

Find the inverse Fourier transform of  $(\pi / 20)\delta_{\pi/4}(\omega)$ .

In the tables we find  $\delta_T(t) \xleftrightarrow{\mathcal{F}} (1/T)\delta_{1/T}(f)$ . We can convert this to the  $\omega$  form by making the change of variable  $f \rightarrow \omega / 2\pi$ . Then

$$\delta_T(t) \xleftrightarrow{\mathcal{F}} (1/T)\delta_{1/T}(\omega / 2\pi)$$

From the definition of the periodic impulse

$$\delta_T(t) \xleftrightarrow{\mathcal{F}} (1/T) \sum_{k=-\infty}^{\infty} \delta(\omega / 2\pi - k / T)$$

$$\delta_T(t) \xleftrightarrow{\mathcal{F}} (1/T) \sum_{k=-\infty}^{\infty} \delta((1/2\pi)(\omega - 2\pi k / T))$$

From the scaling property of the impulse

$$\delta_T(t) \xleftrightarrow{\mathcal{F}} (2\pi / T) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k / T)$$

Then we can recognize the summation as a periodic impulse

$$\delta_T(t) \xleftrightarrow{\mathcal{F}} (2\pi / T)\delta_{2\pi/T}(\omega).$$

From  $(\pi / 20)\delta_{\pi/4}(\omega)$  we want the period to be  $\pi / 4$ . A  $T$  value of 8 makes that happen.

$$\delta_8(t) \xleftrightarrow{\mathcal{F}} (2\pi / 8)\delta_{2\pi/8}(\omega) = (\pi / 4)\delta_{\pi/4}(\omega)$$

Then we divide both sides by 5.

$$(1/5)\delta_8(t) \xleftrightarrow{\mathcal{F}} (\pi / 20)\delta_{\pi/4}(\omega)$$

Alternate Solution:

Derive a scaling property for periodic impulses.

$$\delta_T(at) = \sum_{k=-\infty}^{\infty} \delta(at - kT) = (1/|a|) \sum_{k=-\infty}^{\infty} \delta(t - kT / a) = (1/|a|)\delta_{T/a}(t).$$

Now, start with

$$\delta_1(t) \xleftrightarrow{\mathcal{F}} \delta_1(f)$$

Making the change of variable  $f \rightarrow \omega / 2\pi$  and using the scaling property for the periodic impulse

$$\delta_1(t) \xleftrightarrow{\mathcal{F}} \delta_1(\omega / 2\pi) = 2\pi \delta_{2\pi}(\omega)$$

Now, realizing that we want to change the period from  $2\pi$  to  $\pi / 4$  make the change of variable  $\omega \rightarrow 8\omega$  to compress the  $\omega$  function. The corresponding effect in the time domain is to divide by 8 and make the change of variable  $t \rightarrow t / 8$ .

$$(1 / 8) \delta_1(t / 8) \xleftrightarrow{\mathcal{F}} 2\pi \delta_{2\pi}(8\omega)$$

Now, use the periodic impulse scaling property again on both sides.

$$8 \times (1 / 8) \delta_8(t) \xleftrightarrow{\mathcal{F}} (1 / 8) \times 2\pi \delta_{2\pi/8}(\omega)$$

or

$$\delta_8(t) \xleftrightarrow{\mathcal{F}} (\pi / 4) \delta_{\pi/4}(\omega).$$

Then, dividing both sides by 5

$$(1 / 5) \delta_8(t) \xleftrightarrow{\mathcal{F}} (\pi / 20) \delta_{\pi/4}(\omega)$$