Magnitude and Phase of Complex Functions of Real Variables

$$e^{-(3+j2.3)} = 0.0498e^{-j2.3} \text{ or } 0.0498\angle -2.3 \text{ , } e^{(2-j6)} = 7.3891e^{-j6} = 7.3891\angle 0.2832$$

$$\frac{100}{8+j13} = 6.5512e^{-j1.0191} = 6.5512\angle -1.0191$$

Fundamental Period of a Sum of Two Periodic Signals

$$\underbrace{3\underbrace{\sin(220\pi t)}_{f_0=110,\, T_0=1/110} - 8\underbrace{\cos(120\pi t)}_{f_0=60,\, T_0=1/60}}_{f_0=\text{GCD}(110,60)=10 \Rightarrow T_0=0.1} \qquad \underbrace{\underbrace{\delta_{14}[n]}_{N_0=14} - 6\underbrace{\delta_8[n]}_{N_0=8}}_{N_0=\text{LCM}(14,8)=56}$$

$$-2\cos(3\pi n/12) + 11\cos(14\pi n/10) = \underbrace{-2\underbrace{\cos(2\pi n(1/8))}_{N_0=8} + 11\underbrace{\cos(2\pi n(7/10))}_{N_0=10}}_{N_0=\text{LCM}(8,10)=40}$$

Generalized Derivatives

$$\text{x}(t) = \begin{cases} 4 & , \ t<3 \\ 7t & , \ t>3 \end{cases} \Rightarrow \text{x}'(t) = 17\delta(t-3) + \begin{cases} 0 & , \ t<3 \\ 7 & , \ t>3 \end{cases}$$

Impulses and Periodic Impulses

Sampling Property $\int_{-8}^{22} 8e^{4t}\delta(t-2)\,dt = 8e^{8} = 23{,}848 \;,\; \int_{11}^{82} 3\sin(200t)\delta(t-7)\,dt = 0$

$$\int_{-2}^{10} 39t^2\delta_4(t-1)\,dt = 39\sum_{k=-\infty}^{\infty}\int_{-2}^{10} t^2\delta(t-1-4k)\,dt = 39\sum_{k=-\infty}^{\infty}\begin{Bmatrix}(4k+1)^2 & , & -2<4k+1<10 \\ 0 & , & \text{otherwise}\end{Bmatrix}$$

$$= 39\sum_{k=-\infty}^{\infty}\begin{Bmatrix}(4k+1)^2 & , & -3/4<k<9/4 \\ 0 & , & \text{otherwise}\end{Bmatrix} = 39(1+25+81) = 4173$$

$$\sum_{n=-18}^{33} 38n^2\delta[n+6] = 1368 \;,\; \sum_{n=-4}^{7} -12(0.4)^n\,\mathrm{u}[n]\delta_3[n] = -12\left[(0.4)^0+(0.4)^3+(0.4)^6\right] = -12.8172$$

Equivalence Property $7\delta(t+4)\times(2t^2+5t+1) = 7\times\left[2(-4)^2+5(-4)+1\right]\delta(t+4) = 91\delta(t+4)$

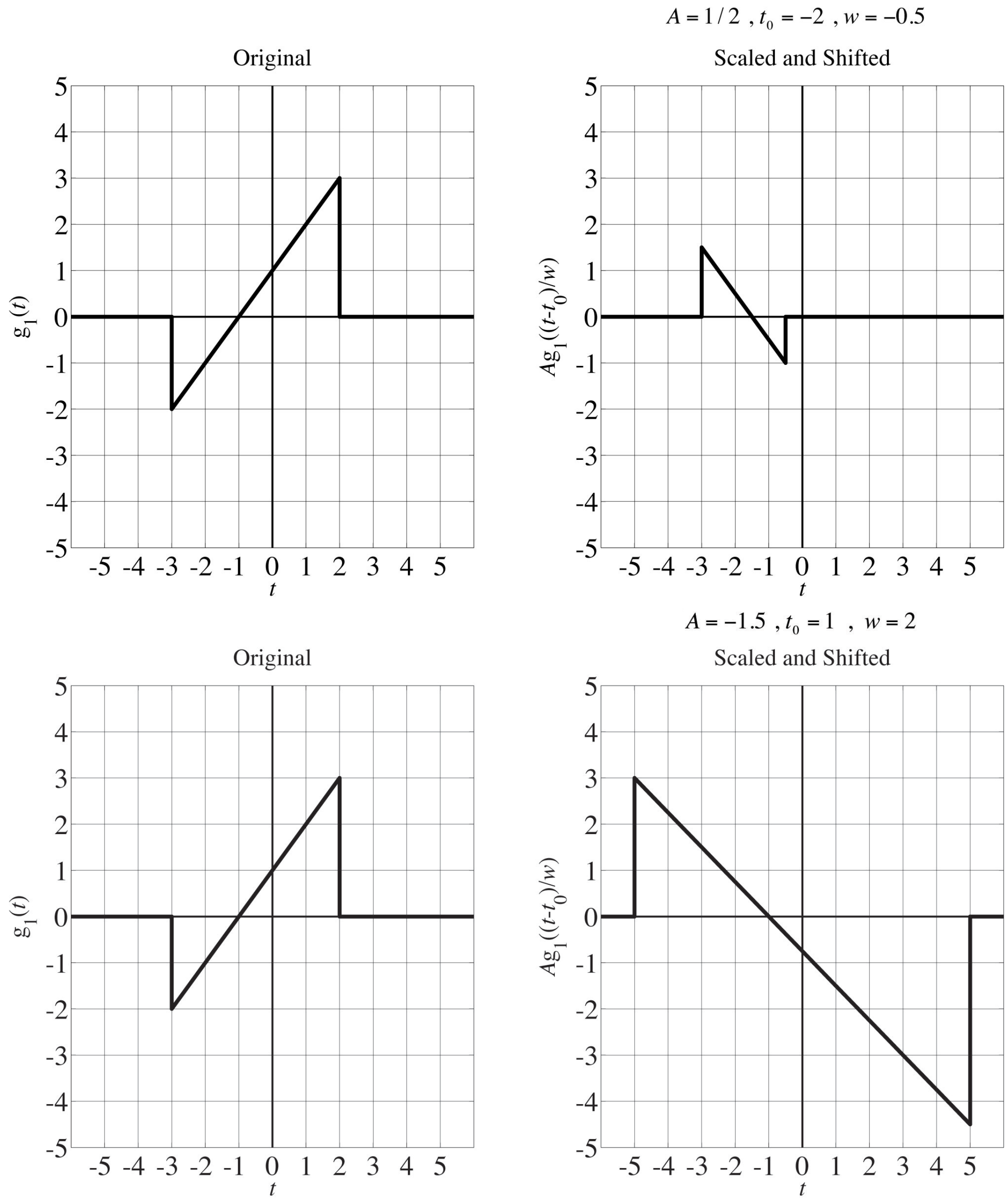
$$27(0.3)^n\delta[n-3] = 27(0.3)^3\delta[n-3] = 0.729\delta[n-3]$$

