CS580 Homework 7

Fall 2024

October 9, 2024

(Due 4:10pm, October 16, 2024)

Email homework assignments to ldojcsak@vols.utk.edu by the beginning of class time.

1. Give a DPDA generating the language $L = \{\ a^n b^n \mid n \geq 1\}$:

   - State all 7 components of the machine $M = (Q, \Sigma, \Gamma, q_0, Z_0, F, \delta)$.
   - δ must be in the form of a transition list.
   - Use the five-element format discussed in class for transitions.
   - There is no need for λ transitions of any sort. Do not use them.
   - Nevertheless, you will sometimes need to pop the top of the stack, and for that remember to use λ, not ϵ, for the empty string.
   - Document your code. Provide a brief introductory statement describing your algorithm and comment each transition.

   a. Accepts by final state.

   b. Accepts by empty stack.

2. Use the pumping lemma for CFLs to show that the following languages are not context-free. Assume that Σ for each language consists of the symbols mentioned in its definition and nothing else.

   a. $L = \{\ a^i b a^i b a^i \mid i \geq 1\}$

   b. $L = \{\ w \in (a + b + c)^* \mid w$ contains the same number of a's and b's and either the same or less number of c's as a's$\}$

   c. $L = \{\ a^j b^{max\{j,k\}} c^k \mid j, k \geq 1\}$

3. Consider the languages $L_1 = \{\ a^j b^k c^j d^k \mid j, k \geq 1\}$ and $L_2 = \{\ a^j b^k c^k d^j \mid j, k \geq 1\}$.

a. Indicate which of the two languages is not context-free. Prove it using the pumping lemma for CFLs.

b. Indicate which of the two languages is context-free. Consider what would happen if you tried to prove that this language is not context-free by applying the pumping lemma as you did in part a. Explain why this approach would fail. Be specific. (Hint: Consider the different cases.) This does NOT prove that the language is context-free. Convince yourself (and me) that it is indeed context-free by writing a brief pseudo-code for a PDA.

c. Is $L_1 \cap L_2$ context-free? Justify your answer. You don't have to prove it.