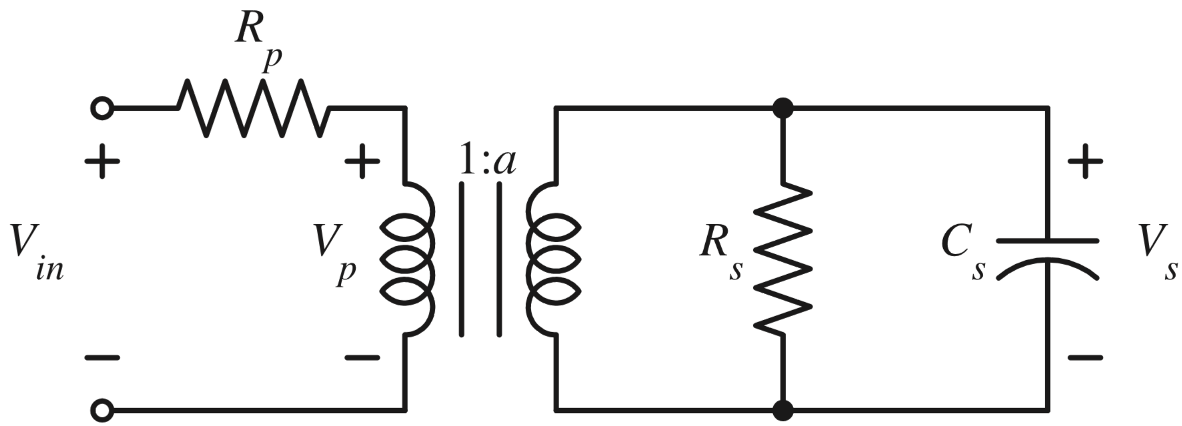
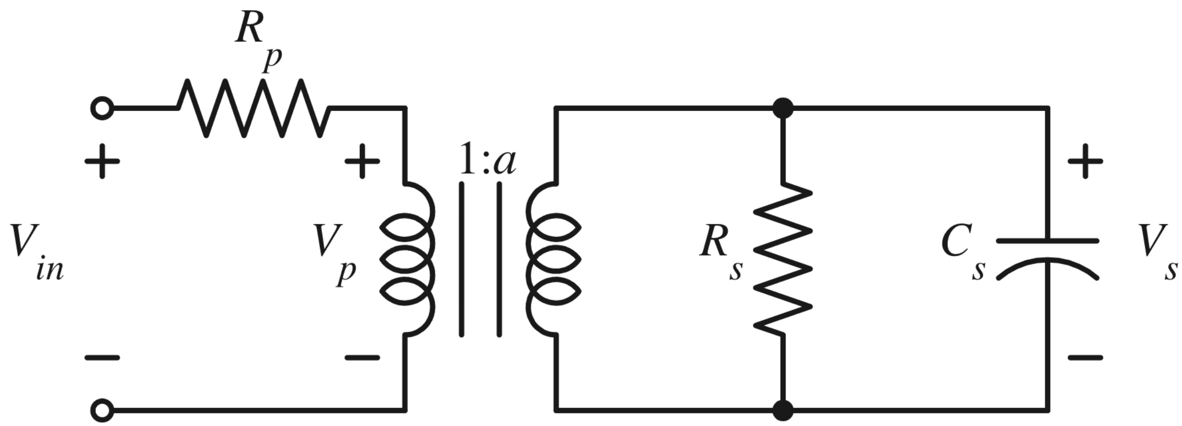


Let $R_p = 2\ \text{k}\Omega$, $a = 4$, $R_s = 6\ \text{k}\Omega$, $C_s = 500\ \text{nF}$

Draw a Bode diagram of the magnitude and phase of the frequency response $\text{H}(j\omega) = \dfrac{V_s(j\omega)}{V_{in}(j\omega)}$.

The secondary impedance is $Z_s(j\omega) = \dfrac{R_s / j\omega C_s}{R_s + 1/j\omega C_s} = \dfrac{R_s}{j\omega R_s C_s + 1}$.

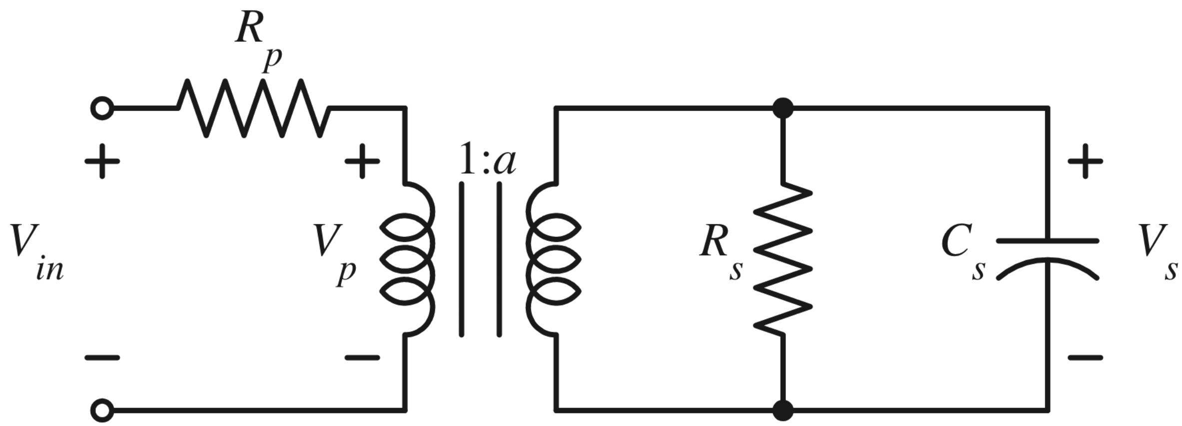


The secondary impedance reflects through the ideal transformer to an equivalent primary impedance of

$Z_p(j\omega) = Z_s(j\omega)/a^2 = \dfrac{R_s/a^2}{j\omega R_s C_s + 1}$. The primary voltage is then

$$V_p(j\omega) = \frac{Z_p(j\omega)}{R_p + Z_p(j\omega)} V_{in}(j\omega) = \frac{\dfrac{R_s/a^2}{j\omega R_s C_s + 1}}{R_p + \dfrac{R_s/a^2}{j\omega R_s C_s + 1}} V_{in}(j\omega)$$

