$\mathrm{y}(t)=\mathrm{x}(\sin(t)) \Rightarrow$ Causal?, Linear?

$\mathrm{y}(t)=\mathrm{x}(\sin(t))$ For any time $t<0$, $\mathrm{y}(t)$ depends on a value of x in the future, because for $t<0$, $\sin(t)>t$. Non-causal.

Let $\mathrm{x}_1(t)=\mathrm{g}(t)$. Then $\mathrm{y}_1(t)=\mathrm{g}(\sin(t)) \rightarrow K\,\mathrm{y}_1(t)=K\,\mathrm{g}(\sin(t))$.

Let $\mathrm{x}_2(t)=K\,\mathrm{g}(t)$. Then $\mathrm{y}_2(t)=K\,\mathrm{g}(\sin(t))=K\,\mathrm{y}_1(t)$ Homogeneous

Let $\mathrm{x}_2(t)=\mathrm{h}(t)$. Then $\mathrm{y}_2(t)=\mathrm{h}(\sin(t))$.

Let $\mathrm{x}_3(t)=\mathrm{g}(t)+\mathrm{h}(t)$.

Then $\mathrm{y}_3(t)=K\,\mathrm{g}(\sin(t))+K\,\mathrm{h}(\sin(t))=K\left[\mathrm{y}_1(t)+\mathrm{y}_2(t)\right]$

Additive $\rightarrow$ Linear

$$y[n] = \sum_{m=n-n_0}^{n+n_0} x[m] \quad , \; n_0 \text{ a finite positive integer}$$

Linear?, Time-Invariant?, Stable?

If the upper bound on $x[n]$ is B, what is the upper bound on C?

Let $x_1[n] = g[n]$. Then $y_1[n] = \sum_{m=n-n_0}^{n+n_0} g[m]$.

Let $x_2[n] = K\,g[n]$. Then $y_2[n] = K \sum_{m=n-n_0}^{n+n_0} g[m] = K\,y_1[n]$

Homogeneous

Let $x_2[n] = h[n]$. Then $y_2[n] = \sum_{m=n-n_0}^{n+n_0} h[m]$

Let $x_3[n] = g[n] + h[n]$. Then $y_3[n] = \sum_{m=n-n_0}^{n+n_0} \left(g[m] + h[m]\right) = y_1[n] + y_2[n]$

Additive $\rightarrow$ Linear

$$y[n] = \sum_{m=n-n_0}^{n+n_0} x[m] \quad , \; n_0 \text{ a finite positive integer}$$

Let $x_1[n] = g[n]$. Then $y_1[n] = \sum_{m=n-n_0}^{n+n_0} g[m]$.

Let $x_2[n] = g[n-n_0]$. Then $y_2[n] = \sum_{m=n-n_0}^{n+n_0} g[m-n_0]$

Let $q = m - n_0 \Rightarrow m = q + n_0$. Then $y_2[n] = \sum_{q=n-2n_0}^{n} g[q]$.

$y_1[n-n_0] = \sum_{m=n-2n_0}^{n} g[m] = y_2[n] \rightarrow$ Time-Invariant

If the upper bound on $x[n]$ is B, what is the upper bound on $y[n]$?

If $x[n] \le B$, then $y[n] \le \sum_{k=n-n_0}^{n+n_0} B = (2n_0 + 1)B$.

Upper bound on y is $(2n_0+1)B \rightarrow$ BIBO Stable

$y(t) = t^2 x(t-1)$, Linear?, Time-Invariant?

Let $x_1(t) = g(t)$. Then $y_1(t) = t^2 g(t-1)$.

Let $x_2(t) = K g(t)$. Then $y_2(t) = t^2 K g(t-1) = K y_1(t) \rightarrow$ Homogeneous

Let $x_2(t) = h(t)$. Then $y_2(t) = t^2 h(t-1)$.

Let $x_3(t) = g(t) + h(t)$.

Then $y_3(t) = t^2 \left[g(t-1) + h(t-1) \right] = y_1(t) + y_2(t) \rightarrow$ Additive $\rightarrow$ Linear

$y_1(t - t_0) = (t - t_0)^2 g(t - t_0 - 1)$.

Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = t^2 K g(t - t_0 - 1) \neq y_1(t - t_0)$

Time Variant

$y[n] = x^2[n-2]$, Linear?, Time-Invariant?

Let $x_1[n] = g[n]$. Then $y_1[n] = g^2[n-2]$.

Let $x_2[n] = K\,g[n]$. Then $y_2[n] = \left(K\,g[n-2]\right)^2 \neq K\,y_1[n]$

Inhomogenous $\rightarrow$ Non-Linear

Let $x_2[n] = g[n-n_0]$. Then $y_2[n] = g^2[n-n_0-2] = y_1[n-n_0]$

Time-Invariant

$y[n] = x[n+1] - x[n-1]$, Linear?, Time-Invariant?

Let $x_1[n] = g[n]$. Then $y_1[n] = g[n+1] - g[n-1]$.

Let $x_2[n] = K\,g[n]$. Then $y_2[n] = K\,g[n+1] - K\,g[n-1]$.

$y_2[n] = K\,y_1[n] \rightarrow$ Homogeneous

Let $x_2[n] = h[n]$. Then $y_2[n] = h[n+1] - h[n-1]$.

Let $x_3[n] = g[n] + h[n]$.

Then $y_3[n] = g[n+1] + h[n+1] - \left(g[n-1] + h[n-1]\right)$.

$y_3[n] = y_1[n] + y_2[n] \rightarrow$ Additive $\rightarrow$ Linear

Let $x_2[n] = g[n - n_0]$.

Then $y_2[n] = g[n - n_0 + 1] - g[n - n_0 - 1]$.

$y_2[n] = y_1[n - n_0] \rightarrow$ Time-Invariant

$\mathrm{y}(t)=\mathrm{x}(t-2)+\mathrm{x}(2-t)$, Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $\mathrm{x}_1(t)=\mathrm{g}(t)$. Then $\mathrm{y}_1(t)=\mathrm{g}(t-2)+\mathrm{g}(2-t)$.

Let $\mathrm{x}_2(t)=K\,\mathrm{g}(t)$. Then $\mathrm{y}_2(t)=K\left[\mathrm{g}(t-2)+\mathrm{g}(2-t)\right]=K\,\mathrm{y}_1(t)$

Homogeneous

Let $\mathrm{x}_2(t)=\mathrm{h}(t)$. Then $\mathrm{y}_2(t)=\mathrm{h}(t-2)+\mathrm{h}(2-t)$.

Let $\mathrm{x}_3(t)=\mathrm{g}(t)+\mathrm{h}(t)$. Then $\mathrm{y}_3(t)=\left[\mathrm{g}(t-2)+\mathrm{h}(t-2)\right]+\left[\mathrm{g}(2-t)+\mathrm{h}(2-t)\right]$.

$\mathrm{y}_3(t)=\mathrm{y}_1(t)+\mathrm{y}_2(t). \rightarrow$ Additive $\rightarrow$ Linear

Let $\mathrm{x}_2(t)=\mathrm{g}(t-t_0)$. Then $\mathrm{y}_2(t)=\mathrm{g}(t-2-t_0)+\mathrm{g}(2-t-t_0)$.