Scaling Property $5\delta(3(t-1)) = (5/3)\delta(t-1) \;,\; -9\delta_{11}(5t) = (-9/5)\delta_{11/5}(t)$

$13\delta[3n] = 13\delta[n]$, (No scaling property for discrete-time impulses)

$$22\delta_3[4n] = 22\sum_{k=-\infty}^{\infty}\delta[4n-3k] = \begin{Bmatrix}22 & , & n/3 \text{ an integer} \\ 0 & , & \text{otherwise}\end{Bmatrix}$$

Scaling and Shifting



Even and odd functions

$$\mathrm{x}(t)=\frac{t^3-4t^2}{e^{j8t}} \Rightarrow \mathrm{x}_e(t)=\frac{\mathrm{x}(t)+\mathrm{x}(-t)}{2}=\frac{\dfrac{t^3-4t^2}{e^{j8t}}+\dfrac{(-t)^3-4(-t)^2}{e^{-j8t}}}{2}$$

$$\mathrm{x}_e(t)=\frac{t^3e^{-j8t}-4t^2e^{-j8t}-t^3e^{j8t}-4t^2e^{j8t}}{2}$$

$$=\frac{t^3\left(e^{-j8t}-e^{j8t}\right)-4t^2\left(e^{-j8t}+e^{j8t}\right)}{2}$$

$$=\frac{-j2t^3\sin(8t)-8t^2\cos(8t)}{2}=-t^2\left[jt\sin(8t)+4\cos(8t)\right]$$

$$\mathrm{x}_o(t)=\frac{\dfrac{t^3-4t^2}{e^{j8t}}-\dfrac{(-t)^3-4(-t)^2}{e^{-j8t}}}{2}=\frac{t^3\left(e^{-j8t}+e^{j8t}\right)-4t^2\left(e^{-j8t}-e^{j8t}\right)}{2}$$

$$=t^2\left[t\cos(8t)+j4\sin(8t)\right]$$

Signal Energy and Signal Power

$$x[n] = n(-1.3)^n (u[n] - u[n-4]) \Rightarrow E_x = \sum_{n=-\infty}^{\infty} \left| n(-1.3)^n (u[n] - u[n-4]) \right|^2$$

$$E_x = \sum_{n=0}^{3} n^2 (1.3)^{2n} = 0 + 1.3^2 + 4 \times 1.3^4 + 9 \times 1.3^6 = 56.5557$$

$x[n]$ is periodic and one period of $x[n]$ is described by

$$x[n] = n(1-n) \ , \ 3 \le n < 6$$

$$P_x = \frac{1}{N} \sum_{n=\langle N \rangle} \left| x[n] \right|^2 \ ,$$

n	3	4	5
$x[n]$	–6	–12	–20

$$P_x = \frac{1}{3}[36 + 144 + 400] = \frac{580}{3} = 193.333\cdots$$

Signal Energy and Signal Power

$$\mathrm{x}(t)=\begin{Bmatrix} |t|-1 & ,\ |t|<1 \\ 0 & ,\ \text{otherwise} \end{Bmatrix} \Rightarrow E_{\mathrm{x}}=\int_{-\infty}^{\infty}|\mathrm{x}(t)|^{2}\,dt=\int_{-1}^{1}(|t|-1)^{2}\,dt=2\int_{0}^{1}(t-1)^{2}\,dt$$

$$E_{\mathrm{x}}=2\int_{0}^{1}\left(t^{2}-2t+1\right)dt=2\left[t^{3}/3-t^{2}+t\right]_{0}^{1}=2(1/3-1+1)=2/3$$

$\mathrm{x}(t)$ is periodic and one period of $\mathrm{x}(t)$ is described by

$$\mathrm{x}(t)=t(1-t)\ ,\ 1<t<5$$

$$P_{\mathrm{x}}=\frac{1}{T}\int_{T}|\mathrm{x}(t)|^{2}\,dt=\frac{1}{4}\int_{4}|t(1-t)|^{2}\,dt=\frac{1}{4}\int_{1}^{5}\left(t^{2}-2t^{3}+t^{4}\right)dt$$

$$P_{\mathrm{x}}=\frac{1}{4}\left[t^{3}/3-t^{4}/2+t^{5}/5\right]_{1}^{5}$$

$$P_{\mathrm{x}}=\frac{1250-9375+18750-10+15-6}{4\times 30}=\frac{1328}{15}=88.5333$$