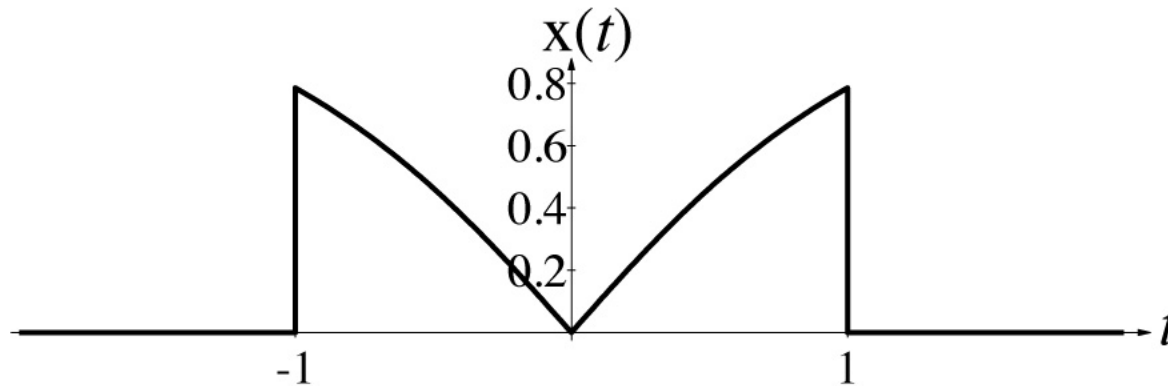


A signal $x(t)$ is described by

$$x(t) = \begin{cases} \tan^{-1}(-t) & , \quad -1 < t < 0 \\ \tan^{-1}(t) & , \quad 0 < t < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

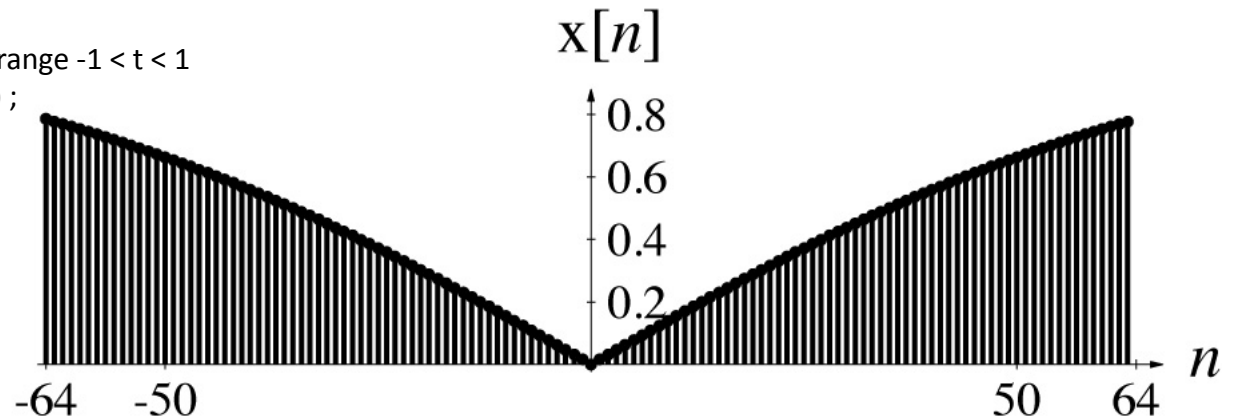


Graph its CTFT over a range of frequencies from -10 Hz to +10 Hz.

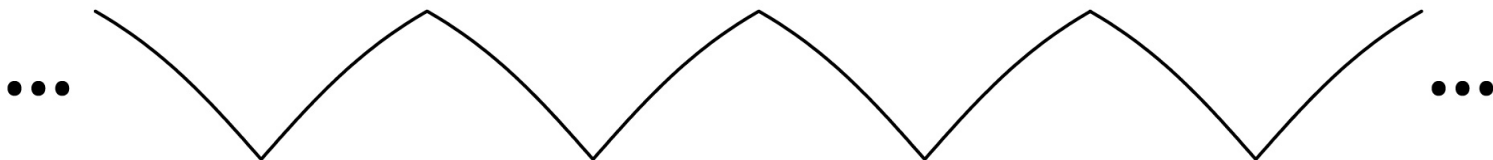
This CTFT is difficult, if not impossible, analytically. We can approximate it using the DFT.

Let's take 128 samples covering the non-zero part of the signal.