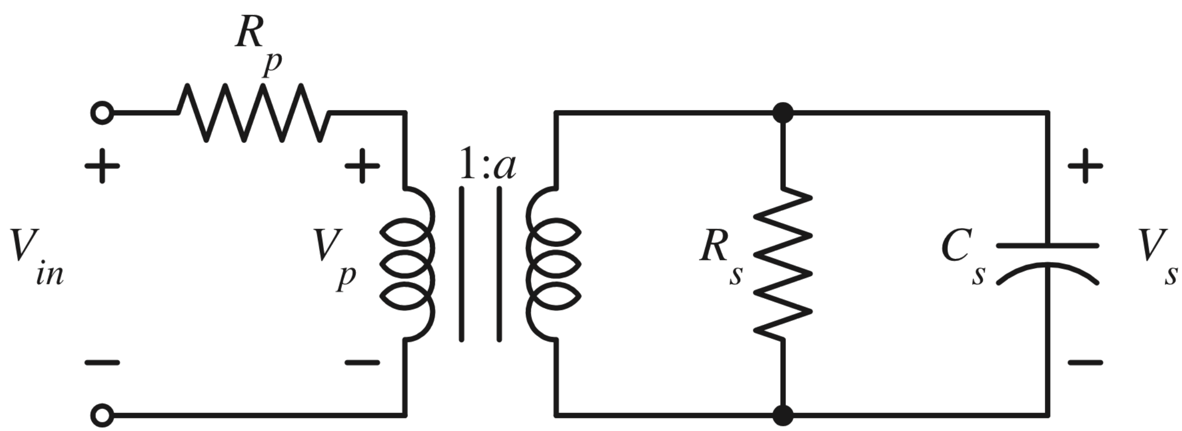
$$= \frac{R_s/a^2}{j\omega R_p R_s C_s + \left(R_p + R_s/a^2\right)} V_{in}(j\omega)$$



$$V_p(j\omega) == \frac{R_s / a^2}{j\omega R_p R_s C_s + \left(R_p + R_s / a^2\right)} V_{in}(j\omega)$$

$$V_s(j\omega) = aV_p(j\omega) \Rightarrow V_s(j\omega) = \frac{R_s / a}{j\omega R_p R_s C_s + \left(R_p + R_s / a^2\right)} V_{in}(j\omega)$$

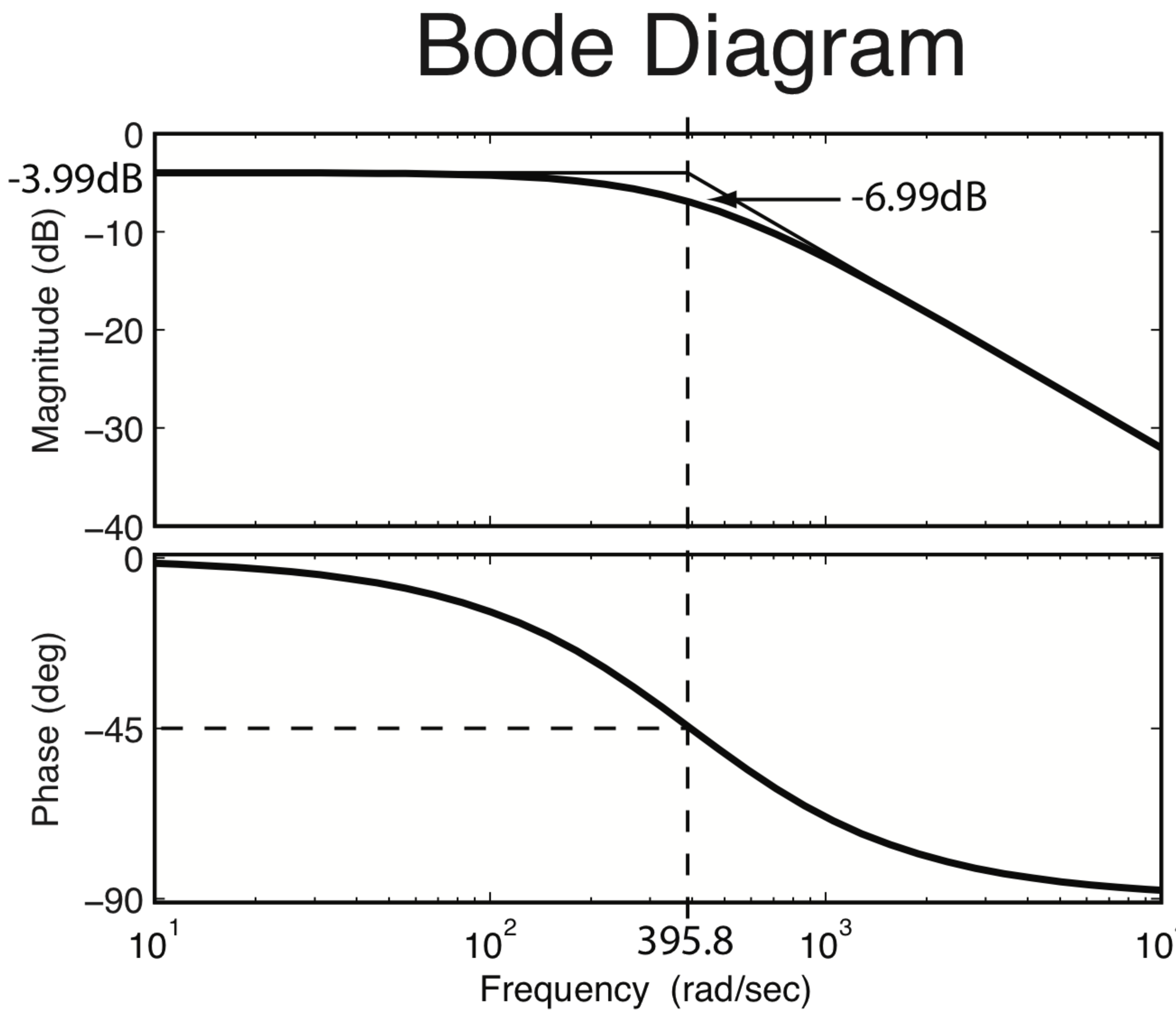
$$\mathrm{H}(j\omega) = \frac{R_s / a}{j\omega R_p R_s C_s + \left(R_p + R_s / a^2\right)} = \frac{1}{aR_p C_s} \frac{1}{j\omega + \dfrac{R_p + R_s / a^2}{R_p R_s C_s}}$$