$\mathrm{y}_2(t)\neq\mathrm{y}_1(t-t_0)\rightarrow$ Time Variant

$\mathrm{y}(t)$ depends on x at other times $\rightarrow$ Dynamic

For $t<1$, $\mathrm{y}(t)$ depends on $\mathrm{x}(t)$ at future times. $\rightarrow$ Non-Causal

Since y is a simple linear combination of shifted versions of x,

if x is bounded, so is y. $\rightarrow$ BIBO Stable

$\mathrm{y}(t) = \mathrm{x}(t)\cos(3t)$, Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $\mathrm{x}_1(t) = \mathrm{g}(t)$. Then $\mathrm{y}_1(t) = \mathrm{g}(t)\cos(3t)$.

Let $\mathrm{x}_2(t) = K\,\mathrm{g}(t)$. Then $\mathrm{y}_2(t) = K\,\mathrm{g}(t)\cos(3t) = K\,\mathrm{y}_1(t)$

Homogeneous

Let $\mathrm{x}_2(t) = \mathrm{h}(t)$. Then $\mathrm{y}_2(t) = \mathrm{h}(t)\cos(3t)$.

Let $\mathrm{x}_3(t) = \mathrm{g}(t) + \mathrm{h}(t)$. Then $\mathrm{y}_3(t) = \left[\mathrm{g}(t) + \mathrm{h}(t)\right]\cos(3t) = \mathrm{y}_1(t) + \mathrm{y}_2(t)$

Additive $\rightarrow$ Linear

Let $\mathrm{x}_2(t) = \mathrm{g}(t - t_0)$. Then $\mathrm{y}_2(t) = \mathrm{g}(t - t_0)\cos(3t)$.

$\mathrm{y}_1(t - t_0) = \mathrm{g}(t - t_0)\cos\left(3(t - t_0)\right) \neq \mathrm{y}_2(t)$

Time <u>Variant</u>

$\mathrm{y}(t)$ depends on x only at time $t \rightarrow$ Static

Static $\rightarrow$ Causal

If $\mathrm{x}(t)$ is bounded, then $\mathrm{x}(t)\cos(3t)$ is also. $\rightarrow$ Stable

$$\mathrm{y}(t)=\int_{-\infty}^{2t}\mathrm{x}(\tau)\,d\tau, \text{ Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?}$$

Let $\mathrm{x}_1(t)=\mathrm{g}(t)$. Then $\mathrm{y}_1(t)=\int_{-\infty}^{2t}\mathrm{g}(\tau)\,d\tau$.

Let $\mathrm{x}_2(t)=K\,\mathrm{g}(t)$. Then $\mathrm{y}_2(t)=K\int_{-\infty}^{2t}\mathrm{g}(\tau)\,d\tau=K\,\mathrm{y}_1(t)$.

Homogeneous

Let $\mathrm{x}_2(t)=\mathrm{h}(t)$. Then $\mathrm{y}_2(t)=\int_{-\infty}^{2t}\mathrm{h}(\tau)\,d\tau$

Let $\mathrm{x}_3(t)=\mathrm{g}(t)+\mathrm{h}(t)$. Then $\mathrm{y}_3(t)=\int_{-\infty}^{2t}\left[\mathrm{g}(\tau)+\mathrm{h}(\tau)\right]d\tau=\mathrm{y}_1(t)+\mathrm{y}_2(t)$

Additive $\rightarrow$ Linear

Let $\mathrm{x}_2(t)=\mathrm{g}(t-t_0)$. Then $\mathrm{y}_2(t)=\int_{-\infty}^{2t}\mathrm{g}(\tau-t_0)\,d\tau$. Let $\lambda=\tau-t_0$.

Then $\mathrm{y}_2(t)=\int_{-\infty}^{2t-t_0}\mathrm{g}(\lambda)\,d\lambda$. $\mathrm{y}_1(t-t_0)=\int_{-\infty}^{2(t-t_0)}\mathrm{g}(\tau)\,d\tau\rightarrow$ Time <u>Variant</u>

$$\mathrm{y}(t)=\int_{-\infty}^{2t}\mathrm{x}(\tau)\,d\tau, \text{ Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?}$$

$\mathrm{y}(t)$ depends on values of $\mathrm{x}(t)$ at other times $\rightarrow$ Dynamic

For any $t>0$, $\mathrm{y}(t)$ depends on all values of $\mathrm{x}(t)$ up to time $2t. \rightarrow$ Non-Causal

If $\mathrm{x}(t)$ is a constant, there is no upper bound on $\mathrm{y}(t)$.

Unstable

Also, by Leibniz's rule for differentiating an integral

$\mathrm{y}'(t)=\mathrm{x}(2t) \Rightarrow$ Eigenvalue is 0. Real part is not negative.

Unstable

$$y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2) & , t \geq 0 \end{cases}$$ Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $x_1(t) = g(t)$. Then $y_1(t) = \begin{cases} 0, & t < 0 \\ g(t) + g(t-2) & , t \geq 0 \end{cases}$.

Let $x_2(t) = K\,g(t)$. Then $y_2(t) = \begin{cases} 0, & t < 0 \\ K\,g(t) + K\,g(t-2) & , t \geq 0 \end{cases} = K\,y_1(t)$

Homogeneous

Let $x_2(t) = h(t)$. Then $y_2(t) = \begin{cases} 0, & t < 0 \\ h(t) + h(t-2) & , t \geq 0 \end{cases}$

Let $x_3(t) = g(t) + h(t)$. Then $y_3(t) = \begin{cases} 0, & t < 0 \\ g(t) + h(t) + g(t-2) + h(t-2) & , t \geq 0 \end{cases}$

$y_3(t) = y_1(t) + y_2(t) \rightarrow \text{Additive} \rightarrow \text{Linear}$

Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = \begin{cases} 0, & t < 0 \\ g(t - t_0) + g(t - t_0 - 2) & , t \geq 0 \end{cases}$

$$y_1(t - t_0) = \begin{cases} 0, & t - t_0 < 0 \\ g(t - t_0) + g(t - t_0 - 2) & , t - t_0 \geq 0 \end{cases} \Rightarrow y_2(t) \neq y_1(t - t_0)$$

Time <u>Variant</u>

$$y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2) & , t \geq 0 \end{cases}$$

Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

$y(t)$ depends on $x(t-2) \rightarrow$ Dynamic

$y(t)$ depends only on x at the same or earlier times $\rightarrow$ Causal

If $x(t)$ is bounded, $x(t-2)$ is bounded and $y(t)$ is bounded. $\rightarrow$ Stable

$$y(t)=\begin{cases}0, & x(t)<0\\ x(t)+x(t-2), & x(t)\geq 0\end{cases}$$

Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $x_1(t)=g(t)$. Then $y_1(t)=\begin{cases}0, & g(t)<0\\ g(t)+g(t-2), & g(t)\geq 0\end{cases}$.

