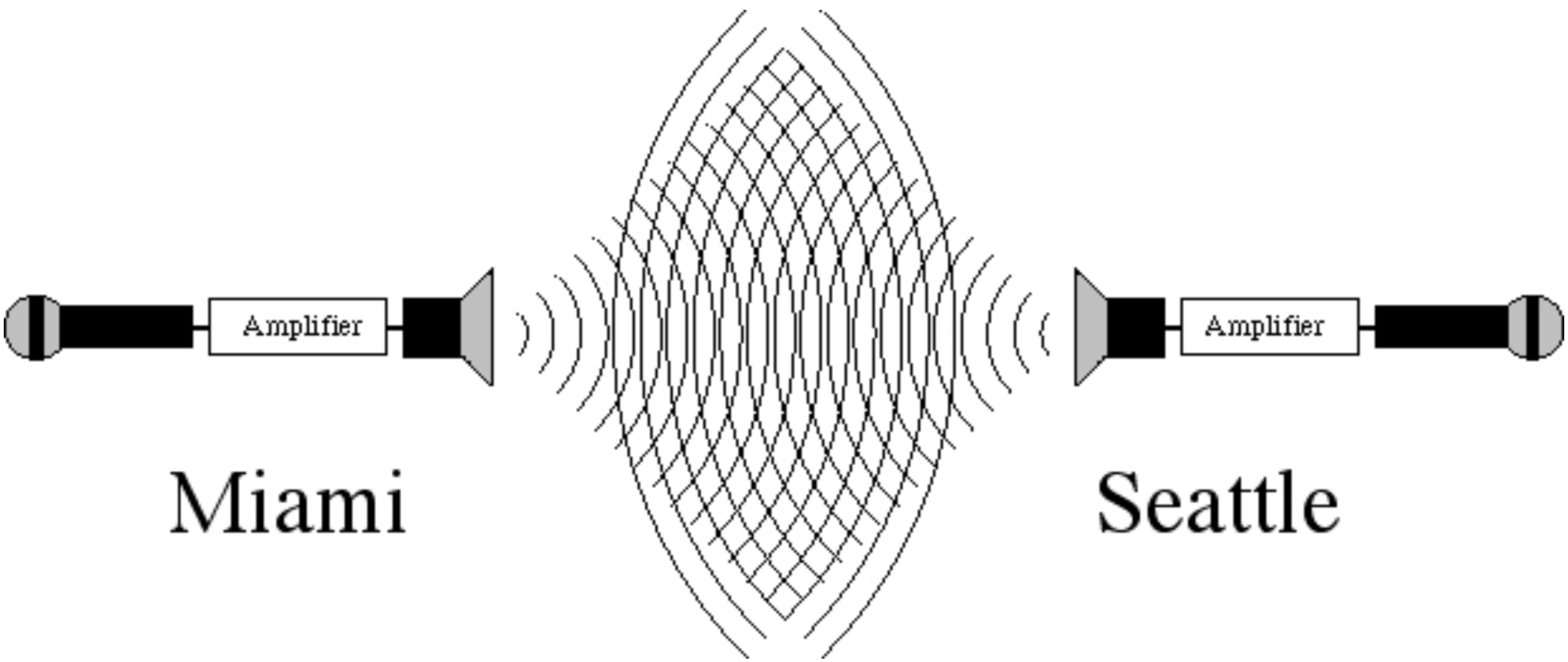
Communication System Analysis

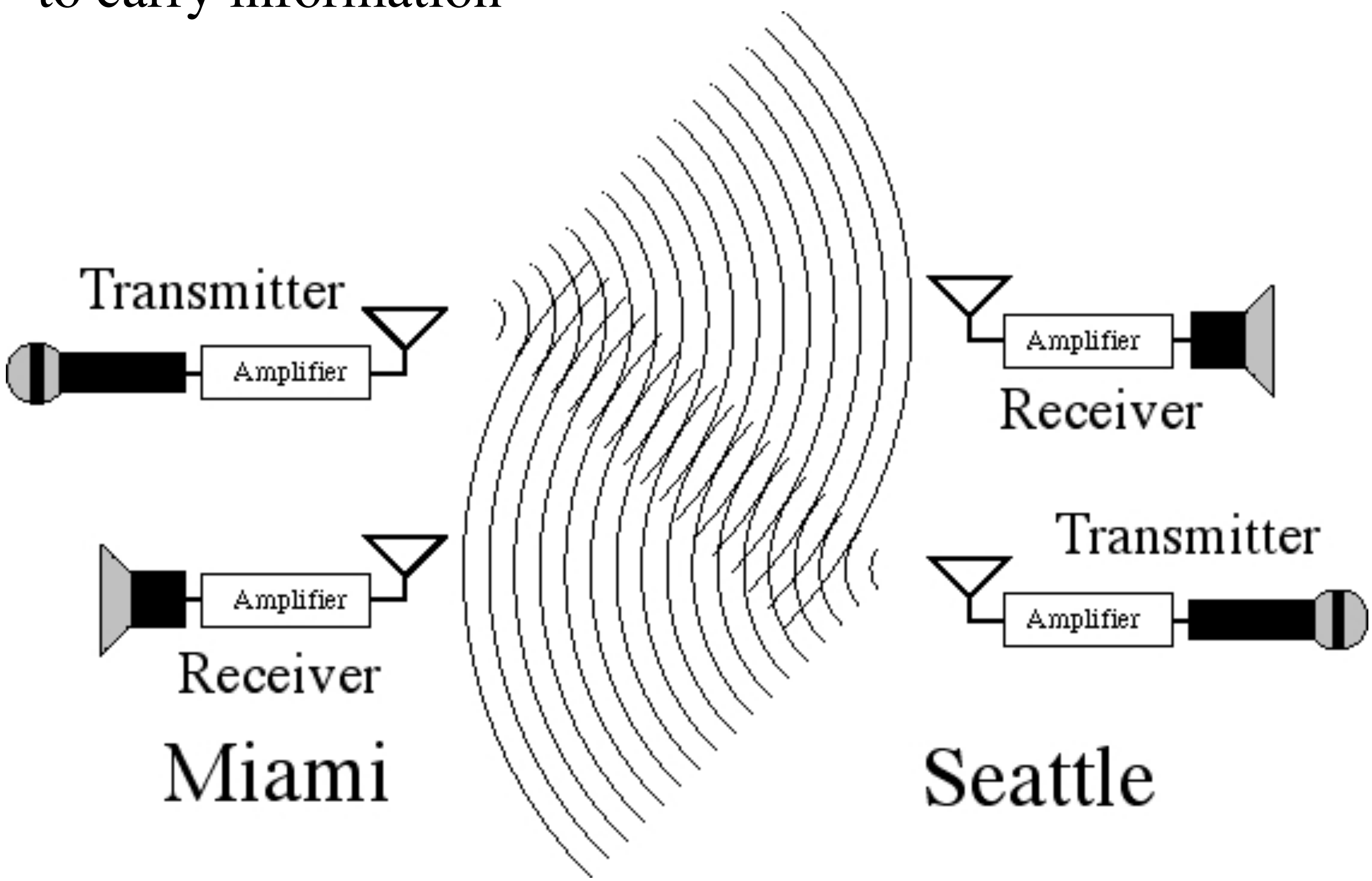
Communication Systems

A naïve, absurd communication system



Communication Systems

A better communication system using electromagnetic waves to carry information