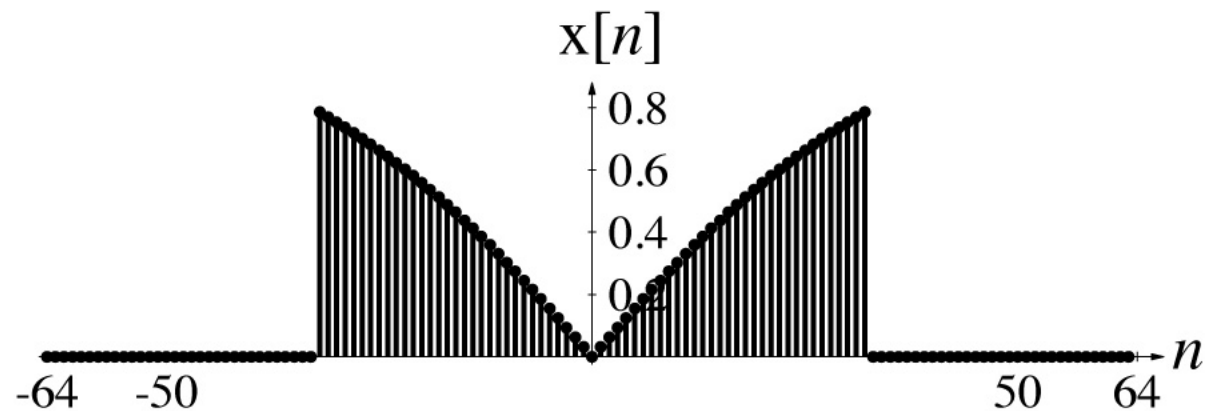
```
N = 128; % Number of samples
T = 2; % Total sampling time
Ts = T/N; % Time between samples
n = [-N/2:N/2-1]'; % Vector of discrete times for graphing samples
fs = 1/Ts; % Sampling rate
% Compute samples of the signal over the time range -1 < t < 1
x = atan(-n*Ts).*(n*Ts<0) + atan(n*Ts).*(n*Ts>=0);
```



This is good sampling of the non-zero part of the signal but if these are all the samples we take, the DFT we get will be from a periodic extension of this set of samples and our approximation will be for a periodic signal of this shape.

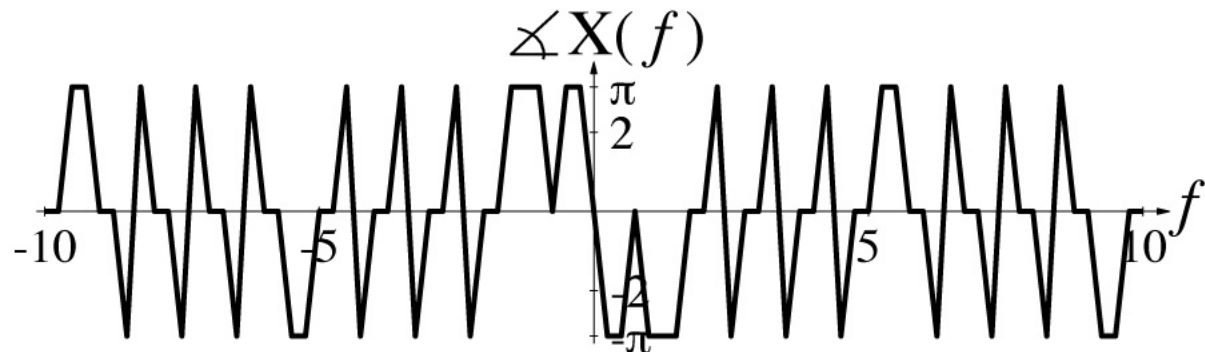
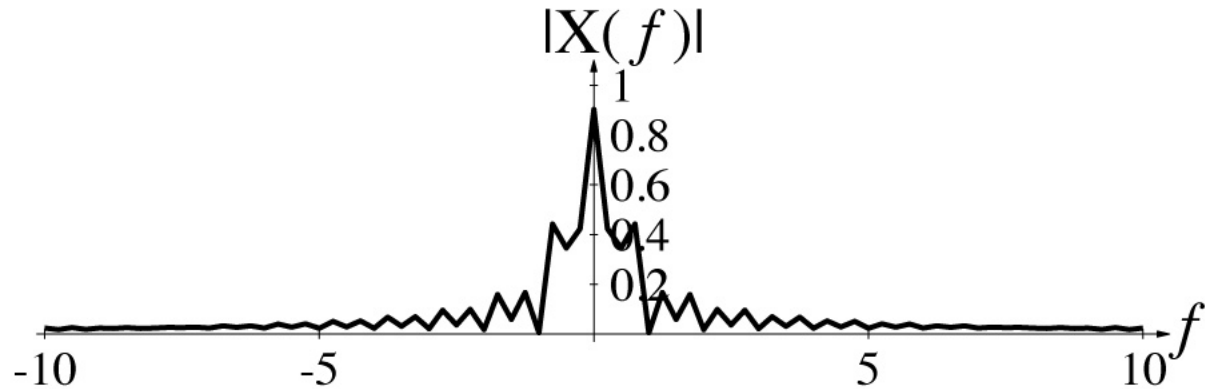


We will get a better approximation if we take a set of samples more representative of the actual shape, including the sudden transition to zero at both ends.