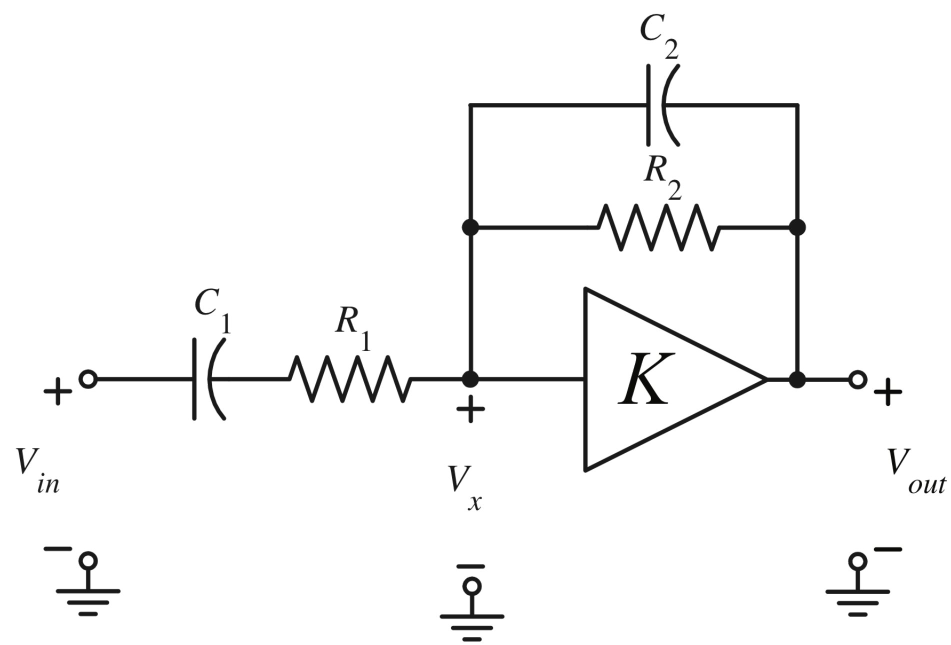
Putting the frequency response into a form that illustrates the simple system components,

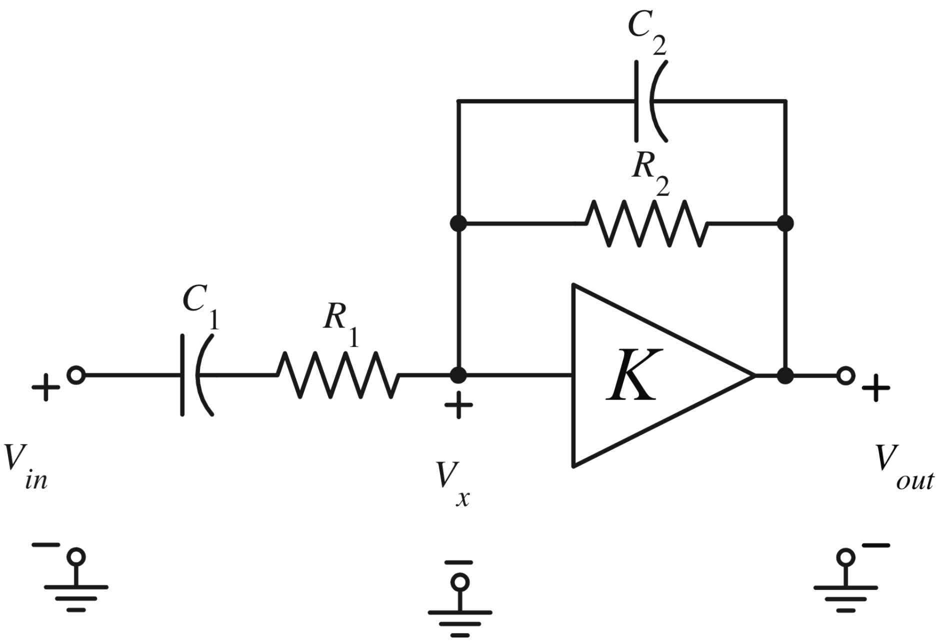
$$\mathrm{H}(j\omega) = \frac{1}{aR_pC_s}\frac{1}{j\omega + \frac{R_p + R_s/a^2}{R_pR_sC_s}} = \underbrace{\frac{R_s}{aR_p + R_s/a}}_{\text{Frequency Independent Gain}} \underbrace{\frac{1}{1 - \frac{j\omega}{-\frac{R_p + R_s/a^2}{R_pR_sC_s}}}}_{\text{Single-Real-Pole System}}$$

$$= 0.63158\frac{1}{1 - \frac{j\omega}{-395.8}}.$$ The single real pole is at $j\omega = -395.8$.


Bode Diagram
Magnitude (dB)
0
-3.99dB
-6.99dB
−10
−20
−30
−40
Phase (deg)
0
−45
−90
10^1
10^2
395.8
10^3
10^4
Frequency (rad/sec)

Let $C_1 = 20\ \text{nF}$, $R_1 = 20\ \text{k}\Omega$, $C_2 = 100\ \text{nF}$, $R_2 = 5\ \text{k}\Omega$ and $K = 8$. Draw a Bode diagram of the magnitude and phase of the frequency response $\text{H}(j\omega) = \dfrac{V_{out}(j\omega)}{V_{in}(j\omega)}$ of this Sallen-Key bandpass filter.

