Find the CTFT of $15\text{rect}\left(4\left(t-3\right)\right)*\delta_3\left(t\right)$ in the f form

$$\text{rect}\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\text{sinc}\left(f\right)$$

$$\text{Time scaling, } t\to 4t\Rightarrow \text{rect}\left(4t\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(1/4\right)\text{sinc}\left(f/4\right)$$

$$\text{Linearity}\Rightarrow 15\,\text{rect}\left(4t\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/4\right)\text{sinc}\left(f/4\right)$$

$$\text{Time shifting, } t\to t-3\Rightarrow \ 15\,\text{rect}\left(4\left(t-3\right)\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/4\right)\text{sinc}\left(f/4\right)e^{-j6\pi f}$$

$$\text{Mult-Conv Duality}\Rightarrow 15\,\text{rect}\left(4\left(t-3\right)\right)*\delta_3\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/4\right)\text{sinc}\left(f/4\right)e^{-j6\pi f}\times\left(1/3\right)\delta_{1/3}\left(f\right)$$

$$15\,\text{rect}\left(4\left(t-3\right)\right)*\delta_3\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/12\right)\text{sinc}\left(f/4\right)e^{-j6\pi f}\delta_{1/3}\left(f\right)$$

Find the CTFT of $15\text{rect}\left(4t-3\right)*\delta_3\left(t\right)$ in the ω form

$$\text{Time shifting, } t\to t-3\Rightarrow \ 15\,\text{rect}\left(t-3\right)\overset{\mathcal{F}}{\longleftrightarrow}15\,\text{sinc}\left(f\right)e^{-j6\pi f}$$

$$\text{Time scaling, } t\to 4t\Rightarrow 15\,\text{rect}\left(4t-3\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/4\right)\text{sinc}\left(f/4\right)e^{-j3\pi f/2}$$

$$\text{Mult-Conv Duality}\Rightarrow 15\,\text{rect}\left(4t-3\right)*\delta_3\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/4\right)\text{sinc}\left(f/4\right)e^{-j3\pi f/2}\times\left(1/3\right)\delta_{1/3}\left(f\right)$$

$$15\,\text{rect}\left(4t-3\right)*\delta_3\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/12\right)\text{sinc}\left(f/4\right)e^{-j3\pi f/2}\delta_{1/3}\left(f\right)$$

$$f=\omega/2\pi\Rightarrow 15\,\text{rect}\left(4t-3\right)*\delta_3\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(15/12\right)\text{sinc}\left(\omega/8\pi\right)e^{-j3\omega/4}\delta_{1/3}\left(\omega/2\pi\right)$$

$$\text{Periodic Impulse Scaling, } \delta_a\left(x/b\right)=b\delta_{ab}\left(f\right)\Rightarrow \quad \left(5\pi/2\right)\text{sinc}\left(\omega/8\pi\right)e^{-j3\omega/4}\delta_{2\pi/3}\left(\omega\right)$$

Find the inverse CTFT of $95\left[\text{sinc}^2\left(5\left(f-10\right)\right)+\text{sinc}^2\left(5\left(f+10\right)\right)\right]$

$$\text{tri}\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\text{sinc}^2\left(f\right)$$

$$\text{Frequency scaling},f\to 5f\Rightarrow\left(1/5\right)\text{tri}\left(t/5\right)\overset{\mathcal{F}}{\longleftrightarrow}\text{sinc}^2\left(5f\right)$$

$$\text{Frequency Shifting},f\to f-10\Rightarrow\left(1/5\right)\text{tri}\left(t/5\right)e^{j20\pi t}\overset{\mathcal{F}}{\longleftrightarrow}\text{sinc}^2\left(5\left(f-10\right)\right)$$

$$\text{Frequency Shifting},f\to f+10\Rightarrow\left(1/5\right)\text{tri}\left(t/5\right)e^{-j20\pi t}\overset{\mathcal{F}}{\longleftrightarrow}\text{sinc}^2\left(5\left(f+10\right)\right)$$

$$\text{Linearity}\Rightarrow\left(95/5\right)\text{tri}\left(t/5\right)e^{j20\pi t}+\left(95/5\right)\text{tri}\left(t/5\right)e^{-j20\pi t}\overset{\mathcal{F}}{\longleftrightarrow}95\left[\text{sinc}^2\left(5\left(f-10\right)\right)+\text{sinc}^2\left(5\left(f+10\right)\right)\right]$$

$$19\,\text{tri}\left(t/5\right)\left(e^{j20\pi t}+e^{-j20\pi t}\right)\overset{\mathcal{F}}{\longleftrightarrow}95\left[\text{sinc}^2\left(5\left(f-10\right)\right)+\text{sinc}^2\left(5\left(f+10\right)\right)\right]$$

$$38\,\text{tri}\left(t/5\right)\cos\left(20\pi t\right)\overset{\mathcal{F}}{\longleftrightarrow}95\left[\text{sinc}^2\left(5\left(f-10\right)\right)+\text{sinc}^2\left(5\left(f+10\right)\right)\right]$$