From the last sampling, the graphs of the magnitude and phase of the CTFT look like this.

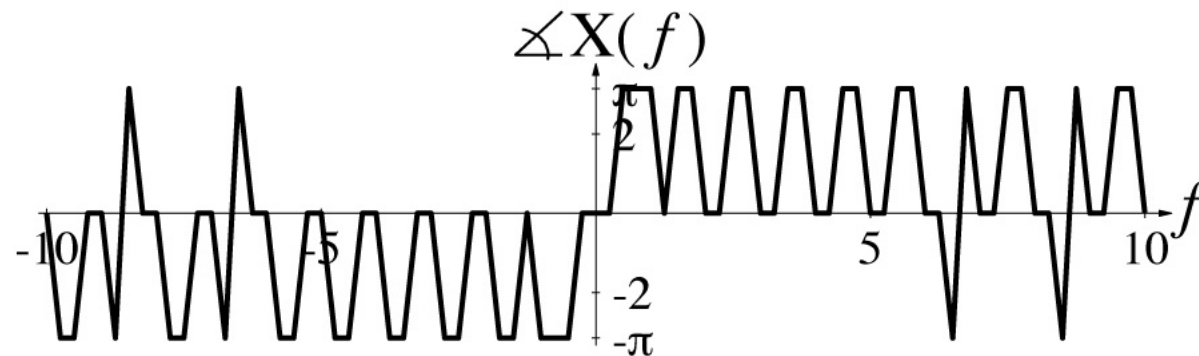
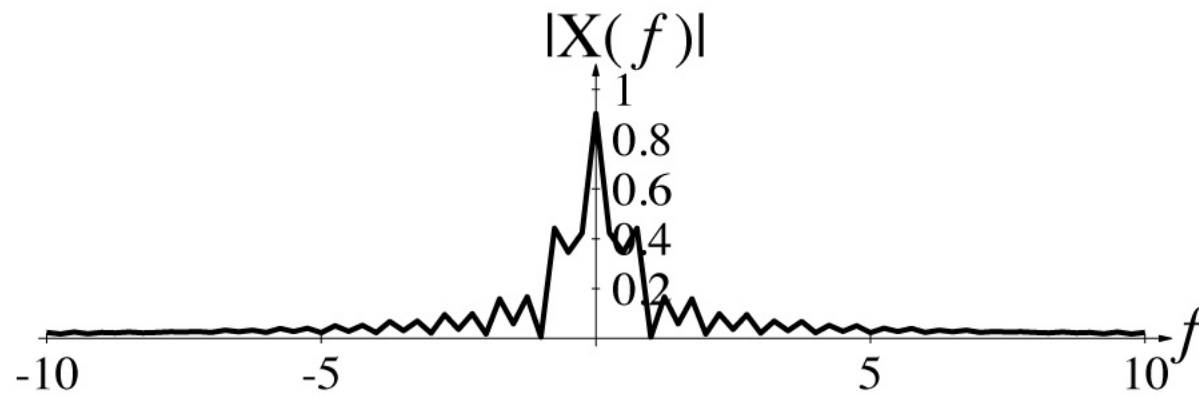
```
X = Ts*fftshift(fft(x));  
k = n ; f = k*fs/N ;
```