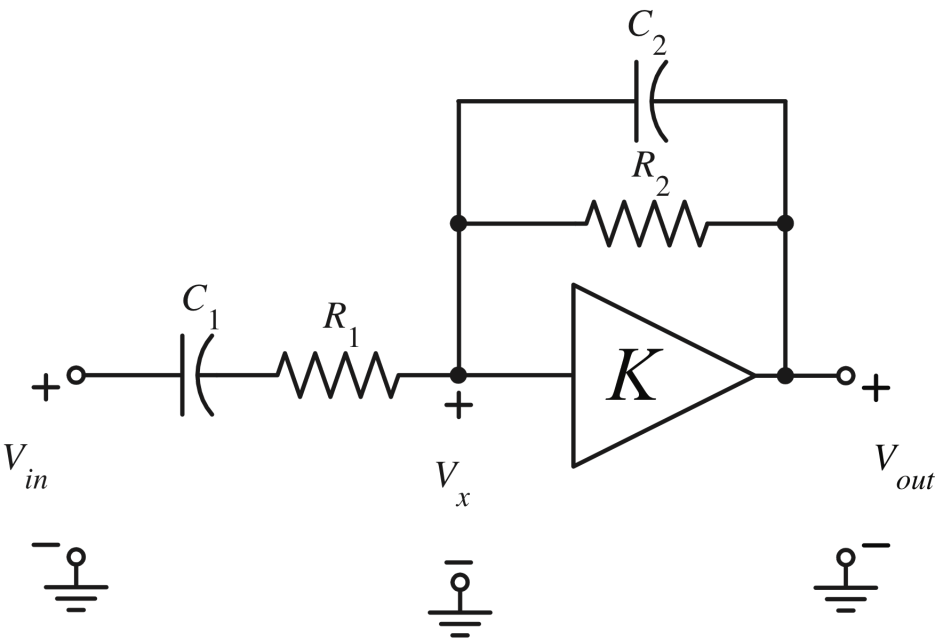




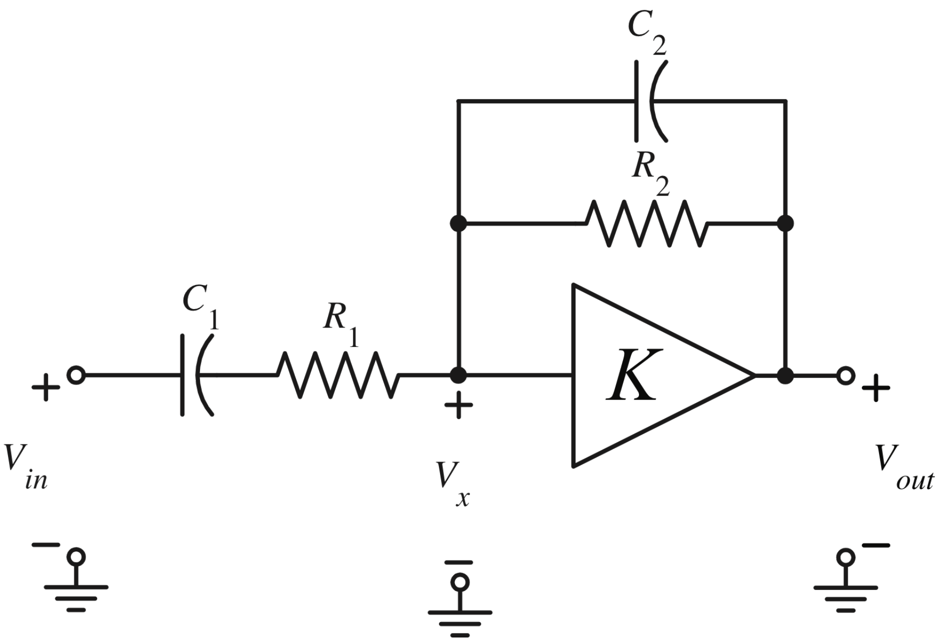
$$\left[\frac{1}{R_1 + 1/j\omega C_1} + \left(j\omega C_2 + G_2\right)\right] V_x(j\omega) - \left(\frac{1}{R_1 + 1/j\omega C_1}\right) V_{in}(j\omega) - \left(j\omega C_2 + G_2\right) V_{out}(j\omega) = 0$$

$$V_{out}(j\omega) = K V_x(j\omega)$$

$$\begin{bmatrix} \dfrac{1}{R_1 + 1/j\omega C_1} + \left(j\omega C_2 + G_2\right) & -\left(j\omega C_2 + G_2\right) \\ -K & 1 \end{bmatrix} \begin{bmatrix} V_x(j\omega) \\ V_{out}(j\omega) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R_1 + 1/j\omega C_1} V_{in}(j\omega) \\ 0 \end{bmatrix}$$

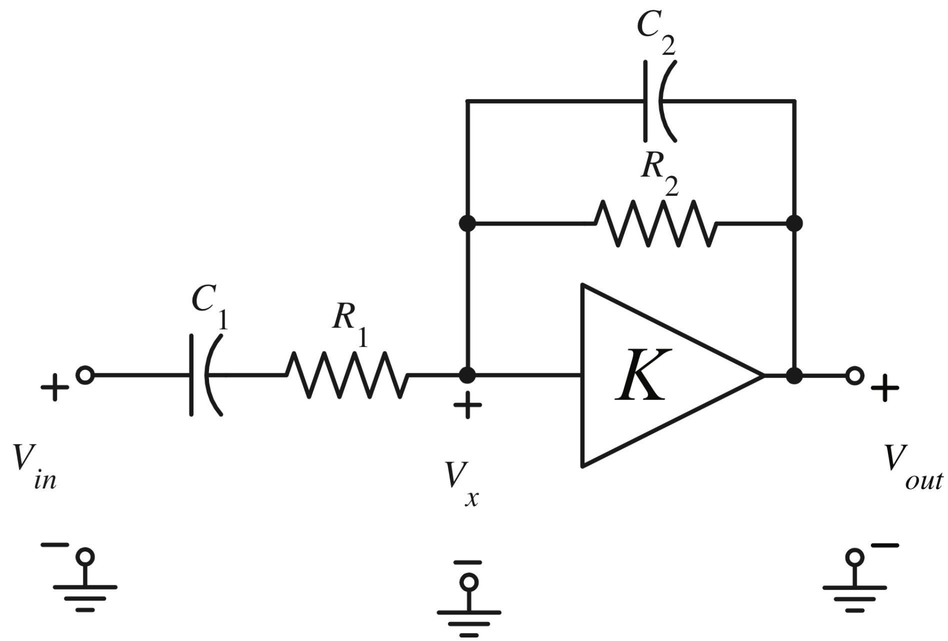


$$\mathrm{H}(j\omega)=\frac{V_{out}(j\omega)}{V_{in}(j\omega)}=\frac{\begin{vmatrix} \dfrac{1}{R_1+1/j\omega C_1}+\left(j\omega C_2+G_2\right) & \dfrac{1}{R_1+1/j\omega C_1} \\ -K & 0 \end{vmatrix}}{\begin{vmatrix} \dfrac{1}{R_1+1/j\omega C_1}+\left(j\omega C_2+G_2\right) & -\left(j\omega C_2+G_2\right) \\ -K & 1 \end{vmatrix}}$$



$$\mathrm{H}(j\omega) = \frac{\dfrac{K}{R_1 + 1/j\omega C_1}}{\dfrac{1}{R_1 + 1/j\omega C_1} + (j\omega C_2 + G_2) - K(j\omega C_2 + G_2)}$$

$$\mathrm{H}(j\omega) = \frac{1}{R_1 C_2} \frac{j\omega K/(1-K)}{(j\omega)^2 + j\omega\left[1/R_1 C_2(1-K) + 1/R_2 C_2 + 1/R_1 C_1\right] + 1/R_1 R_2 C_1 C_2}$$

