

# CTFS Examples

$$x(t) = A \left\{ 2 \operatorname{rect} \left( 2(t - T_0/4) / T_0 \right) * \delta_{T_0}(t) - 1 \right\}$$

Find the CTFS harmonic function using  $T = T_0$ .

$$\operatorname{rect}(t/w) * \delta_{T_0}(t) \xrightarrow{\mathcal{FS}} (w/T_0) \operatorname{sinc}(wk/mT_0) \delta_m[k]$$

Since  $T = T_0 \Rightarrow m = 1$ . If  $\operatorname{rect}(t/w) = \operatorname{rect}(2t/T_0)$  then  $w = T_0/2$ .

$$\operatorname{rect}(2t/T_0) * \delta_{T_0}(t) \xrightarrow{\mathcal{FS}} (1/2) \operatorname{sinc}(k/2) \underbrace{\delta_1[k]}_{=1 \text{ for any integer } k}$$

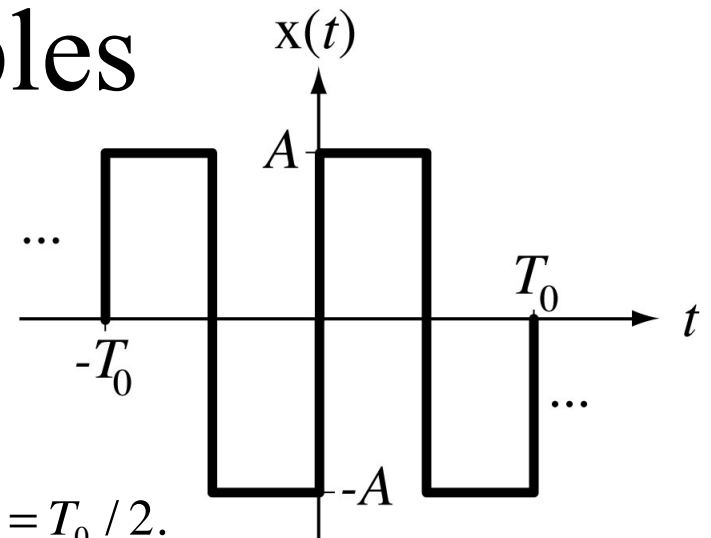
$$2 \operatorname{rect}(2t/T_0) * \delta_{T_0}(t) \xrightarrow{\mathcal{FS}} \operatorname{sinc}(k/2)$$

$$\text{Using the time shifting property, } x(t - t_0) \xrightarrow{\mathcal{FS}} e^{-j2\pi kt_0/T} c_x[k]$$

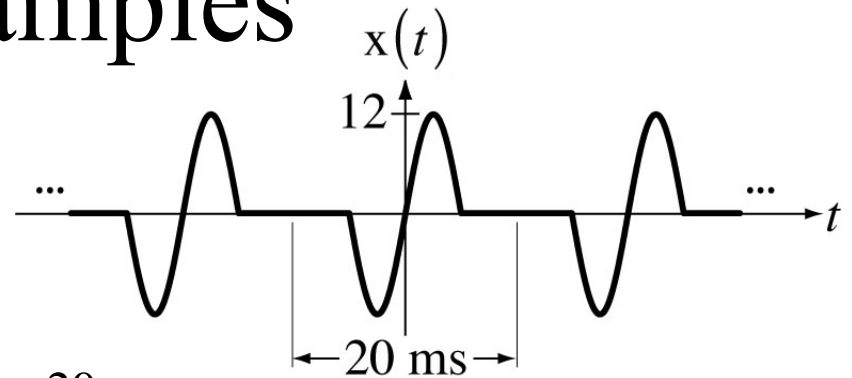
$$2 \operatorname{rect} \left( 2(t - T_0/4) / T_0 \right) * \delta_{T_0}(t) \xrightarrow{\mathcal{FS}} \operatorname{sinc}(k/2) e^{-j\pi k/2}$$

Then, using  $1 \xrightarrow{\mathcal{FS}} \delta[k]$ ,  $T$  arbitrary,  $1 \xrightarrow{\mathcal{FS}} \delta[k]$  and

$$A \left\{ 2 \operatorname{rect} \left( 2(t - T_0/4) / T_0 \right) * \delta_{T_0}(t) - 1 \right\} \xrightarrow{\mathcal{FS}} A \left\{ \operatorname{sinc}(k/2) e^{-j\pi k/2} - \delta[k] \right\} = c_x[k]$$