Let $x_2(t)=K\,g(t)$. Then $y_2(t)=\begin{cases}0, & K\,g(t)<0\\ K\,g(t)+K\,g(t-2), & K\,g(t)\geq 0\end{cases}$.

$$Ky_1(t)=\begin{cases}0, & g(t)<0\\ K\,g(t)+K\,g(t-2), & g(t)\geq 0\end{cases}\neq y_2(t)$$

For example, let $g(t)=1$. Then $y_1(t)=2$.

Let $K=-1$. Then $y_2(t)=0\neq -2$.

Inhomogeneous $\rightarrow$ Non-Linear

$$y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2) \ , & x(t) \geq 0 \end{cases}$$

Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $x_1(t) = g(t)$. Then $y_1(t) = \begin{cases} 0, & g(t) < 0 \\ g(t) + g(t-2) \ , & g(t) \geq 0 \end{cases}$.

Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = \begin{cases} 0, & g(t - t_0) < 0 \\ g(t - t_0) + g(t - t_0 - 2) \ , & g(t - t_0) \geq 0 \end{cases}$.

$$y_1(t - t_0) = \begin{cases} 0, & g(t - t_0) < 0 \\ K\,g(t - t_0) + K\,g(t - t_0 - 2) \ , & g(t - t_0) \geq 0 \end{cases} = y_2(t)$$

Time Invariant

$y(t)$ depends on $x(t-2) \rightarrow$ Dynamic

$y(t)$ depends only on x at the same or earlier times $\rightarrow$ Causal

If $x(t)$ is bounded, $x(t-2)$ is bounded and $y(t)$ is bounded. $\rightarrow$ Stable

$y[n] = x[-n]$ Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $x_1[n] = g[n]$. Then $y_1[n] = g[-n]$.

Let $x_2[n] = K\,g[n]$. Then $y_2[n] = K\,g[-n] = K\,y_1[n]. \rightarrow$ Homogeneous

Let $x_2[n] = h[n]$. Then $y_2[n] = h[-n]$.

Let $x_3[n] = g[n] + h[n]$. Then $y_3[n] = g[-n] + h[-n]$.

$y_3[n] = y_1[n] + y_2[n] \rightarrow$ Additive $\rightarrow$ Linear

$y_1[n - n_0] = g[-(n - n_0)]$

Let $x_2[n] = g[n - n_0]$. Then $y_2[n] = g[-n - n_0] \neq y_1[n - n_0]. \rightarrow$ Time <u>Variant</u>

y at any time n depends on x at time $-n \rightarrow$ Dynamic

y at any time n depends on x at time $-n$. For negative n, $-n$ is the future.

Non-Causal

If x is bounded, y is bounded. $\rightarrow$ BIBO Stable

$\mathrm{y}[n]=\mathrm{x}[n-2]-2\,\mathrm{x}[n-8]$ Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $\mathrm{x}_1[n]=\mathrm{g}[n]$. Then $\mathrm{y}_1[n]=\mathrm{g}[n-2]-2\,\mathrm{g}[n-8]$.

Let $\mathrm{x}_2[n]=K\,\mathrm{g}[n]$. Then $\mathrm{y}_2[n]=K\left(\mathrm{g}[n-2]-2\,\mathrm{g}[n-8]\right)=K\,\mathrm{y}_1[n]$.

Homogeneous

Let $\mathrm{x}_2[n]=\mathrm{h}[n]$. Then $\mathrm{y}_2[n]=\mathrm{h}[n-2]-2\,\mathrm{h}[n-8]$.

Let $\mathrm{x}_3[n]=\mathrm{g}[n]+\mathrm{h}[n]$. Then $\mathrm{y}_3[n]=\mathrm{g}[n-2]+\mathrm{h}[n-2]-2\left(\mathrm{g}[n-8]+\mathrm{h}[n-8]\right)$.

$\mathrm{y}_3[n]=\mathrm{y}_1[n]+\mathrm{y}_2[n]\rightarrow$ Additive $\rightarrow$ Linear

Let $\mathrm{x}_2[n]=\mathrm{g}[n-n_0]$. Then $\mathrm{y}_2[n]=\mathrm{g}[n-n_0-2]-2\,\mathrm{g}[n-n_0-8]=\mathrm{y}_1[n-n_0]$.

Time Invariant

y at any time n depends on x at other times $\rightarrow$ Dynamic

y at any time n depends only on x at earlier times. $\rightarrow$ Causal

If x is bounded, y is bounded. $\rightarrow$ BIBO Stable

$$\mathrm{y}[n]=\begin{cases}\mathrm{x}[n] & , \ n\ge 1\\ 0 & , \ n=0\\ \mathrm{x}[n+1] & , \ n\le -1\end{cases}$$

Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $\mathrm{x}_1[n]=\mathrm{g}[n]$. Then $\mathrm{y}_1[n]=\begin{cases}\mathrm{g}[n] & , \ n\ge 1\\ 0 & , \ n=0\\ \mathrm{g}[n+1] & , \ n\le -1\end{cases}$.

Let $\mathrm{x}_2[n]=K\,\mathrm{g}[n]$. Then $\mathrm{y}_2[n]=\begin{cases}K\,\mathrm{g}[n] & , \ n\ge 1\\ 0 & , \ n=0\\ K\,\mathrm{g}[n+1] & , \ n\le -1\end{cases}=K\,\mathrm{y}_1[n]$.

Homogeneous

$$y[n] = \begin{cases} x[n] & , \ n \geq 1 \\ 0 & , \ n = 0 \\ x[n+1] & , \ n \leq -1 \end{cases}$$

Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $x_1[n] = g[n]$. Then $y_1[n] = \begin{cases} g[n] & , \ n \geq 1 \\ 0 & , \ n = 0 \\ g[n+1] & , \ n \leq -1 \end{cases}$.

Let $x_2[n] = h[n]$. Then $y_2[n] = \begin{cases} h[n] & , \ n \geq 1 \\ 0 & , \ n = 0 \\ h[n+1] & , \ n \leq -1 \end{cases}$.

Let $x_3[n] = g[n] + h[n]$. Then $y_3[n] = \begin{cases} g[n] + h[n] & , \ n \geq 1 \\ 0 & , \ n = 0 \\ g[n+1] + h[n+1] & , \ n \leq -1 \end{cases}$.

$y_3[n] = y_1[n] + y_2[n] \rightarrow$ Additive $\rightarrow$ Linear

Let $x_1[n] = g[n]$. Then $y_1[n] = \begin{cases} g[n] & , n \geq 1 \\ 0 & , n = 0 \\ g[n+1] & , n \leq -1 \end{cases}$.

Let $x_2[n] = g[n - n_0]$. Then $y_2[n] = \left\{ \begin{array}{ll} g[n - n_0] & , n \geq 1 \\ 0 & , n = 0 \\ g[n - n_0 + 1] & , n \leq -1 \end{array} \right\} \neq y_1[n - n_0]$.