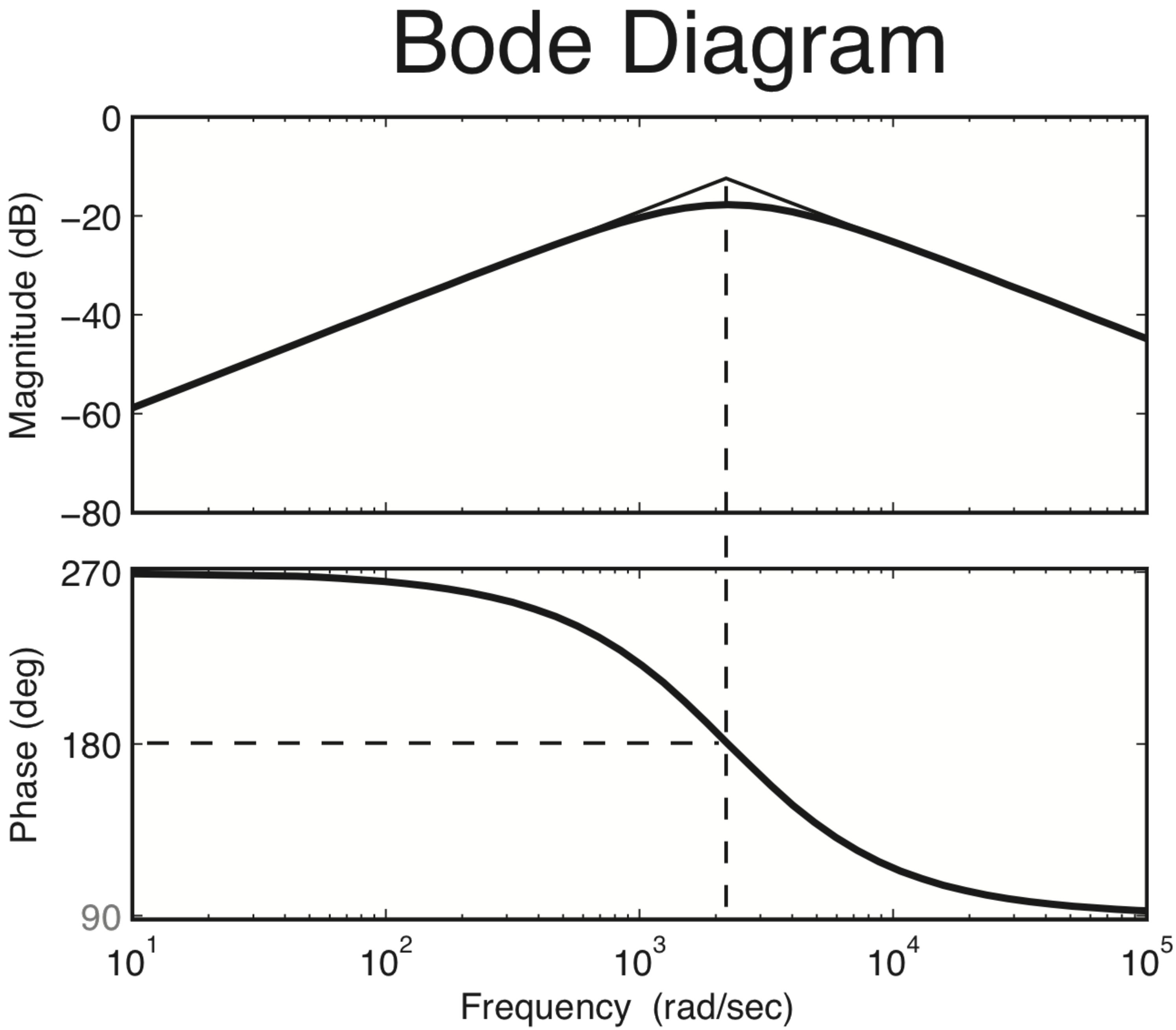


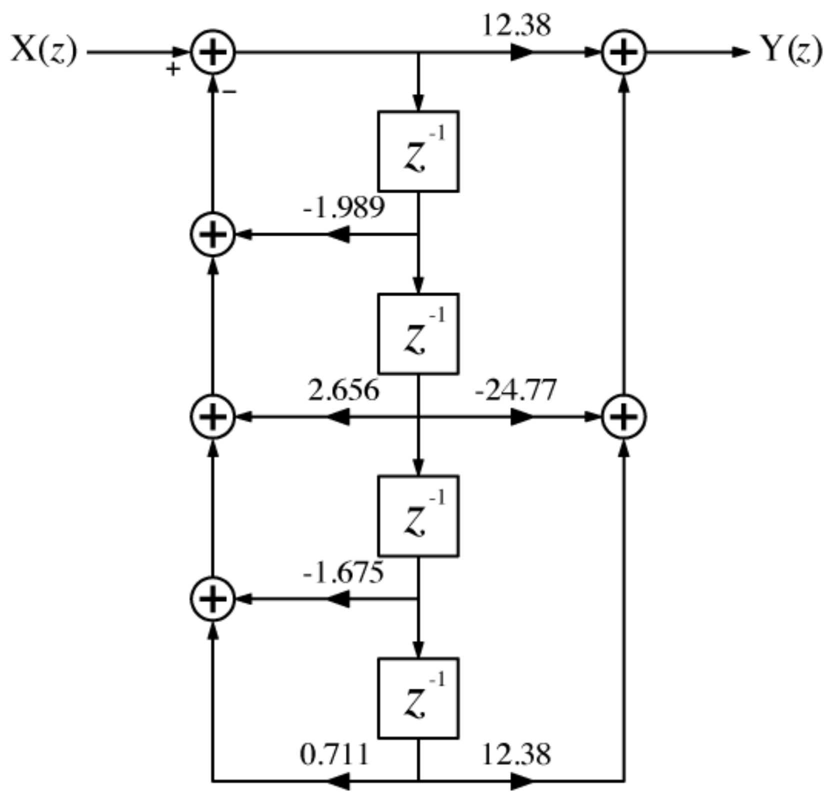
$$\mathrm{H}(j\omega)=\frac{1}{R_1C_2}\frac{j\omega K/(1-K)}{(j\omega)^2+j\omega\left[1/R_1C_2(1-K)+1/R_2C_2+1/R_1C_1\right]+1/R_1R_2C_1C_2}$$

