Starting with the DTFT transform pair

$$\text{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\text{rect}(wF) * \delta_1(F)$$

make the change of variable $F = \Omega / 2\pi$ to verify the transform pair

$$\text{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\text{rect}(w\Omega / 2\pi) * \delta_{2\pi}(\Omega) \ .$$

We are making a change of variable in both functions that are being convolved. Therefore we must use the convolution property

$$\text{If } \mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) \text{ then } \mathrm{y}(at) = |a|\,\mathrm{x}(at) * \mathrm{h}(at) \quad \text{(Chapter 5)}$$

Using that property

$$\text{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\text{rect}(wF) * \delta_1(F) \xrightarrow{F=\Omega/2\pi} \text{sinc}(n/w) \xleftrightarrow{\mathcal{F}} (1/2\pi)\, w\,\text{rect}(w\Omega / 2\pi) * \delta_1(\Omega / 2\pi)$$

Now, using the scaling property of the periodic impulse

$$\delta_a(bt) = |1/b|\,\delta_{a/b}(t) \quad \text{(Chapter 2)}$$

we get

$$\text{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\text{rect}(w\Omega / 2\pi) * \delta_{2\pi}(\Omega) \ .$$

Now, just to prove that this result is correct, we can derive the omega-form transform directly from the definition of the inverse transform.

$$\mathcal{F}^{-1}\left(w\,\text{rect}(w\Omega / 2\pi) * \delta_{2\pi}(\Omega)\right) = \frac{1}{2\pi}\int_{2\pi}\left[w\,\text{rect}(w\Omega / 2\pi) * \delta_{2\pi}(\Omega)\right]e^{j\Omega n}\,d\Omega$$

There is only one rectangle centered at zero in the range $-\pi < \Omega < \pi$ and its full width is $2\pi / w$.

$$\mathcal{F}^{-1}\left(w\,\text{rect}(w\Omega / 2\pi) * \delta_{2\pi}(\Omega)\right) = \frac{w}{2\pi}\int_{-\pi/w}^{\pi/w} e^{j\Omega n}\,d\Omega = \frac{w}{2\pi}\left[\frac{e^{j\Omega n}}{jn}\right]_{-\pi/w}^{\pi/w} = \frac{w}{j2\pi}\frac{e^{j\pi n/w} - e^{-j\pi n/w}}{n}$$

$$= \frac{1}{j2}\frac{e^{j\pi n/w} - e^{-j\pi n/w}}{\pi n / w} = \frac{\sin(\pi n / w)}{\pi n / w} = \text{sinc}(n/w)$$