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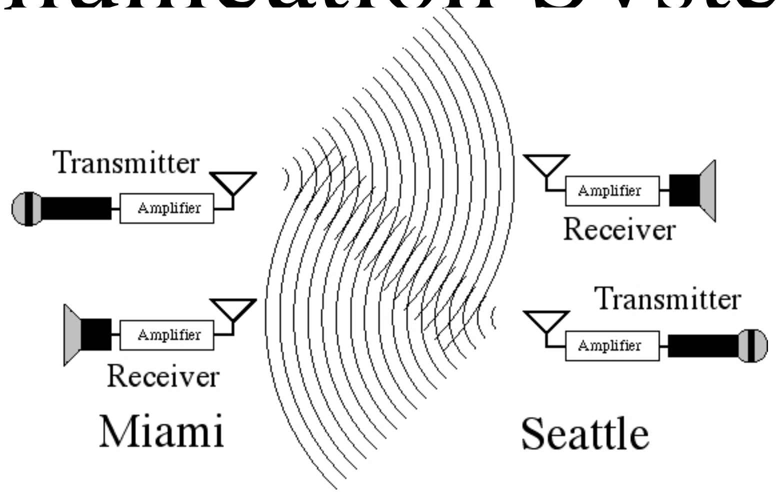


Problems

Antenna inefficiency at audio frequencies

All transmissions from all transmitters are in the same bandwidth, thereby interfering with each other

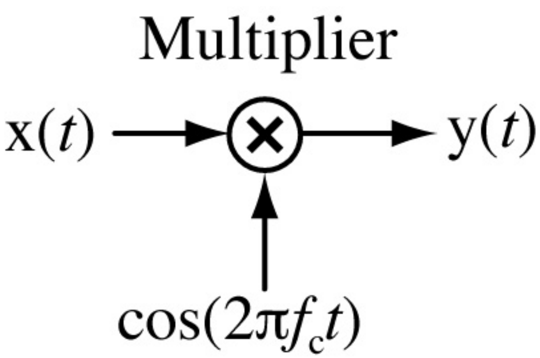
Solution **Frequency multiplexing** using modulation