Find the CTFT of $e^{-\pi\left(4\left(t+1\right)\right)^2}$

$$e^{-\pi t^2}\overset{\mathcal{F}}{\longleftrightarrow}e^{-\pi f^2}$$

$$\text{Time scaling},t\to 4t\Rightarrow e^{-\pi\left(4t\right)^2}\overset{\mathcal{F}}{\longleftrightarrow}\left(1/4\right)e^{-\pi\left(f/4\right)^2}=\left(1/4\right)e^{-\pi f^2/16}$$

$$\text{Time Shifting},t\to t+1\Rightarrow e^{-\pi\left(4\left(t+1\right)\right)^2}\overset{\mathcal{F}}{\longleftrightarrow}\left(1/4\right)e^{-\pi f^2/16}e^{j2\pi f}$$

Find the CTFT of $\text{rect}\left(t/6\right)*\cos\left(\pi t/3\right)$

$$\text{rect}\left(t\right)\overset{\mathcal{F}}{\longleftrightarrow}\text{sinc}\left(f\right)\quad,\quad\cos\left(\pi t/3\right)\overset{\mathcal{F}}{\longleftrightarrow}\left(1/2\right)\left[\delta\left(f-1/6\right)+\delta\left(f+1/6\right)\right]$$

$$\text{Time Scaling},t\to t/6\Rightarrow\text{rect}\left(t/6\right)\overset{\mathcal{F}}{\longleftrightarrow}6\,\text{sinc}\left(6f\right)$$

$$\text{Mult-Conv Duality}\Rightarrow\text{rect}\left(t/6\right)*\cos\left(\pi t/3\right)\overset{\mathcal{F}}{\longleftrightarrow}6\,\text{sinc}\left(6f\right)\times\left(1/2\right)\left[\delta\left(f-1/6\right)+\delta\left(f+1/6\right)\right]$$

$$3\left[\text{sinc}\left(6f\right)\delta\left(f-1/6\right)+\text{sinc}\left(6f\right)\delta\left(f+1/6\right)\right]$$

$$3\left[\underbrace{\text{sinc}\left(-1\right)}_{=0}\delta\left(f-1/6\right)+\underbrace{\text{sinc}\left(1\right)}_{=0}\delta\left(f+1/6\right)\right]=0$$

A signal $\mathrm{x}(t)=5\sin\left(4\times10^{6}\pi t\right)$ excites a system with transfer function $\mathrm{H}(s)=\dfrac{10^{6}}{s+10^{6}}$.

Find the response $\mathrm{y}(t)$.

$$\mathrm{Y}(j\omega)=\mathrm{H}(j\omega)\mathrm{X}(j\omega)$$

$$\mathrm{X}(j\omega)=j5\pi\left[\delta\left(\omega+4\pi\times10^{6}\right)-\delta\left(\omega-4\pi\times10^{6}\right)\right]\ ,\ \mathrm{H}(j\omega)=\frac{10^{6}}{j\omega+10^{6}}$$

$$\mathrm{Y}(j\omega)=\frac{10^{6}}{j\omega+10^{6}}j5\pi\left[\delta\left(\omega+4\pi\times10^{6}\right)-\delta\left(\omega-4\pi\times10^{6}\right)\right]$$

$$\mathrm{Y}(j\omega)=j5\times10^{6}\pi\left[\frac{\delta\left(\omega+4\pi\times10^{6}\right)}{j\omega+10^{6}}-\frac{\delta\left(\omega-4\pi\times10^{6}\right)}{j\omega+10^{6}}\right]$$

$$\mathrm{Y}(j\omega)=j5\times10^{6}\pi\left[\frac{\delta\left(\omega+4\pi\times10^{6}\right)}{j\left(-4\pi\times10^{6}\right)+10^{6}}-\frac{\delta\left(\omega-4\pi\times10^{6}\right)}{j\left(4\pi\times10^{6}\right)+10^{6}}\right]$$

$$\mathrm{Y}(j\omega)=j5\times10^{6}\pi\left\{\frac{\delta\left(\omega+4\pi\times10^{6}\right)\left[j\left(4\pi\times10^{6}\right)+10^{6}\right]-\delta\left(\omega-4\pi\times10^{6}\right)\left[j\left(-4\pi\times10^{6}\right)+10^{6}\right]}{\left(4\pi\times10^{6}\right)^{2}+10^{12}}\right\}$$

$$\mathrm{Y}(j\omega)=j5\times10^{6}\pi\left\{\frac{j\left(4\pi\times10^{6}\right)\left[\delta\left(\omega+4\pi\times10^{6}\right)+\delta\left(\omega-4\pi\times10^{6}\right)\right]+10^{6}\left[\delta\left(\omega+4\pi\times10^{6}\right)-\delta\left(\omega-4\pi\times10^{6}\right)\right]}{\left[(4\pi)^{2}+1\right]\times10^{12}}\right\}$$

$$\mathrm{Y}(j\omega)=\frac{5\pi}{(4\pi)^{2}+1}\left\{j\left[\delta\left(\omega+4\pi\times10^{6}\right)-\delta\left(\omega-4\pi\times10^{6}\right)\right]-4\pi\left[\delta\left(\omega+4\pi\times10^{6}\right)+\delta\left(\omega-4\pi\times10^{6}\right)\right]\right\}$$

$$\mathrm{y}(t)=\frac{5\sin\left(4\pi\times10^{6}t\right)-20\pi\cos\left(4\pi\times10^{6}t\right)}{(4\pi)^{2}+1}=0.3966\cos\left(4\pi\times10^{6}t-3.062\right)$$