Fourier Series of a Square Wave.

Let one period of a periodic signal of fundamental period T_0 be defined by

$$\mathrm{x}(t)=\begin{cases} A \quad , |t|<T_0/4 \\ -A \ , \ T_0/4<|t|<T_0/2 \end{cases}$$

Its CTFS harmonic function (with $T=T_0$) is

$$\mathrm{c_x}[k]=1/T_0\int_{-T_0/2}^{T_0/2}\mathrm{x}(t)e^{-j2\pi kt/T_o}\,dt=1/T_0\int_{-T_0/2}^{T_0/2}\mathrm{x}(t)\left[\cos(2\pi kt/T_0)-j\sin(2kt/T_0)\right]dt$$

$$\mathrm{c_x}[k]=2/T_0\int_{0}^{T_0/2}\mathrm{x}(t)\cos(2\pi kt/T_0)\,dt=\frac{2A}{T_0}\left[\int_{0}^{T_0/4}\cos(2\pi kt/T_0)\,dt-\int_{T_0/4}^{T_0/2}\cos(2\pi kt/T_0)\,dt\right]$$

$$\mathrm{c_x}[k]=\frac{2A}{T_0}\left[\frac{\sin(2\pi kt/T_0)}{2\pi k/T_0}\bigg|_0^{T_0/4}-\frac{\sin(2\pi kt/T_0)}{2\pi k/T_0}\bigg|_{T_0/4}^{T_0/2}\right]$$

$$\mathrm{c_x}[k]=A\left[\frac{\sin(\pi k/2)}{\pi k/2}-\frac{\sin(\pi k)}{\pi k}\right]=A\left[\mathrm{sinc}(\pi k/2)-\mathrm{sinc}(\pi k)\right]$$

$$\mathrm{c_x}[0]=0$$

This is the average value of the function and must be zero.