Karnaugh Maps & Sum-of-Products

Representation Forms [1]

- Sum-of-products form
  - Boolean expression consisting of a sum of terms where each term is a "product" containing exactly one instance of every variable, e.g.,

$$F_1(A,B,C) = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}$$

Sum-of-Products

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Can use to express a function using variables that yield a function value of 1.

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

001 010 100

$$= \sum m(1,2,4)$$

Representation Forms [2]

- Product-of-sums form
  - Boolean expression consisting of a product of terms where each term is a "sum" containing exactly one instance of every variable, e.g.,

$$F_2(A,B,C) = (\bar{A}+\bar{B}+C)\ (A+\bar{B}+\bar{C})\ (A+\bar{B}+C)$$

$$\overline{F_1(A,B,C)} = F_2(A,B,C)$$ (Verify yourself)

Product-of-Sums

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Can use to express a function using variables that yield a function value of 0.

$$F = (A+B+C)\,(\bar{A}+\bar{B}+\bar{C})$$

000 111

$$= \prod M(0,7)$$

Karnaugh Maps [1]

- Invented by Maurice Karnaugh
- Logic expressions quickly become complex to read and understand
- Simplification is not easy
  - We have to remember all of the laws we can apply to an expression
- Karnaugh maps provide easy mechanism to simplify expressions
  - Uses a graphical method
  - Does NOT cover all laws

Karnaugh Maps [2]

- Feasible for up to 4-6 variables (harder to display graphically beyond 4)
- Map is an array of 2^n cells, representing the possible combinations of the values of n binary variables
- Karnaugh map is derived from a truth table (sop form of function)

2-Variable K-Map [1]

- Simple example...
- One cell in the map for each row in the truth table
- e.g.- Expression: $F(X,Y) = X\bar{Y} + \bar{X}Y + XY$

Truth Table:

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	1

K-Map:

XY	00	01	11	10
		1	1	1

Note the order of the list of values for XY – adjacent cells change the value of only one variable.

2-Variable K-Map [2]

K-Map:

XY	00	01	11	10
		1	**1**	**1**

Group adjacent cells with 1's into power-of-two sized groupings. Include all 1's. May re-use a cell. Each group of more than 1 cell represents a simplification.

$m_1 + m_3 = \overline{X}Y + XY = (\overline{X} + X)Y = Y$

$m_3 + m_2 = XY + X\overline{Y} = X(Y + \overline{Y}) = X$

Thus, F(X,Y) = X + Y

3-Variable K-Map [1]

- Example 1: $F(X,Y,Z) = \overline{X}\overline{Y}\overline{Z} + XYZ + \overline{X}\overline{Y}Z + \overline{X}YZ$

X \ YZ	00	01	11	10
0	1	1	1	
1			1	

$m_0 + m_1 = \overline{X}\overline{Y}\overline{Z} + \overline{X}\overline{Y}Z = \overline{X}\overline{Y}$

$m_3 + m_7 = XYZ + \overline{X}YZ = YZ$

$F(X,Y,Z) = \overline{X}\overline{Y} + YZ$

3-Variable K-Map [2]

- Example 2: $F(A,B,C) = \overline{A}C + \overline{A}B + A\overline{B}C + BC$

Note that function is not in SOP form; can expand. For example, $\overline{A}C$ expands to $\overline{A}BC + \overline{A}\overline{B}C$.

A \ BC	00	01	11	10
0		1	1	1
1		1	1	

$m_1 + m_3 + m_5 + m_7 = \overline{A}\overline{B}C + \overline{A}BC + A\overline{B}C + ABC$
$= \overline{A}C + AC = C$

$m_3 + m_2 = \overline{A}BC + \overline{A}B\overline{C} = \overline{A}B$

$F(A,B,C) = \overline{A}B + C$

4-Variable K-Map [1]

- Example 1: $F(W,X,Y,Z) = \Sigma\, m(0,2,5,8,10,13)$

WX \ YZ	00	01	11	10
00	1			1
01		1		
11		1		
10	1			1

$m_0 + m_2 + m_8 + m_{10} = \overline{W}\overline{X}\overline{Y}\overline{Z} + \overline{W}\overline{X}Y\overline{Z} + W\overline{X}\overline{Y}\overline{Z} + W\overline{X}Y\overline{Z}$
$= \overline{X}\overline{Z}$

$m_5 + m_{13} = \overline{W}X\overline{Y}Z + WX\overline{Y}Z = X\overline{Y}Z$

$F(W,X,Y,Z) = X\overline{Y}Z + \overline{X}\overline{Z}$

4-Variable K-Map [2]

- Example 2: $F(W,X,Y,Z) = \sum m(0,1,2,4,6,14,15)$

WX \ YZ	00	01	11	10
00	1	1		1
01	1			1
11			1	1
10				

$m_0 + m_2 + m_4 + m_6 = \overline{W}\overline{X}\overline{Y}\overline{Z} + \overline{W}\overline{X}Y\overline{Z} + \overline{W}X\overline{Y}\overline{Z} + \overline{W}XY\overline{Z}$

$= \overline{W}\overline{Z}$

$m_{14} + m_{15} = WXYZ + WXY\overline{Z} = WXY$

$m_0 + m_1 = \overline{W}\overline{X}\overline{Y}\overline{Z} + \overline{W}\overline{X}\overline{Y}Z = \overline{W}\overline{X}\overline{Y}$

$F(W,X,Y,Z) = \overline{W}\overline{Z} + \overline{W}\overline{X}\overline{Y} + WXY$

Don't Care Conditions

$F = \sum m(1,3,7,11,15)$

$x = \sum m(0,5)$

$\overline{A}\overline{B}\overline{C}\overline{D}$ & $\overline{A}B\overline{C}D$ never occur; thus, don't care what value is associated with them. Use those cells to your advantage (can ignore or use as 1).

AB \ CD	00	01	11	10
00	x	1	1	
01		x	1	
11			1	
10			1	

$F = (\overline{A} + C)\,D$

5-Variable K-Map

BC \ DE	A=0				A=1			
	0	1	3	2	16	17	19	18
	4	5	7	6	20	21	23	22
	12	13	15	14	28	29	31	30
	8	9	11	10	24	25	27	26

6-Variable K-Map