CS580: Foundations, Spring 2004
Due: Tuesday, January 20, 2004

Problem Set 1

Since each problem will be graded separately, please turn in each on a separate page with your name.

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1. If M is a DFA accepting language L, then exchanging the final and non-final states in M gives a new DFA accepting the complement of L. Show, by giving an example, that this is not true in general for NFAs.

2. Give state diagrams of DFAs recognizing the following languages. In all cases, the alphabet is {0, 1}.

- **a.** { w | w contains at least 3 1s}.
- **b.** { w | w does not contain the substring 110}.
- **c.** { w | the length of w is at most 5}.
- **d.** { w | w contains an even number of 0s or exactly two 1s}.
- **e.** { w | w contains at most one pair of consecutive 0's and at most one pair of consecutive 1's}.
- **f.** { w | w, when interpreted as an integer, is divisible by 5}. (The most significant digit is the first to be read.)

3. Give regular expressions generating the languages of problems 2a-e above. Provide justification that each regular expression is correct.

4. Below is the transition table of an NFA with start state p and accepting states q and s. Use subset construction to find an equivalent DFA.

	0	**1**
p	$\{q, s\}$	$\{q\}$
q	$\{r\}$	$\{q,r\}$
r	$\{s\}$	$\{p\}$
s	$\{\ \}$	$\{p\}$

5. Design an NFA to recognize the language below, where the alphabet is {0, 1}. Your NFA should have no more than 13 states and 15 arcs. You may represent your NFA by a transition diagram.

{w | w ends in 010 and has 011 somewhere preceding, or w ends in 101 and has 100 somewhere preceding}.

6. Prove that every NFA can be converted to an equivalent one that has a single accept state.

7. Give a counterexample to show that the following construction fails to prove the closure of the class of regular languages under Kleene closure (or "star operation"). (In other words, you must present a finite automaton, N_1, for which the constructed automaton N does not recognize the star of N_1's language, A_1^*.

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct N = $(Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

- The states of N are the states of N_1.
- The start state of N is the same as the start state of N_1.
- $F = \{q_1\} \cup F_1$.
- Define δ so that for any $q \in$ Q and any a in Σ^*,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \end{cases}$$