Substituting in numerical values

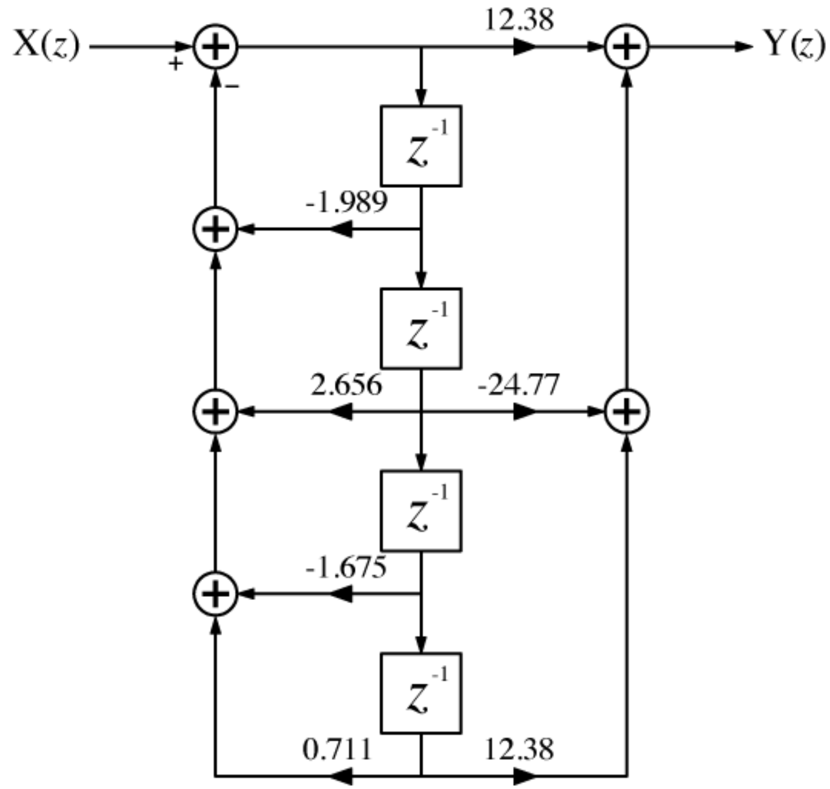
$$\mathrm{H}(j\omega)=-571.4\frac{j\omega}{(j\omega)^2+4429(j\omega)+5\times10^6}$$

Poles at $s=-2214.5\pm j309.82\Rightarrow$ Underdamped


Bode Diagram
Magnitude (dB)
0
−20
−40
−60
−80
Phase (deg)
270
180
90
10^1
10^2
10^3
10^4
10^5
Frequency (rad/sec)



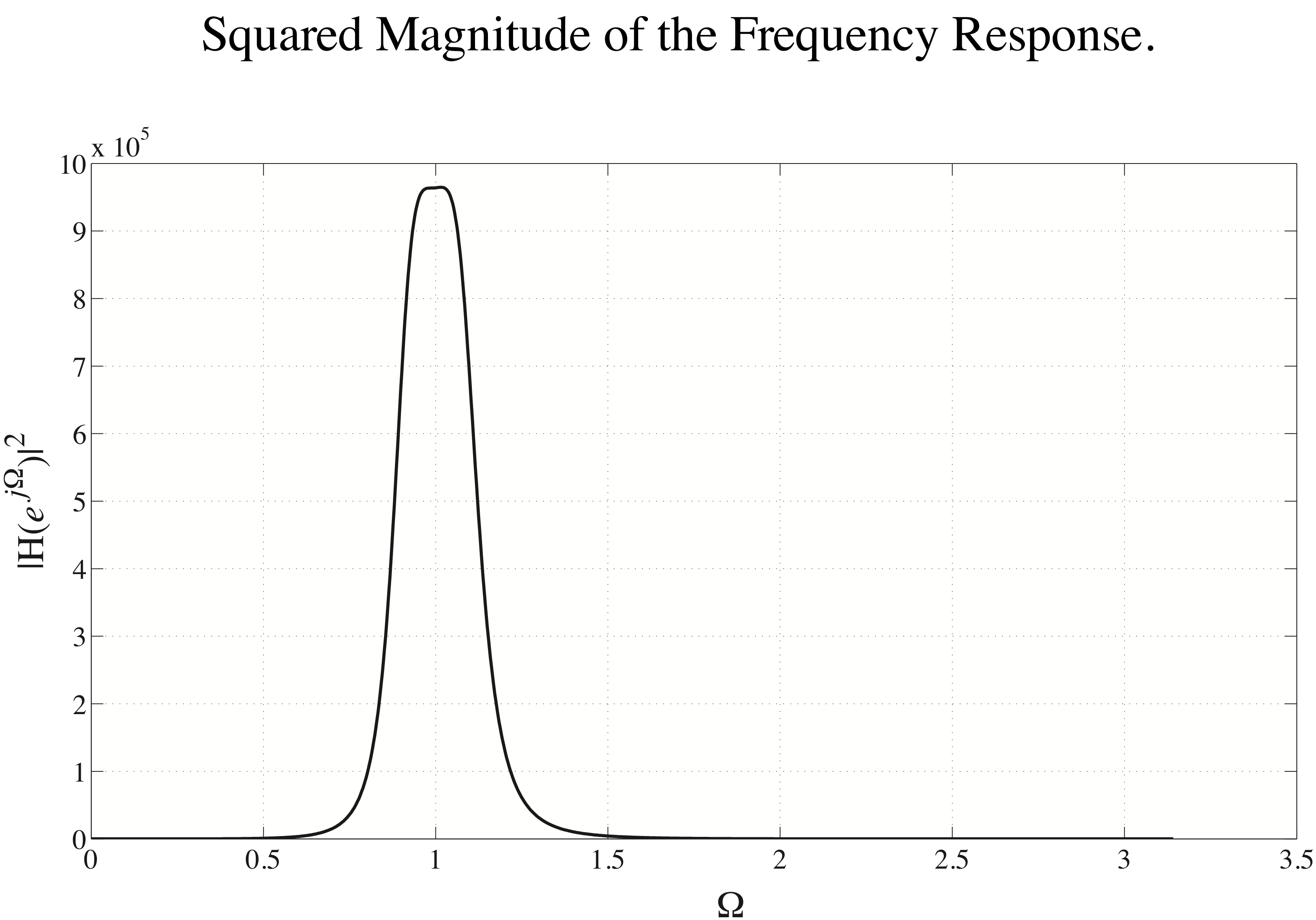
A signal generator generates a constant-amplitude, variable-frequency analog sinusoid that is sampled at a rate of 1 kHz to form the input signal for this bandpass digital filter. Find the frequency of the signal generator that corresponds to the center frequency of the filter. Also find the effective -3dB bandwidth of the filter by finding the signal frequencies at which the power of the output signal is half of its value at the center frequency.



