If $\mathrm{x}(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \delta(f-8) + \delta(f+8)$ and $\mathrm{x}(t) \xleftrightarrow[1]{\mathcal{FS}} \mathrm{c_x}[k]$, find $\mathrm{c_x}[k]$.

The relationship between the CTFT and the CTFS harmonic function is

$$\mathrm{x}(t) = \sum_{k=-\infty}^{\infty} \mathrm{c_x}[k] e^{j2\pi kt/T} \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \sum_{k=-\infty}^{\infty} \mathrm{c_x}[k] \delta(f - k/T)$$

In this case, setting $T = 1$,

$$\mathrm{x}(t) = \sum_{k=-\infty}^{\infty} \mathrm{c_x}[k] e^{j2\pi kt} \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \sum_{k=-\infty}^{\infty} (\delta[k-8] + \delta[k+8]) \delta(f-k)$$

Therefore $\mathrm{c_x}[k] = \delta[k-8] + \delta[k+8]$.

If we instead set $T = 1/8$,

$$\mathrm{x}(t) = \sum_{k=-\infty}^{\infty} \mathrm{c_x}[k] e^{j16\pi kt} \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \sum_{k=-\infty}^{\infty} (\delta[k-1] + \delta[k+1]) \delta(f-8k)$$

and $\mathrm{x}(t) \xleftrightarrow[1/8]{\mathcal{FS}} \delta[k-1] + \delta[k+1]$. Then, using the CTFS property

$$\begin{array}{r} \mathrm{x}(t) \xleftrightarrow[T]{\mathcal{FS}} \mathrm{c}_x[k] \\ \text{and } \mathrm{x}(t) \xleftrightarrow[mT]{\mathcal{FS}} c_{xm}[k] \end{array} \Rightarrow \mathrm{c}_{xm}[k] = \begin{cases} \mathrm{c}_x[k/m] & , k/m \text{ an integer} \\ 0 & , \text{otherwise} \end{cases}$$

$$\mathrm{x}(t) \xleftrightarrow[1]{\mathcal{FS}} \left. \begin{cases} \delta[k/8-1] + \delta[k/8+1] & , k/m \text{ an integer} \\ 0 & , \text{otherwise} \end{cases} \right\} = \delta[k-8] + \delta[k+8].$$

A continuous-time system has a transfer function

$$\mathrm{H}(s) = \frac{2\times 10^6}{s^2 + 2000s + 2\times 10^6}$$

and therefore a frequency response

$$\mathrm{H}(j\omega) = \frac{2\times 10^6}{(j\omega)^2 + j2000\omega + 2\times 10^6} = \frac{2\times 10^6}{(j\omega + 1000)^2 + 10^6}.$$

Find its impulse response.

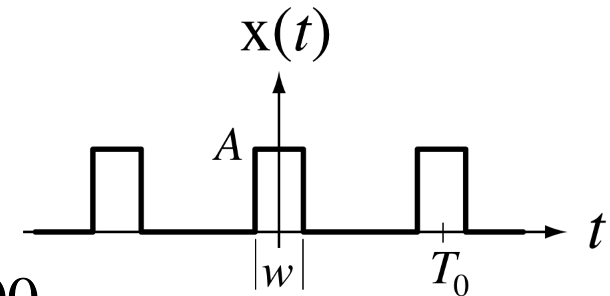
$$e^{-\alpha t}\sin(\omega_n t)\mathrm{u}(t) \xleftrightarrow{\mathscr{F}} \frac{\omega_n}{(j\omega+\alpha)^2 + \omega_n^2}\ , \mathrm{Re}(\alpha) > 0$$

$$e^{-1000t}\sin(1000t)\mathrm{u}(t) \xleftrightarrow{\mathscr{F}} \frac{1000}{(j\omega+1000)^2 + (1000)^2}$$

$$2000e^{-1000t}\sin(1000t)\mathrm{u}(t) \xleftrightarrow{\mathscr{F}} \frac{2\times 10^6}{(j\omega+1000)^2 + (1000)^2}$$

Therefore $\mathrm{h}(t) = 2000e^{-1000t}\sin(1000t)\mathrm{u}(t)$

This rectangular pulse train excites an amplifier with transfer function $\mathrm{H}(s) = \dfrac{500{,}000}{s + 50{,}000}$ and therefore frequency response $\mathrm{H}(f) = \dfrac{500{,}000}{j2\pi f + 50{,}000}$.



If $T_0 = 0.5$ ms, $w = 0.1$ ms and $A = 100$ mV find the average signal power of the amplifier response (using MATLAB where necessary).