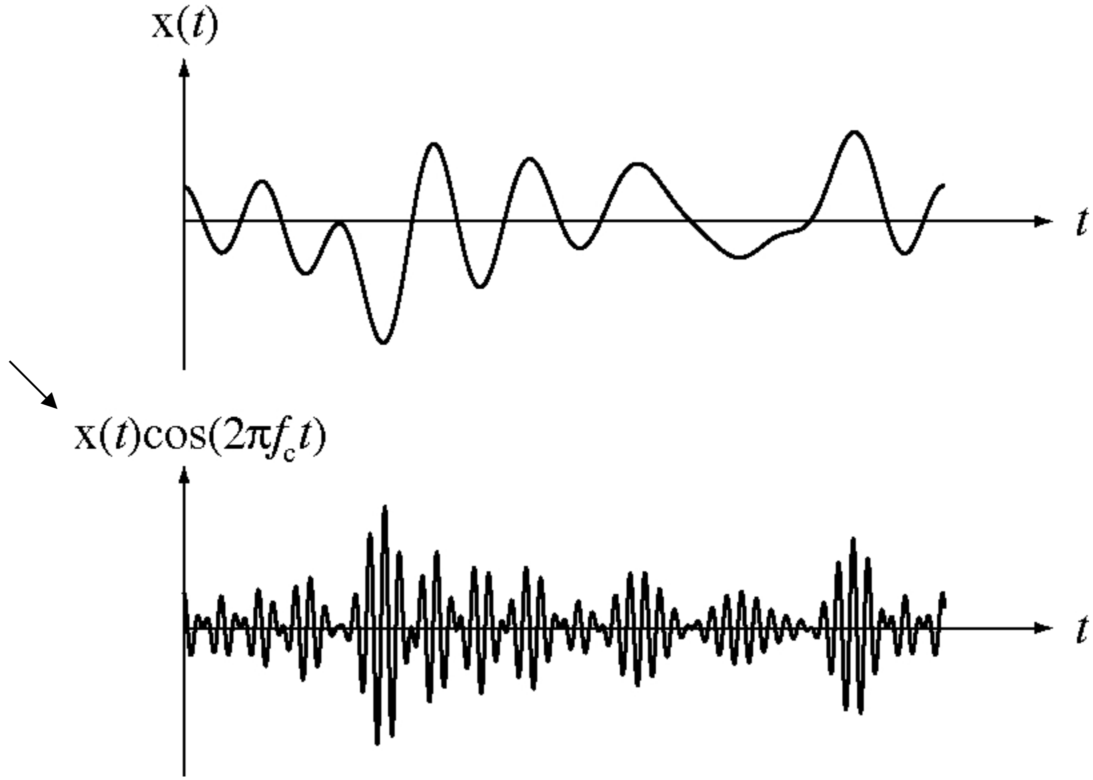
Communication Systems

Double-Sideband Suppressed-Carrier (DSBSC) Modulation

$$y(t) = x(t)\cos(2\pi f_c t)$$

Modulator

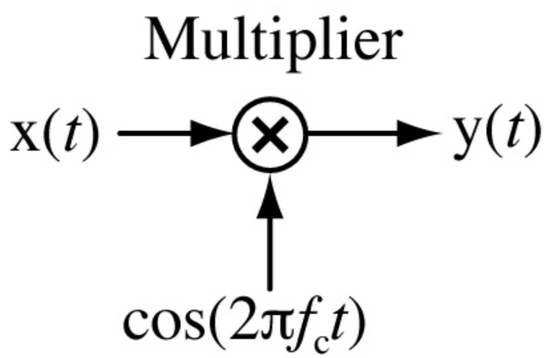


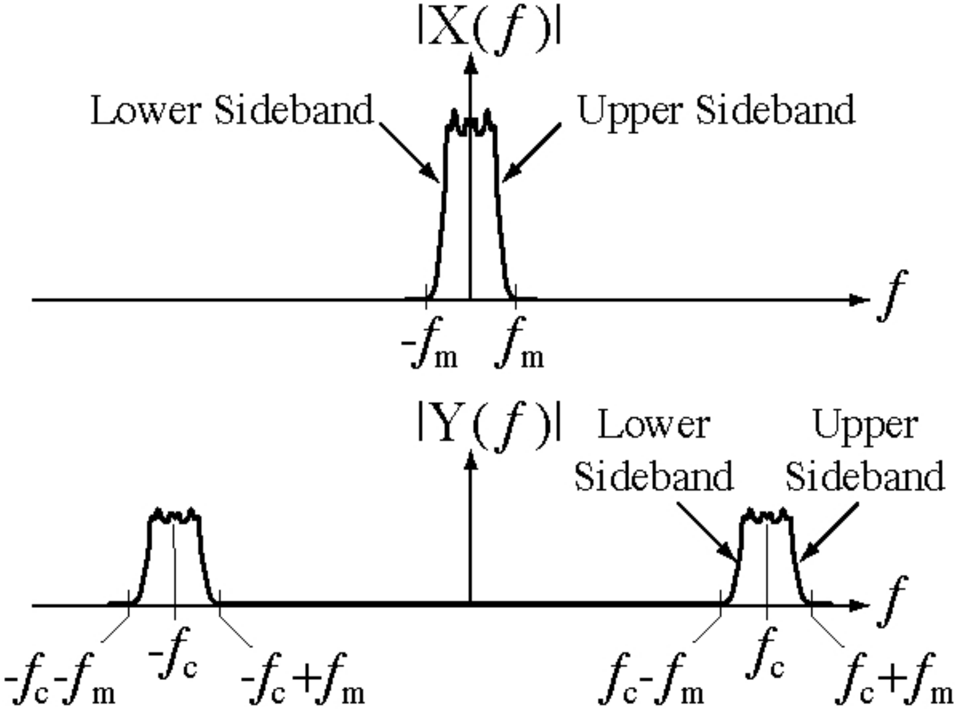


Communication Systems

Double-Sideband Suppressed-Carrier (DSBSC) Modulation

$$\mathrm{Y}(f) = \mathrm{X}(f) * \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$





Frequency multiplexing is using a different carrier frequency f_c for each transmitter.

