating set in a graph G is a set S of vertices such that every vertex in G is either in S or is adjacent to a vertex in S. The decision version of the dominating set problem is defined as follows:

   - Input: A graph G and positive integer k.
   - Output: Yes or no, depending on whether G has a dominating set of size k.

Answer the following:

a. Define the search version of the dominating set problem.

b. Define the optimization version of the dominating set problem.

c. Assume we have algorithm A to solve the decision version of the problem. How would we use A to solve the search version?

d. How would we use A to solve your optimization version of the problem?