Time <u>Variant</u>

y at any time n depends on x at other times $\rightarrow$ Dynamic

y at negative times n depends on x at time $n+1$. $\rightarrow$ Non-Causal

If x is bounded, y is bounded. $\rightarrow$ BIBO Stable

$\mathrm{y}[n]=\mathrm{x}[4n+1]$ Linear?, Time-Invariant?, Dynamic?, Causal?, Stable?

Let $\mathrm{x}_1[n]=\mathrm{g}[n]$. Then $\mathrm{y}_1[n]=\mathrm{g}[4n+1]$.

Let $\mathrm{x}_2[n]=K\,\mathrm{g}[n]$. Then $\mathrm{y}_2[n]=K\,\mathrm{g}[4n+1]=K\,\mathrm{y}_1[n]$.

Homogeneous

Let $\mathrm{x}_2[n]=\mathrm{h}[n]$. Then $\mathrm{y}_2[n]=\mathrm{h}[4n+1]$.

Let $\mathrm{x}_3[n]=\mathrm{g}[n]+\mathrm{h}[n]$. Then $\mathrm{y}_3[n]=\mathrm{g}[4n+1]+\mathrm{h}[4n+1]$.

$\mathrm{y}_3[n]=\mathrm{y}_1[n]+\mathrm{y}_2[n]\rightarrow$ Additive $\rightarrow$ Linear

Let $\mathrm{x}_2[n]=\mathrm{g}[n-n_0]$. Then $\mathrm{y}_2[n]=\mathrm{g}[4n+1-n_0]\neq\mathrm{y}_1[n-n_0]=\mathrm{g}[4(n-n_0)+1]$.

Time <u>Variant</u>

y at any time n depends on x at other times $\rightarrow$ Dynamic

y at any time n depends on x at time $4n+1$.

For $n>0$ that is in the future. $\rightarrow$ Non-Causal

If x is bounded, y is bounded. $\rightarrow$ BIBO Stable

$$\mathrm{y}(t)=\frac{1}{\mathrm{x}(t)}\left(\frac{d\,\mathrm{x}(t)}{dt}\right)^2$$, Homogeneous?, Additive?

Let $\mathrm{x}_1(t)=\mathrm{g}(t)$. Then $\mathrm{y}_1(t)=\frac{1}{\mathrm{g}(t)}\left(\frac{d\,\mathrm{g}(t)}{dt}\right)^2 \to K\,\mathrm{y}_1(t)=\frac{K}{\mathrm{g}(t)}\left(\frac{d\,\mathrm{g}(t)}{dt}\right)^2$

Let $\mathrm{x}_2(t)=K\,\mathrm{g}(t)$. Then $\mathrm{y}_2(t)=\frac{1}{K\,\mathrm{g}(t)}\left(\frac{d\left(K\,\mathrm{g}(t)\right)}{dt}\right)^2=\frac{K}{\mathrm{g}(t)}\left(\frac{d\,\mathrm{g}(t)}{dt}\right)^2=K\,\mathrm{y}_1(t)$

Homogeneous

Let $\mathrm{x}_2(t)=\mathrm{h}(t)$. Then $\mathrm{y}_2(t)=\frac{1}{\mathrm{h}(t)}\left(\frac{d\,\mathrm{h}(t)}{dt}\right)^2$

Let $\mathrm{x}_3(t)=\mathrm{g}(t)+\mathrm{h}(t)$. Then $\mathrm{y}_3(t)=\frac{1}{\mathrm{g}(t)+\mathrm{h}(t)}\left[\frac{d\left(\mathrm{g}(t)+\mathrm{h}(t)\right)}{dt}\right]^2$

$$\mathrm{y}_3(t)=\frac{1}{\mathrm{g}(t)+\mathrm{h}(t)}\left[\frac{d\,\mathrm{g}(t)}{dt}+\frac{d\,\mathrm{h}(t)}{dt}\right]^2=\frac{1}{\mathrm{g}(t)+\mathrm{h}(t)}\left[\left(\frac{d\,\mathrm{g}(t)}{dt}\right)^2+\left(\frac{d\,\mathrm{h}(t)}{dt}\right)^2+2\frac{d\,\mathrm{g}(t)}{dt}\frac{d\,\mathrm{h}(t)}{dt}\right]$$

$\mathrm{y}_3(t)\neq \mathrm{y}_1(t)+\mathrm{y}_2(t)\to$ Not Additive $\to$ Non-Linear

For example, let $\mathrm{g}(t)=t^2$ and let $\mathrm{h}(t)=3t+2$. Then $\frac{d\,\mathrm{g}(t)}{dt}=2t$ and $\frac{d\,\mathrm{h}(t)}{dt}=3$.

Then $\mathrm{y}_1(t)=\frac{4t^2}{t^2}=4$ and $\mathrm{y}_2(t)=\frac{9}{3t+2}$ and $\mathrm{y}_1(t)+\mathrm{y}_2(t)=\frac{12t+8+9}{3t+2}=4\frac{t+17/12}{t+2/3}$.

$$\mathrm{y}_3(t)=\frac{4t^2+9+2\times 2t\times 3}{t^2+3t+2}=\frac{4t^2+12t+9}{t^2+3t+2}=4\frac{t^2+3t+9/4}{t^2+3t+2}\neq 4\frac{t+17/12}{t+2/3}.$$

$$\mathrm{y}[n]=\frac{\mathrm{x}[n]\mathrm{x}[n-2]}{\mathrm{x}[n-1]},\ \text{Homogeneous?, Additive?}$$

$$\text{Let } \mathrm{x}_1[n]=\mathrm{g}[n].\ \text{Then } \mathrm{y}_1[n]=\frac{\mathrm{g}[n]\mathrm{g}[n-2]}{\mathrm{g}[n-1]}\rightarrow K\,\mathrm{y}_1[n]=K\frac{\mathrm{g}[n]\mathrm{g}[n-2]}{\mathrm{g}[n-1]}$$

$$\text{Let } \mathrm{x}_2[n]=K\,\mathrm{g}[n].\ \text{Then } \mathrm{y}_2[n]=\frac{K\,\mathrm{g}[n]K\,\mathrm{g}[n-2]}{K\,\mathrm{g}[n-1]}=K\frac{\mathrm{g}[n]\mathrm{g}[n-2]}{\mathrm{g}[n-1]}=K\,\mathrm{y}_1[n]$$

Homogeneous

$$\text{Let } \mathrm{x}_2[n]=\mathrm{h}[n].\ \text{Then } \mathrm{y}_2[n]=\frac{\mathrm{h}[n]\mathrm{h}[n-2]}{\mathrm{h}[n-1]}.$$

$$\text{Let } \mathrm{x}_3[n]=\mathrm{g}[n]+\mathrm{h}[n].\ \text{Then } \mathrm{y}_3[n]=\frac{(\mathrm{g}[n]+\mathrm{h}[n])(\mathrm{g}[n-2]+\mathrm{h}[n-2])}{\mathrm{g}[n-1]+\mathrm{h}[n-1]}.$$

$$\mathrm{y}_3[n]\neq\mathrm{y}_1[n]+\mathrm{y}_2[n]\rightarrow\text{Not Additive}\rightarrow\text{Non-Linear}$$

$\mathrm{y}(t)=\mathrm{x}(t-4)$, Invertible?