# CTFS Examples



$$x(t) = 12 \sin(2\pi t / 0.01) [\text{rect}(t / 0.01) * \delta_{0.02}(t)]$$

$$x(t) = 12 \sin(200\pi t) [\text{rect}(100t) * \delta_{0.02}(t)]$$

Find the CTFS harmonic function of  $x(t)$  with  $T = 20$  ms.

$$\sin(2\pi q t / T_0) \xrightarrow{\mathcal{FS}} (j/2) (\delta[k + mq] - \delta[k - mq])$$

$$\sin(200\pi t) \xrightarrow{\mathcal{FS}} (j/2) (\delta[k + 2 \times 1] - \delta[k - 2 \times 1])$$

$$\text{rect}(t / w) * \delta_{T_0}(t) \xrightarrow{\mathcal{FS}} (w / T_0) \text{sinc}(wk / mT_0) \delta_m[k]$$

$$\text{rect}(100t) * \delta_{0.02}(t) \xrightarrow{\mathcal{FS}} (1/2) \text{sinc}(k/2)$$

Using  $x(t)y(t) \xrightarrow{\mathcal{FS}} c_x[k] * c_y[k]$ ,

$$12 \sin(200\pi t) [\text{rect}(100t) * \delta_{0.02}(t)] \xrightarrow{\mathcal{FS}} 12 (j/2) (\delta[k + 2] - \delta[k - 2]) * (1/2) \text{sinc}(k/2)$$

$$12 \sin(200\pi t) [\text{rect}(100t) * \delta_{0.02}(t)] \xrightarrow{\mathcal{FS}} j3 (\text{sinc}((k + 2)/2) - \text{sinc}((k - 2)/2))$$

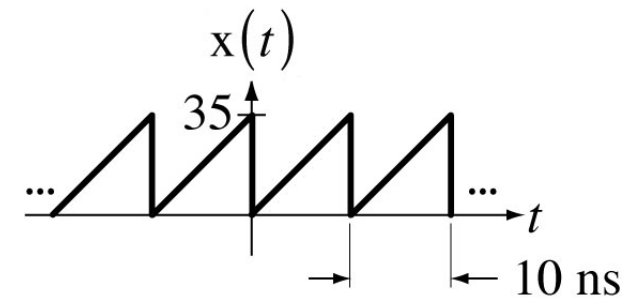
# CTFS Examples

Find the CTFS harmonic function of  $x(t)$  with  $T = 10^{-8}$ .

$$c_x[k] = (1/T) \int_T x(t) e^{-j2\pi kt/T} dt \Rightarrow c_x[0] = 10^8 \int_0^{10^{-8}} (35 \times 10^8 t) dt = 35/2$$

$$c_x[k] = 10^8 \int_0^{10^{-8}} (35 \times 10^8 t) e^{-j2\pi \times 10^8 kt} dt = 35 \times 10^{16} \int_0^{10^{-8}} t e^{-j2\pi \times 10^8 kt} dt$$

$$c_x[k] = 35 \times 10^{16} \left\{ \left[ t \frac{e^{-j2\pi \times 10^8 kt}}{-j2\pi \times 10^8 k} \right]_0^{10^{-8}} - \int_0^{10^{-8}} \left( \frac{e^{-j2\pi \times 10^8 kt}}{-j2\pi \times 10^8 k} \right) dt \right\}$$



$$c_x[k] = 35 \times 10^{16} \left\{ -\frac{e^{-j2\pi k} \times 10^{-8}}{j2\pi \times 10^8 k} - \left[ \frac{e^{-j2\pi \times 10^8 kt}}{(j2\pi \times 10^8 k)^2} \right]_0^{10^{-8}} \right\}$$

$$c_x[k] = 35 \times 10^{16} \left[ -\frac{10^{-16} e^{-j2\pi k}}{j2\pi k} + \frac{1 - e^{-j2\pi k}}{(j2\pi k)^2 \times 10^{16}} \right] = 35 \frac{1 - e^{-j2\pi k} - j2\pi k e^{-j2\pi k}}{(j2\pi k)^2}$$

$$c_x[k] = 35 \begin{cases} 1/2, & k = 0 \\ \frac{j}{2\pi k}, & k \neq 0 \end{cases}$$