

CS580 Homework 3  
Fall 2024  
September 4, 2024  
(Due 4:10pm, September 11, 2024)

Email homework assignments to ldojcsak@vols.utk.edu by the beginning of class time.

1. Show whether each of the following languages is regular or not by either drawing a DFA or by applying the pumping lemma for regular sets. For each problem let  $\Sigma = \{a, b\}$ .
  - a.  $\{ b^{4n+1} \mid n \geq 0 \}$
  - b.  $\{ b^n a^m b^{n+m} \mid n, m \geq 1 \}$
  - c.  $\{ a^n b^m \mid 0 \leq n < 2 < m \}$
  - d.  $\{ a^n b^m \mid n, m > 0 \text{ and either } n \text{ or } m \text{ is even, but not both} \}$
  - e.  $\{ a^p \mid p \text{ is prime} \}$
  - f.  $\{ x \mid x \text{ has an equal number of } a\text{'s and } b\text{'s} \}$
  - g.  $\{ x \mid x \text{ has an equal number of } ab \text{ and } ba \text{ substrings} \}$
  - h.  $\{ xwx^R \mid x \in (a+b)^*, w \in (a+b), x^R \text{ is the reverse of } x \}$
  - i.  $\{ xwx^R \mid x, w \in (a+b)^*, x^R \text{ is the reverse of } x \}$
  - j.  $\{ xx \mid x \in (a+b)^* \}$
2. Prove each statement using closure properties of regular sets, or give a counter-example.
  - a. Let  $X, Y$ , and  $Z$  denote sets. Prove/disprove  $X - (Y \cup Z) = (X - Y) \cap (X - Z)$ .
  - b. If  $L_1 \cup L_2$  is regular and  $L_2$  is regular, then  $L_1$  is regular.
  - c. If all proper subsets of  $L$  are regular, then  $L$  is regular.