

Sum of torques equals rotational moment of inertia times angular acceleration.  $\tau_m$  is the motor torque in N-m,  $I$  is the rotational moment of inertia of the telescope in  $\text{kg}\cdot\text{m}^2$ ,  $K_f$  is the proportionality constant relating friction and resisting torque in  $\text{N}\cdot\text{m}/(\text{rad}/\text{s})$  and  $\omega$  is its angular position in radians.

$$\underbrace{\tau_m(t)}_{\text{motor torque}} - \underbrace{K_f \omega'_T(t)}_{\text{resisting torque due to friction}} = I \omega''_T(t) \Rightarrow \omega''_T(t) + (K_f / I) \omega'_T(t) = (1 / I) \tau_m(t)$$

$$s^2 \Omega_T(s) + s(K_f / I) \Omega_T(s) = (1 / I) T_m(s)$$

Let the transfer function relating applied dc voltage to torque for the motor be

$$H_m(s) = \frac{K_\tau \omega_{cm}}{s + \omega_{cm}}$$

where  $K_\tau$  is in  $\text{N}\cdot\text{m}/\text{V}$  and  $\omega_{cm}$  is the radian corner frequency of the motor's frequency response. The telescope transfer function, angular position in radians divided by torque in  $\text{N}\cdot\text{m}$  is

$$H_T(s) = \frac{\Omega_T(s)}{T_m(s)} = \frac{1 / I}{s(s + K_f / I)}$$

The forward-path transfer function of the unity-gain feedback system for positioning the telescope is

$$H_1(s) = K_A H_m(s) H_T(s) K_\omega$$

where  $K_A$  is the voltage amplifier gain in  $\text{V}/\text{V}$  and  $K_\omega$  is the proportionality constant relating telescope angular position to voltage of the position-measuring system in  $\text{V}/\text{rad}$ .

Overall system feedback transfer function.

$$H(s) = \frac{H_1(s)}{1 + H_1(s)}$$

Let the initial position of the telescope be at zero radians and let the desired position be one radian. Then the forcing function is  $v_{set} = K_\omega u(t)$  volts and  $V_{set}(s) = K_\omega / s$ . Simulate in MATLAB.