14.5 A system is excited by the signal, $x(t) = 3u(t)$, and the response is $y(t) = 0.961e^{-1.5t}\sin(3.122t)u(t)$. Write a set of state equations and output equations using a minimum number of states.

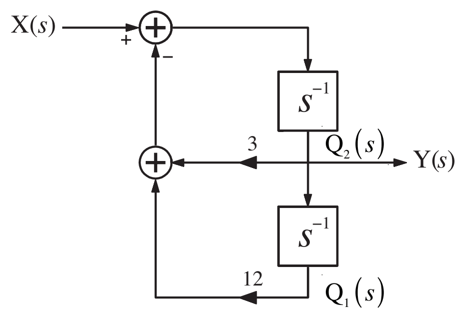
$$X(s) = \frac{3}{s}$$

$$Y(s) = 0.961\frac{3.122}{(s+1.5)^2 + 3.122^2} = \frac{3}{s^2 + 3s + 12}.$$

Then the transfer function is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{3}{s^2+3s+12}}{\frac{3}{s}} = \frac{s}{s^2 + 3s + 12}.$$

This can be realized in Direct Form II.



Let the responses of the integrators be the states $Q_1(s)$ and $Q_2(s)$ as indicated on the block diagram. Then

$$s^2 Q_1(s) = X(s) - 3sQ_1(s) - 12Q_1(s) \Rightarrow X(s) = Q_1(s)(s^2 + 3s + 12)$$
$$Y(s) = sQ_1(s)$$

and we can solve for the transfer function

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s^2 + 3s + 12}$$

confirming that the block diagram does indeed represent the system. Then, putting the equations in standard state-space form

$$sQ_1(s) = Q_2(s)$$
$$sQ_2(s) = -12Q_1(s) - 3Q_2(s) + X(s)$$
$$Y(s) = Q_2(s)$$

and we can write matrix state equations as

$$\begin{bmatrix} sQ_1(s) \\ sQ_2(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -12 & -3 \end{bmatrix}\begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} X(s)$$

$$Y(s) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix}$$