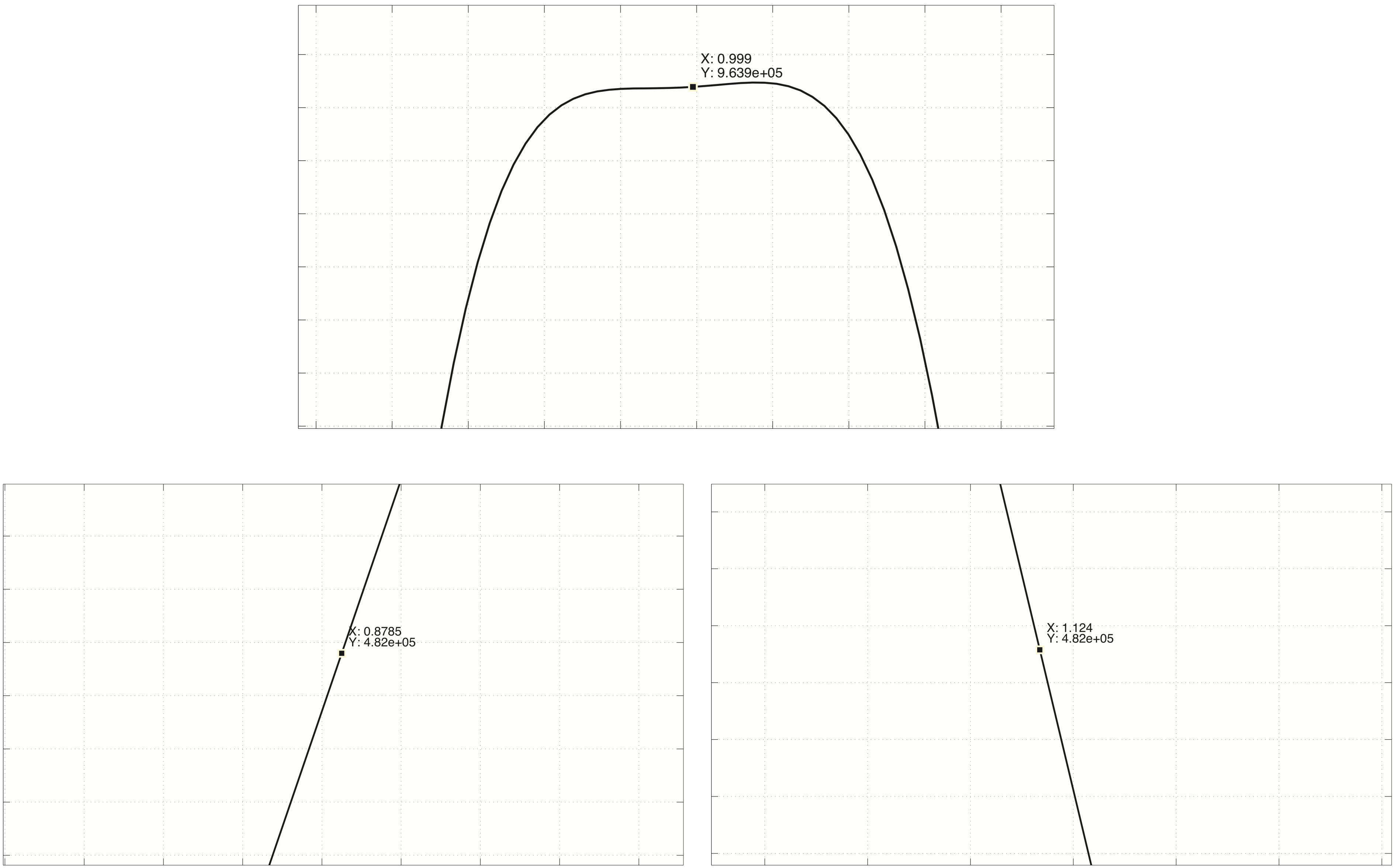
This filter's block diagram is drawn in Direct Form II so the transfer function

is $H(z) = \dfrac{12.38z^4 - 24.77z^2 + 12.38}{z^4 - 1.989z^3 + 2.656z^2 - 1.675z + 0.711}$ and the frequency

response is $H(e^{j\Omega}) = \dfrac{12.38e^{j4\Omega} - 24.77e^{j2\Omega} + 12.38}{e^{j4\Omega} - 1.989e^{j3\Omega} + 2.656e^{j2\Omega} - 1.675e^{j\Omega} + 0.711}$.



Squared Magnitude of the Frequency Response.
x 10^5
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3
3.5
|H(e^{jΩ})|^2
Ω


X: 0.999
Y: 9.639e+05
X: 0.8785
Y: 4.82e+05
X: 1.124
Y: 4.82e+05

The digital center frequency is $\Omega = 0.999$ radians/sample. That corresponds to a signal generator frequency of $\omega = 999$ radians/second or 159 Hz. The -3dB points are at

$$\Omega = 0.8785 \text{ radians/sample} \Rightarrow 139.82 \text{ Hz}$$

$$\Omega = 1.124 \text{ radians/sample} \Rightarrow 178.90 \text{ Hz}$$

for a half-power bandwidth of about 39.1 Hz.

A signal $\mathrm{x}(t) = 4\cos(2000\pi t)\cos(200\pi t)$ is sampled at its Nyquist rate.

What is the signal power of the resultant discrete-time signal $\mathrm{x}[n]$?