Let $t \to t+4$. Then $\mathrm{y}(t+4)=\mathrm{x}(t) \to$ Invertible

$\mathrm{y}(t)=\cos(\mathrm{x}(t))$, Invertible?

$\mathrm{x}(t)=\cos^{-1}(\mathrm{y}(t))$ The $\cos^{-1}$ function is multiple-valued.

Not Invertible.

$\mathrm{y}[n]=n\,\mathrm{x}[n]$, Invertible?

$\mathrm{x}[n]=\mathrm{y}[n]/n$. When $n=0$, $\mathrm{x}[n]$ is undefined. $\to$ Not Invertible.

If $\mathrm{y}[n]$ is zero, that can be because $\mathrm{x}[n]=0$ or because $n=0$ (or both).

So when $\mathrm{y}[n]=0$ we cannot determine $\mathrm{x}[n]$ from $\mathrm{y}[n]$.

$$y[n] = \begin{cases} x[n-1] & , n \geq 1 \\ 0 & , n = 0 \\ x[n] & , n \leq -1 \end{cases} \quad , \text{ Invertible?}$$

When $n = 0$, $y[n] = 0$ regardless of the value of $x[n]$. So, when $n = 0$, $x[n]$ cannot be determined by knowledge of $y[n]$. $\rightarrow$ Not Invertible

$y[n] = x[n]x[n-1]$, Invertible?

$y[n] = x[n]x[n-1] \rightarrow x[n] = y[n]/x[n-1]$

$y[n] = x[n]x[n-1] \rightarrow x[n-1] = y[n]/x[n] \rightarrow x[n] = y[n+1]/x[n+1]$

So we cannot determine $x[n]$ without first determining either $x[n-1]$ or $x[n+1]$. $\rightarrow$ Not Invertible

Example: Let $x[n] = (-1)^n$. Then $y[n] = (-1)^n (-1)^{n-1} = -1$.

$y[n]$ always equals -1 and that can occur with $x[n] = 1$ and $x[n-1] = -1$ or with $x[n] = -1$ and $x[-n] = 1$.

$$y[n]=x[1-n],\ \text{Invertible?}$$

$$y[n]=x[1-n]\rightarrow y[n-1]=x[-n]\rightarrow x[n]=y[-(n-1)]=y[1-n]$$

Invertible

$$y[n]=\sum_{m=-\infty}^{n}(1/2)^{n-m}x[m],\ \text{Invertible?}$$

$$y[n]-y[n-1]=\sum_{m=-\infty}^{n}(1/2)^{n-m}x[m]-\sum_{m=-\infty}^{n-1}(1/2)^{n-1-m}x[m]$$

$$y[n]-y[n-1]=\underbrace{(1/2)^{n-n}}_{=1}x[n]+\underbrace{\sum_{m=-\infty}^{n-1}(1/2)^{n-1-m}x[m]-\sum_{m=-\infty}^{n-1}(1/2)^{n-1-m}x[m]}_{=0}$$

$$x[n]=y[n]-y[n-1]\rightarrow \text{Invertible}$$

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\, d\tau, \text{ Invertible?}$$

Leibniz's rule for differentiating an integral is $\frac{d}{dx}\int_{-\infty}^{x} g(\lambda)\, d\lambda = g(x)$.

Applying it to this case, $y'(t) = e^{-(t-t)} x(t) = x(t) \rightarrow$ Invertible

$$y(t) = x'(t) \quad , \text{ Invertible?}$$

If $x(t) = \int_{-\infty}^{t} y(\tau)\, d\tau + K$, then $x'(t) = y(t)$.

Therefore, $y(t) = x'(t) \Rightarrow x(t) = \int_{-\infty}^{t} y(\tau)\, d\tau + K$

We can determine $x(t)$ to within an additive constant K,

but not exactly. $\rightarrow$ Not Invertible

$$\mathrm{y}(t) = \mathrm{x}(2t) \quad , \text{ Invertible?}$$

$$\mathrm{y}(t/2) = \mathrm{x}(t) \to \text{ Invertible}$$

$$\mathrm{y}[n] = \begin{cases} \mathrm{x}[n/2], & n \text{ even} \\ 0 & , n \text{ odd} \end{cases} \quad , \text{ Invertible?}$$

$$\mathrm{y}[2n] = \begin{cases} \mathrm{x}[n], & 2n \text{ even} \\ 0 & , \quad 2n \text{ odd} \leftarrow 2n \text{ can never be odd.} \end{cases}$$

Therefore

$$\mathrm{x}[n] = \mathrm{y}[2n] \to \text{Invertible}$$

If $y'(t)-3y(t)=4x'(t)+7x(t)$ find the impulse response $h(t)$.

$h'(t)-3h(t)=4\delta'(t)+7\delta(t) \Rightarrow$ Eigenvalue is 3.

$h(t)=Ke^{3t}\,u(t)+K_\delta\delta(t)$

$$\int_{0^-}^{0^+} h'(t)\,dt-3\int_{0^-}^{0^+} h(t)\,dt=4\int_{0^-}^{0^+}\delta'(t)\,dt+7\int_{0^-}^{0^+}\delta(t)\,dt$$

$$\underbrace{h(0^+)}_{=K}-\underbrace{h(0^-)}_{=0}-3\left\{\underbrace{\left[Ke^{3t}/3\right]_0^{0^+}}_{=0}+K_\delta\left[\underbrace{u(0^+)}_{=1}-\underbrace{u(0^-)}_{=0}\right]\right\}=4\left[\underbrace{\delta(0^+)}_{=0}-\underbrace{\delta(0^-)}_{=0}\right]+7\left[\underbrace{u(0^+)}_{=1}-\underbrace{u(0^-)}_{=0}\right]$$

$K-3K_\delta=7$

$$\int_{0^-}^{0^+}\underbrace{\int_{-\infty}^{t} h'(\lambda)\,d\lambda}_{=h(t)}\,dt-3\int_{0^-}^{0^+}\underbrace{\int_{-\infty}^{t} h(\lambda)\,d\lambda}_{=Ke^{3t}\,u(t)/3+K_\delta\,u(t)}\,dt=4\int_{0^-}^{0^+}\underbrace{\int_{-\infty}^{t}\delta'(\lambda)\,d\lambda}_{=\delta(t)}\,dt+7\int_{0^-}^{0^+}\underbrace{\int_{-\infty}^{t}\delta(\lambda)\,d\lambda}_{=u(t)}\,dt$$

$K_\delta-3(0)=4+7(0)\Rightarrow K_\delta=4\Rightarrow K=19$

$h(t)=19e^{3t}\,u(t)+4\delta(t)\Rightarrow h'(t)=19e^{3t}\delta(t)+57e^{3t}\,u(t)+4\delta'(t)$

$$h'(t)-3h(t)=4\delta'(t)+7\delta(t)\Rightarrow 19\underbrace{e^{3t}\delta(t)}_{=\delta(t)}+57e^{3t}\,u(t)+4\delta'(t)-57e^{3t}\,u(t)-12\delta(t)=4\delta'(t)+7\delta(t)$$

$4\delta'(t)+7\delta(t)=4\delta'(t)+7\delta(t)$ Check.