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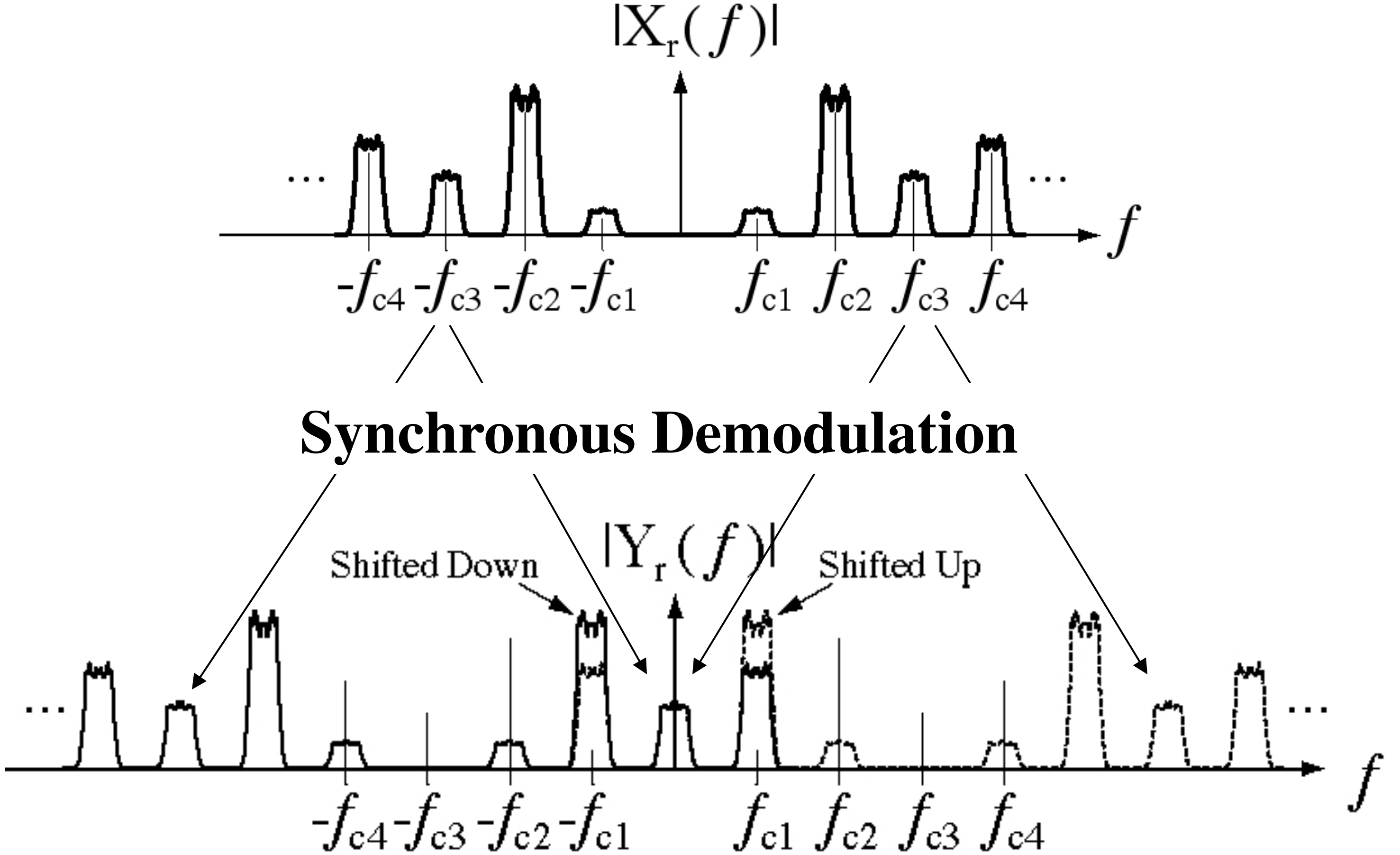
Double-Sideband Suppressed-Carrier (DSBSC) Modulation