The Nyquist rate is 2200 samples/second. Therefore

$$\mathrm{x}[n] = 4\cos(2\pi(5/11)n)\cos(2\pi(1/22)n) = 2[\cos(2\pi(9/22)n) + \cos(\pi n)]$$

$$P_x = \frac{1}{N_0}\sum_{n=\langle N_0\rangle}|\mathrm{x}[n]|^2 = \frac{4}{22}\sum_{n=\langle 22\rangle}|[\cos(2\pi(9/22)n) + \cos(\pi n)]|^2$$

$$P_x = \frac{2}{11}\sum_{n=\langle 22\rangle}[\cos^2(2\pi(9/22)n) + \cos^2(\pi n) + 2\cos(2\pi(9/22)n)\cos(\pi n)]$$

$$P_x = \frac{2}{11}\left[\sum_{n=\langle 22\rangle}\cos^2(2\pi(9/22)n) + \sum_{n=\langle 22\rangle}\cos^2(\pi n) + 2\underbrace{\sum_{n=\langle 22\rangle}\cos(2\pi(9/22)n)\cos(\pi n)}_{=0}\right]$$

$$P_x = \frac{2}{11}\{(1/2)22 + 22\} = 6$$

Alternate Solution:

A signal $\mathrm{x}(t) = 4\cos(2000\pi t)\cos(200\pi t)$ is sampled at its Nyquist rate.

What is the signal power of the resultant discrete-time signal $\mathrm{x}[n]$?

The Nyquist rate is 2200 samples/second. Therefore

$$\mathrm{x}[n] = 4\cos(2\pi(5/11)n)\cos(2\pi(1/22)n)$$

$$\mathrm{X}(F) = [\delta_1(F-5/11) + \delta_1(F+5/11)] * [\delta_1(F-1/22) + \delta_1(F+1/22)]$$

$$\mathrm{X}(F) = \underbrace{\delta_1(F-1/2)}_{=\delta_1(F+1/2)} + \delta_1(F-9/22) + \delta_1(F+9/22) + \underbrace{\delta_1(F+1/2)}_{=\delta_1(F-1/2)}$$

$$\mathrm{X}(F) = 2\delta_1(F-1/2) + \delta_1(F-9/22) + \delta_1(F+9/22)$$

$$P_{\mathrm{x}} = 2^2 + 1^2 + 1^2 = 6$$

A signal $x(t) = 4\cos(2000\pi t)\cos(200\pi t)$ is sampled at twice its Nyquist rate.

What is the signal power of the resultant discrete-time signal $x[n]$?

The Nyquist rate is 2200 samples/second. Therefore

$$x[n] = 4\cos(2\pi(5/22)n)\cos(2\pi(1/44)n) = 2\left[\cos(2\pi(9/44)n) + \cos(\pi n/2)\right]$$

$$P_x = \frac{1}{N_0}\sum_{n=\langle N_0\rangle}|x[n]|^2 = \frac{4}{44}\sum_{n=\langle 44\rangle}\left|\left[\cos(2\pi(9/44)n) + \cos(\pi n/2)\right]\right|^2$$

$$P_x = \frac{1}{11}\sum_{n=\langle 44\rangle}\left[\cos^2(2\pi(9/44)n) + \cos^2(\pi n/2) + 2\cos(2\pi(9/44)n)\cos(\pi n/2)\right]$$

$$P_x = \frac{1}{11}\left[\sum_{n=\langle 44\rangle}\cos^2(2\pi(9/44)n) + \sum_{n=\langle 44\rangle}\cos^2(\pi n/2) + 2\sum_{n=\langle 44\rangle}\cos(2\pi(9/44)n)\cos(\pi n/2)\right]$$

$$P_x = \frac{1}{11}\{(1/2)44 + (1/2)44\} = 4$$

It can be shown that this answer is the same for any sampling rate that is greater than the Nyquist rate. Why is this signal power less than the previous signal power?

A signal $\mathrm{x}(t) = 4\cos(2000\pi t)\sin(200\pi t)$ is sampled at twice its Nyquist rate. What is the signal power of the resultant discrete-time signal $\mathrm{x}[n]$?

The Nyquist rate is 2200 samples/second. Therefore

$$\mathrm{x}[n] = 4\cos(2\pi(5/22)n)\sin(2\pi(1/44)n) = 2[\cos(2\pi(9/44)n) + \sin(\pi n/2)]$$

$$P_x = \frac{1}{N_0}\sum_{n=\langle N_0\rangle}|\mathrm{x}[n]|^2 = \frac{4}{44}\sum_{n=\langle 44\rangle}\left|[\sin(2\pi(9/44)n) + \sin(\pi n/2)]\right|^2$$

$$P_x = \frac{1}{11}\sum_{n=\langle 44\rangle}\left[\sin^2(2\pi(9/44)n) + \sin^2(\pi n/2) + 2\sin(2\pi(9/44)n)\sin(\pi n/2)\right]$$

$$P_x = \frac{1}{11}\left[\sum_{n=\langle 44\rangle}\sin^2(2\pi(9/44)n) + \sum_{n=\langle 44\rangle}\sin^2(\pi n/2) + 2\sum_{n=\langle 44\rangle}\sin(2\pi(9/44)n)\sin(\pi n/2)\right]$$

$$P_x = \frac{1}{11}\{(1/2)44 + (1/2)44\} = 4$$

It can be shown that this answer is the same for any sampling rate that is greater than the Nyquist rate. This is the same as the previous signal power.

A signal $x(t) = 4\cos(2000\pi t)\sin(200\pi t)$ is sampled at its Nyquist rate.

What is the signal power of the resultant discrete-time signal $x[n]$?

The Nyquist rate is 2200 samples/second. Therefore

$$x[n] = 4\cos(2\pi(5/11)n)\sin(2\pi(1/22)n) = 2\left[\sin(2\pi(9/22)n) + \sin(\pi n)\right]$$

$$P_x = \frac{1}{N_0}\sum_{n=\langle N_0 \rangle} \left|x[n]\right|^2 = \frac{4}{22}\sum_{n=\langle 22 \rangle} \left\| \left[\sin(2\pi(9/22)n) + \underbrace{\sin(\pi n)}_{=0} \right] \right\|^2$$

$$P_x = \frac{2}{11}\sum_{n=\langle 22 \rangle} \sin^2(2\pi(9/22)n) = 2$$

Why is this signal power less than the previous two signal powers?