If $2\,\mathrm{y}''(t)+5\,\mathrm{y}'(t)=4\,\mathrm{x}(t)$ find the impulse response $\mathrm{h}(t)$.

$2\,\mathrm{h}''(t)+5\,\mathrm{h}'(t)=4\delta(t)\Rightarrow$ Eigenvalues are $-5/2$ and 0.

$$\mathrm{h}(t)=\left(K_1e^{-5t/2}+K_2\right)\mathrm{u}(t)$$

$$2\int_{0^-}^{0^+}\mathrm{h}''(t)\,dt+5\int_{0^-}^{0^+}\mathrm{h}'(t)\,dt=4\int_{0^-}^{0^+}\delta(t)\,dt$$

$$2\left[\underbrace{\mathrm{h}'(0^+)}_{=-5K_1/2}-\underbrace{\mathrm{h}'(0^-)}_{=0}\right]+5\left[\underbrace{\mathrm{h}(0^+)}_{=K_1+K_2}-\underbrace{\mathrm{h}(0^-)}_{0}\right]=4\left[\underbrace{\mathrm{u}(0^+)}_{1}-\underbrace{\mathrm{u}(0^-)}_{0}\right]$$

$$-5K_1+5K_1+5K_2=4\Rightarrow K_2=4/5$$

$$2\int_{0^-}^{0^+}\underbrace{\int_{-\infty}^{t}\mathrm{h}''(\lambda)\,d\lambda}_{=\mathrm{h}'(t)}\,dt+5\int_{0^-}^{0^+}\underbrace{\int_{-\infty}^{t}\mathrm{h}'(\lambda)\,d\lambda}_{=\mathrm{h}(t)}\,dt=4\int_{0^-}^{0^+}\underbrace{\int_{-\infty}^{t}\delta(\lambda)\,d\lambda}_{\mathrm{u}(t)}\,dt$$

$$2(K_1+K_2)+5\underbrace{\left[-2K_1e^{-5t/2}/5+K_2t\right]_0^{0^+}}_{=0}=0\Rightarrow K_1+K_2=0\Rightarrow K_1=-K_2=-4/5$$

$$\mathrm{h}(t)=(4/5)\left(1-e^{-5t/2}\right)\mathrm{u}(t)\Rightarrow\mathrm{h}'(t)=(4/5)\left[\underbrace{\left(1-e^{-5t/2}\right)\delta(t)}_{=0}+\left(5e^{-5t/2}/2\right)\mathrm{u}(t)\right]=2e^{-5t/2}\,\mathrm{u}(t)$$

$$\mathrm{h}''(t)=2\left[\underbrace{e^{-5t/2}\delta(t)}_{=\delta(t)}-\left(5e^{-5t/2}/2\right)\mathrm{u}(t)\right]=2\left[\delta(t)-\left(5e^{-5t/2}/2\right)\mathrm{u}(t)\right]$$

$$2\,\mathrm{h}''(t)+5\,\mathrm{h}'(t)=4\delta(t)\Rightarrow4\left[\delta(t)-\left(5e^{-5t/2}/2\right)\mathrm{u}(t)\right]+10e^{-5t/2}\,\mathrm{u}(t)=4\delta(t)\Rightarrow4\delta(t)=4\delta(t)\ \ \text{Check.}$$

If $2\,\mathrm{y}[n]-\mathrm{y}[n-1]=3\,\mathrm{x}[n-1]+\mathrm{x}[n-2]$ find the impulse response $\mathrm{h}[n]$.

$2\,\mathrm{h}[n]-\mathrm{h}[n-1]=3\delta[n-1]+\delta[n-2]\Rightarrow$ Eigenvalue is $1/2$.

Let $2\mathrm{h}_0[n]-\mathrm{h}_0[n-1]=\delta[n]$. Then $\mathrm{h}_0[n]=K(1/2)^n\,\mathrm{u}[n]$.

$$2\underbrace{\mathrm{h}_0[0]}_{=K}-\underbrace{\mathrm{h}_0[-1]}_{=0}=\underbrace{\delta[0]}_{=1}\Rightarrow K=1/2 \text{ and } \mathrm{h}_0[n]=(1/2)^{n+1}\,\mathrm{u}[n].$$

Using superposition and time invariance, if $\mathrm{h}_0[n]=(1/2)^{n+1}\,\mathrm{u}[n]$

then $\mathrm{h}[n]=3\mathrm{h}_0[n-1]+\mathrm{h}_0[n-2]=3(1/2)^n\,\mathrm{u}[n-1]+(1/2)^{n-1}\,\mathrm{u}[n-2]$

$\mathrm{h}[n]=(1/2)^n\left(3\mathrm{u}[n-1]+2\,\mathrm{u}[n-2]\right)$.

The first few values of $\mathrm{h}[n]$ are

n	0	1	2	3	4
$\mathrm{h}[n]$	0	3/2	5/4	5/8	5/16

We can find these values also by direct iteration on

$\mathrm{h}[n]=(1/2)\left(3\delta[n-1]+\delta[n-2]+\mathrm{h}[n-1]\right)$ and we get

n	0	1	2	3	4
$\mathrm{h}[n]$	0	3/2	5/4	5/8	5/16

confirming the validity of the solution.

If $\mathrm{x}(t)=\delta(t-1)-3\delta(t+2)$ and $\mathrm{h}(t)=4\,\mathrm{rect}(t/5)$ and $\mathrm{y}(t)=\mathrm{x}(t)*\mathrm{h}(t)$ find the signal energy of $\mathrm{y}(t)$ E_{y}.

$$\mathrm{y}(t)=\left[\delta(t-1)-3\delta(t+2)\right]*4\,\mathrm{rect}(t/5)=4\left[\delta(t-1)*\mathrm{rect}(t/5)-3\delta(t+2)*\mathrm{rect}(t/5)\right]$$

$$\mathrm{y}(t)=4\left[\mathrm{rect}\left((t-1)/5\right)-3\,\mathrm{rect}\left((t+2)/5\right)\right]$$

$$E_{\mathrm{y}}=\int_{-\infty}^{\infty}\left|\mathrm{y}(t)\right|^2 dt=16\int_{-\infty}^{\infty}\left[\mathrm{rect}\left((t-1)/5\right)-3\,\mathrm{rect}\left((t+2)/5\right)\right]^2 dt$$

$$E_{\mathrm{y}}=16\int_{-\infty}^{\infty}\left[\mathrm{rect}^2\left((t-1)/5\right)+9\,\mathrm{rect}^2\left((t+2)/5\right)-6\,\mathrm{rect}\left((t-1)/5\right)\mathrm{rect}\left((t+2)/5\right)\right]dt$$

$$E_{\mathrm{y}}=16\left[\int_{-\infty}^{\infty}\mathrm{rect}^2\left((t-1)/5\right)dt+9\int_{-\infty}^{\infty}\mathrm{rect}^2\left((t+2)/5\right)dt-6\int_{-\infty}^{\infty}\mathrm{rect}\left((t-1)/5\right)\mathrm{rect}\left((t+2)/5\right)dt\right]$$