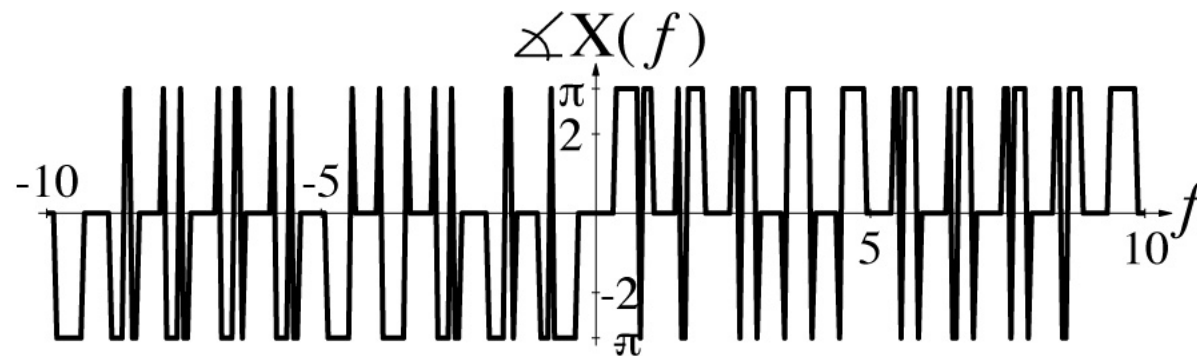
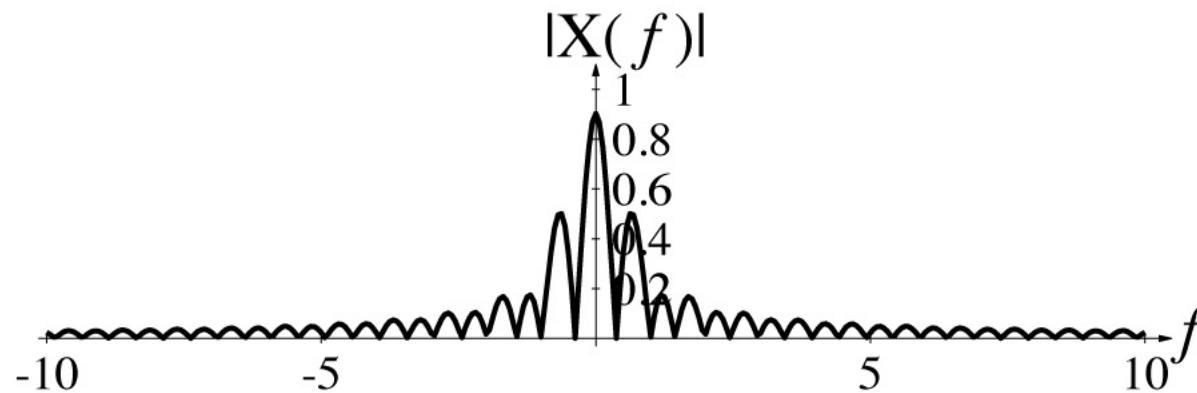
There is still a problem with this approximation to the CTFT. What is it?

The DFT transforms a set of numbers under the assumption that the first number is the sample of the time-domain signal occurring at time $t = 0$. But our sampling started at time $t = -2$. So there is a time shifting effect because of this difference. This affects the phase, but not the magnitude. So the magnitude graph is ok. What we have is the CTFT of our original signal, except delayed by two seconds. To compensate for that delay we can multiply the answer by the complex exponential corresponding to an advance in time by two seconds, $e^{j4\pi f}$. Then the phase will be correct.

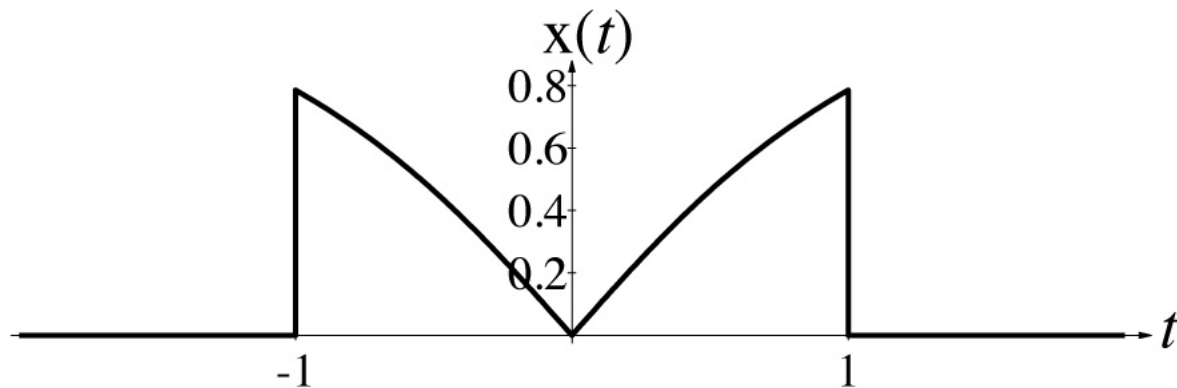
Compensating for the shift we get these graphs of magnitude and phase. These graphs are a little grainy. We can improve the resolution in frequency by using a larger set of samples over a longer time.



For this final CTFT approximation, 512 samples over the time range $-8 < t < 8$ were taken. This is finally a good approximation to the CTFT of the original time-limited, continuous-time signal.



How could we check this CTFT approximation to determine whether it is reasonable? Look again at the original signal.



It looks a lot like $x(t) = 0.7854 [\text{rect}(t/2) - \text{tri}(t)]$ whose CTFT is $X(f) = 0.7854 [2 \text{sinc}(2f) - \text{sinc}^2(f)]$

This graphs are the magnitude and phase of

$$X(f) = 0.7854 [2 \operatorname{sinc}(2f) - \operatorname{sinc}^2(f)]$$