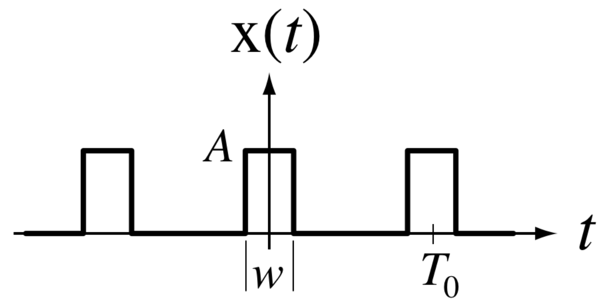
$$\mathrm{rect}(t) \xleftrightarrow{\mathscr{F}} \mathrm{sinc}(f)$$

$$0.1\,\mathrm{rect}(10{,}000t) \xleftrightarrow{\mathscr{F}} \frac{0.1\,\mathrm{sinc}(f/10{,}000)}{10{,}000}$$

$$0.1\,\mathrm{rect}(10{,}000t) * \delta_{0.5\,\mathrm{ms}}(t) \xleftrightarrow{\mathscr{F}} \frac{0.1\,\mathrm{sinc}(f/10{,}000)}{10{,}000} 2000\,\delta_{2000}(f)$$

$$0.1\,\mathrm{rect}(10{,}000t) * \delta_{0.5\,\mathrm{ms}}(t) \xleftrightarrow{\mathscr{F}} \frac{\mathrm{sinc}(f/10{,}000)}{50} \delta_{2000}(f)$$

$$\mathrm{Y}(f) = \mathrm{H}(f)\mathrm{X}(f)$$



$$\mathrm{Y}(f) = \frac{500{,}000}{j2\pi f + 50{,}000} \frac{\mathrm{sinc}(f/10{,}000)}{50} \delta_{2000}(f)$$

Using the definition of the periodic impulse,

$$\mathrm{Y}(f) = (1/50) \sum_{k=-\infty}^{\infty} \frac{500{,}000\,\mathrm{sinc}(f/10{,}000)}{j2\pi f + 50{,}000} \delta(f - 2000k)$$

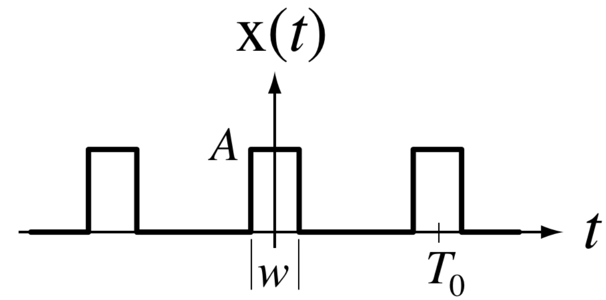
Using the equivalence property of the impulse

$$\mathrm{Y}(f) = (1/50) \sum_{k=-\infty}^{\infty} \frac{500{,}000\,\mathrm{sinc}(k/5)}{j4000\pi k + 50{,}000} \delta(f - 2000k)$$

From Parseval's theorem, $P_{\mathrm{y}} = \sum_{k=-\infty}^{\infty} \left|\mathrm{c}_{\mathrm{y}}[k]\right|^2$.

$$\mathrm{c}_{\mathrm{y}}[k] = \frac{1}{50} \frac{500{,}000\,\mathrm{sinc}(k/5)}{j4000\pi k + 50{,}000} \Rightarrow P_{\mathrm{y}} = (1/2500) \sum_{k=-\infty}^{\infty} \left|\frac{500{,}000\,\mathrm{sinc}(k/5)}{j4000\pi k + 50{,}000}\right|^2$$

$$P_y = (1/2500)\sum_{k=-\infty}^{\infty}\left|\frac{500{,}000\,\mathrm{sinc}(k/5)}{j4000\pi k + 50{,}000}\right|^2$$



We can find this quantity using MATLAB.

...

```
k = [-kmax:kmax]';
Py = sum(abs(5e5*sinc(k/5)./(j*4000*k+5e4)).^2)/2500;
```

For `kmax = 10`, `Py = 0.1845`

For `kmax = 20`, `Py = 0.1867`

For `kmax = 50`, `Py = 0.1872`

For `kmax = 100`, `Py = 0.1873`

For `kmax = 200`, `Py = 0.1873`

If the amplifier had infinite bandwidth the response signal power would be 0.2.

The signal from a pressure sensor in an industrial plant is interfered by radiated EMI (electromagnetic interference) from a periodic rectangular pulse train of fundamental frequency 15 kHz. What would be the impulse response of a filter that would reject this EMI, including all its harmonics?

The source of the EMI is of the form $\mathrm{e}(t) = A\,\mathrm{rect}(t/w) * \delta_{T_0}(t)$.

The mechanism of interference through radiation depends on the first derivative of the EMI. So the received EMI is of the form $\mathrm{e}(t) = A\left[\delta(t + w/2) - \delta(t - w/2)\right] * \delta_{T_0}(t)$. Its CTFT is

$\mathrm{E}(f) = A\left[e^{j2\pi fw/2} - e^{-j2\pi fw/2}\right] f_0 \delta_{f_0}(f) = j2A\sin(2\pi fw/2) f_0 \delta_{f_0}(f)$.

where $f_0 = 1/T_0$. So it has impulses at integer multiples of f_0. An impulse response that averages the signal over exactly one fundamental period T_0 of the EMI would be $\mathrm{h}(t) = B\,\mathrm{rect}(t/T_0)$. Its CTFT is

$\mathrm{H}(f) = BT_0\,\mathrm{sinc}(T_0 f) = BT_0\,\mathrm{sinc}(f/f_0)$. This frequency response has nulls at integer multiples of f_0. So it would reject the EMI, including all its harmonics.