$$E_{\mathrm{y}}=16\left[\int_{-3/2}^{7/2}dt+9\int_{-9/2}^{1/2}dt-6\int_{-3/2}^{1/2}dt\right]=16(5+45-12)=608$$

$$\mathrm{y}(t)=\begin{cases}-12 & , \; -9/2<t<-3/2 \\ -8 & , \; -3/2<t<1/2 \\ 4 & , \; 1/2<t<7/2\end{cases}\Rightarrow \mathrm{y}^2(t)=\begin{cases}144 & , \; -9/2<t<-3/2 \\ 64 & , \; -3/2<t<1/2 \\ 16 & , \; 1/2<t<7/2\end{cases}$$

$$E_{\mathrm{y}}=144\times 3+64\times 2+16\times 3=608 \quad \text{Check.}$$

If $\mathrm{x}(t)=\cos(200\pi t)\mathrm{u}(t)$ and $\mathrm{h}(t)=e^{-100t}\,\mathrm{u}(t)$ and $\mathrm{y}(t)=\mathrm{x}(t)*\mathrm{h}(t)$ find $\mathrm{y}(t)$.

$$\mathrm{y}(t)=\int_{-\infty}^{\infty}\mathrm{x}(\tau)\mathrm{h}(t-\tau)\,d\tau=\int_{-\infty}^{\infty}\cos(200\pi\tau)\mathrm{u}(\tau)e^{-100(t-\tau)}\,\mathrm{u}(t-\tau)\,d\tau$$

$$\mathrm{y}(t)=e^{-100t}\int_{0}^{t}e^{100\tau}\cos(200\pi\tau)\,d\tau=\frac{e^{-100t}}{2}\int_{0}^{t}e^{100\tau}\left(e^{j200\pi\tau}+e^{-j200\pi\tau}\right)d\tau \quad , \ t>0 \ ; \mathrm{y}(t)=0 \ , t<0$$

$$\mathrm{y}(t)=\frac{e^{-100t}}{2}\mathrm{u}(t)\int_{0}^{t}\left(e^{(100+j200\pi)\tau}+e^{(100-j200\pi)\tau}\right)d\tau=\frac{e^{-100t}}{2}\mathrm{u}(t)\left[\frac{e^{(100+j200\pi)\tau}}{100+j200\pi}+\frac{e^{(100-j200\pi)\tau}}{100-j200\pi}\right]_{0}^{t}$$

$$\mathrm{y}(t)=\frac{e^{-100t}}{2}\left(\frac{e^{(100+j200\pi)t}-1}{100+j200\pi}+\frac{e^{(100-j200\pi)t}-1}{100-j200\pi}\right)\mathrm{u}(t)$$

$$=\frac{e^{-100t}}{2}\frac{100\left(e^{(100+j200\pi)t}+e^{(100-j200\pi)t}-2\right)+j200\left(e^{(100-j200\pi)t}-e^{(100+j200\pi)t}\right)}{100^2+(200\pi)^2}\mathrm{u}(t)$$

$$=\frac{e^{-100t}}{2}\frac{200\left(e^{100t}\cos(200\pi t)-1\right)+j200\left(-j2e^{100t}\sin(200\pi t)\right)}{404784.2}\mathrm{u}(t)$$

$$=e^{-100t}\,\frac{100\left(e^{100t}\cos(200\pi t)-1\right)+200e^{100t}\sin(200\pi t)}{404784.2}\mathrm{u}(t)$$

$$=\frac{\cos(200\pi t)+2\sin(200\pi t)-e^{-100t}}{4047.842}\mathrm{u}(t)$$

If $x(t) = e^{-20t}\cos(200\pi t)u(t)$ and $h(t) = e^{-100t}u(t)$ and $y(t) = x(t) * h(t)$ find $y(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} e^{-20\tau}\cos(200\pi\tau)u(\tau)e^{-100(t-\tau)}u(t-\tau)d\tau$$

$$y(t) = e^{-100t}\int_{0}^{t} e^{80\tau}\cos(200\pi\tau)d\tau = \frac{e^{-100t}}{2}\int_{0}^{t} e^{80\tau}\left(e^{j200\pi\tau} + e^{-j200\pi\tau}\right)d\tau \quad , \; t > 0 \; ; y(t) = 0 \; , t < 0$$

$$y(t) = \frac{e^{-100t}}{2}u(t)\int_{0}^{t}\left(e^{(80+j200\pi)\tau} + e^{(80-j200\pi)\tau}\right)d\tau = \frac{e^{-100t}}{2}u(t)\left[\frac{e^{(80+j200\pi)\tau}}{80+j200\pi} + \frac{e^{(80-j200\pi)\tau}}{80-j200\pi}\right]_0^t$$

$$y(t) = \frac{e^{-100t}}{2}\left(\frac{e^{(80+j200\pi)t}-1}{80+j200\pi} + \frac{e^{(80-j200\pi)t}-1}{80-j200\pi}\right)u(t)$$

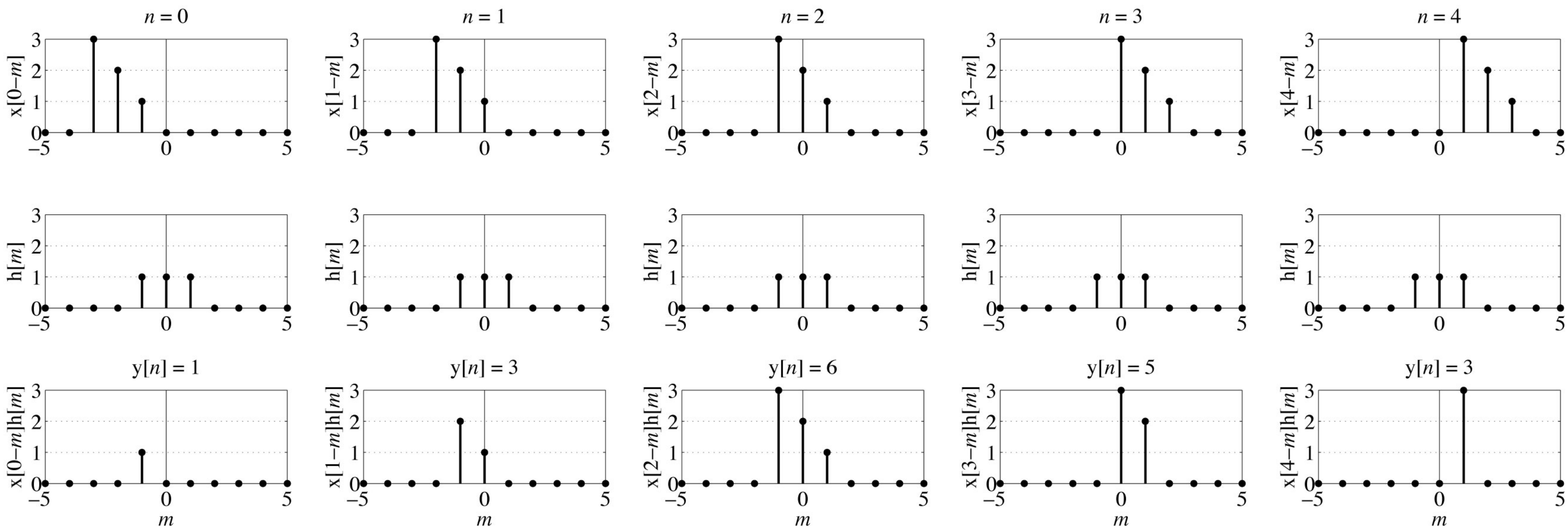
$$= \frac{e^{-100t}}{2}\frac{80\left(e^{(80+j200\pi)t} + e^{(80-j200\pi)t} - 2\right) + j200\left(e^{(80-j200\pi)t} - e^{(80+j200\pi)t}\right)}{80^2 + (200\pi)^2}u(t)$$

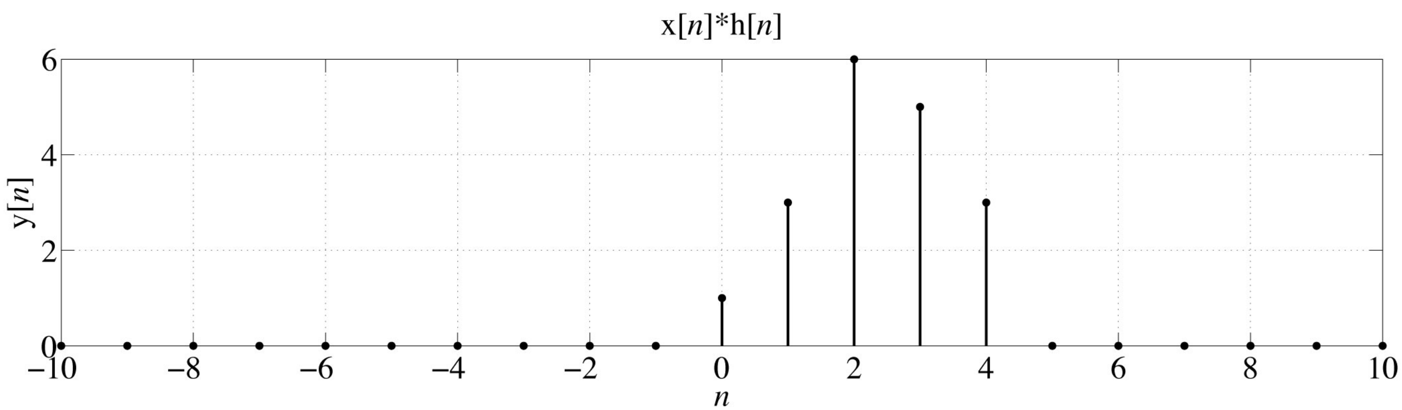
$$= \frac{e^{-100t}}{2}\frac{160\left(e^{80t}\cos(200\pi t) - 1\right) + j200\left(-j2e^{80t}\sin(200\pi t)\right)}{401184.2}u(t)$$

$$= e^{-100t}\frac{80\left(e^{80t}\cos(200\pi t) - 1\right) + 200e^{80t}\sin(200\pi t)}{401184.2}u(t)$$

$$= \frac{e^{-20t}\left[0.8\cos(200\pi t) + 2\sin(200\pi t)\right] - 0.8e^{-100t}}{4011.842}u(t)$$

If $\mathrm{x}[n] = \mathrm{ramp}[n]\mathrm{u}[3-n]$ and $\mathrm{h}[n] = \mathrm{u}[n+1] - \mathrm{u}[n-2]$ and $\mathrm{y}[n] = \mathrm{x}[n] * \mathrm{h}[n]$ find the signal energy of $\mathrm{y}[n]$.





$$E_{\mathrm{y}} = 1^2 + 3^2 + 6^2 + 5^2 + 3^2 = 80$$

If $\mathrm{x}[n]=\mathrm{u}[n+4]$ and $\mathrm{h}[n]=-\mathrm{u}[n-1]$ and $\mathrm{y}[n]=\mathrm{x}[n]*\mathrm{h}[n]$, find $\mathrm{y}[n]$.

$$\mathrm{y}[n]=\sum_{m=-\infty}^{\infty}\mathrm{x}[m]\mathrm{h}[n-m]=\sum_{m=-\infty}^{\infty}\mathrm{u}[m+4]\left(-\mathrm{u}[n-m-1]\right)$$

$$=\left\{\begin{array}{ll}-\sum_{m=-4}^{n-1}1 & , \; n\geq -3 \\ 0 & , \; n<-3\end{array}\right\}=\left(-\sum_{m=-4}^{n-1}1\right)\mathrm{u}[n+3]$$

n	-4	-3	-2	-1	0
$\mathrm{y}[n]$	0	-1	-2	-3	-4

$$\cdots\Rightarrow \mathrm{y}[n]=-\mathrm{ramp}[n+4]$$

If $x[n] = u[n-2] - u[n-6]$ and $h[n] = u[n+3] - u[n-3]$ and $y[n] = x[n] * h[n]$, find $y[n]$.

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

$$= \sum_{m=-\infty}^{\infty} \left(u[m-2] - u[m-6]\right)\left(u[n-m+3] - u[n-m-3]\right)$$

$$= \sum_{m=2}^{5} \left(u[n-m+3] - u[n-m-3]\right)$$

In words, for any value of n, add the impulses in $\left(u[n-m+3] - u[n-m-3]\right)$ for m ranging from 2 to 5. For example, let $n = 0$. Then

$$y[0] = \sum_{m=2}^{5} \left(u[-m+3] - u[-m-3]\right)$$

$$= \left(u[1] - u[-5]\right) + \left(u[0] - u[-6]\right) + \left(u[-1] - u[-7]\right) + \left(u[-2] - u[-8]\right) = 2$$

If $\mathrm{x}[n]=(0.9)^n \mathrm{u}[n]$ and $\mathrm{h}[n]=\mathrm{u}[n-4]$ and $\mathrm{y}[n]=\mathrm{x}[n]*\mathrm{h}[n]$ find $\mathrm{y}[n]$.

$$\mathrm{y}[n]=\sum_{m=-\infty}^{\infty}\mathrm{x}[m]\mathrm{h}[n-m]=\sum_{m=-\infty}^{\infty}(0.9)^m \mathrm{u}[m]\mathrm{u}[n-m-4]$$

$$\mathrm{y}[n]=\left\{\begin{array}{ll}\sum_{m=0}^{n-4}(0.9)^m & , \ n\geq 4 \\ 0 & , \ n<4\end{array}\right\}=\frac{1-(0.9)^{n-3}}{1-0.9}\mathrm{u}[n-4]$$

$$=10\left[1-(0.9)^{n-3}\right]\mathrm{u}[n-4]$$

This starts with value 1 at time $n=4$ and approaches, in a decaying exponential form, a final value of 10 as $n\to\infty$.