Typical signal received by an antenna

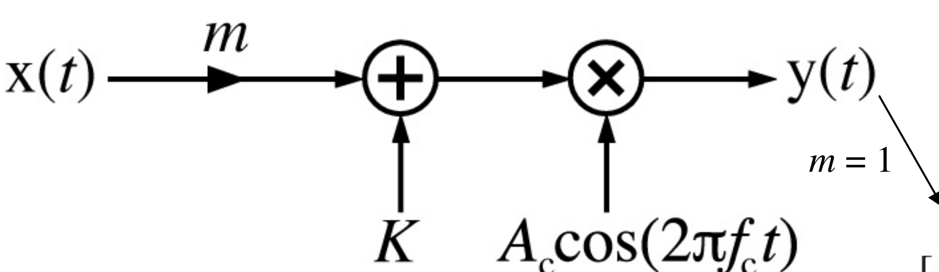


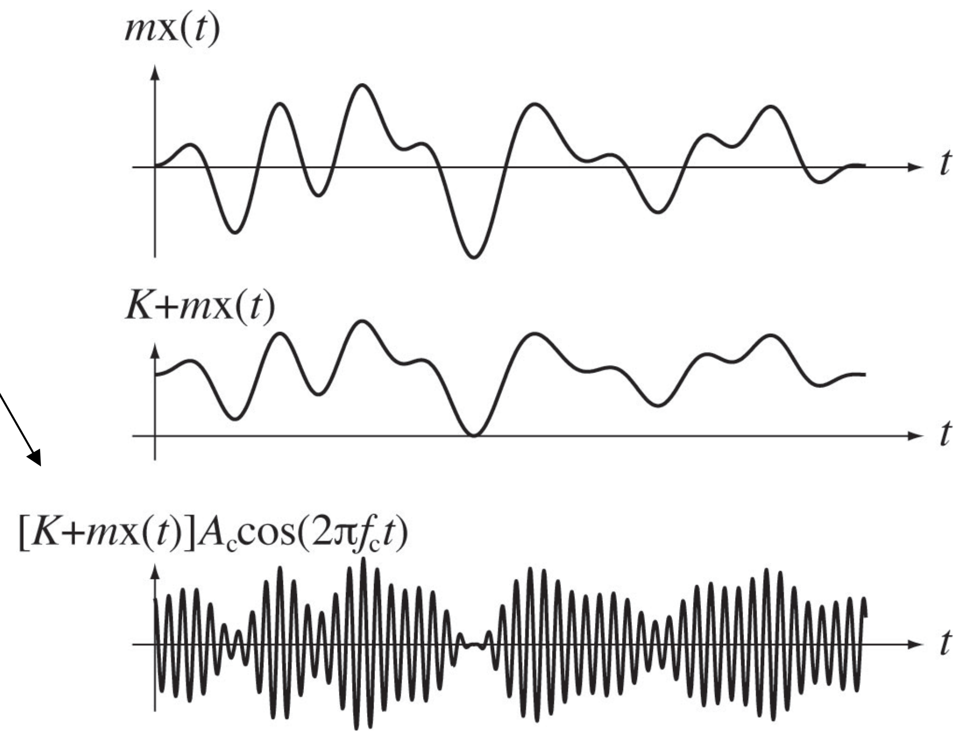
Communication Systems

Double-Sideband Transmitted-Carrier (DSBTC) Modulation

$$y(t) = \left[K + m\,x(t) \right] A_c \cos(2\pi f_c t)$$

Modulator





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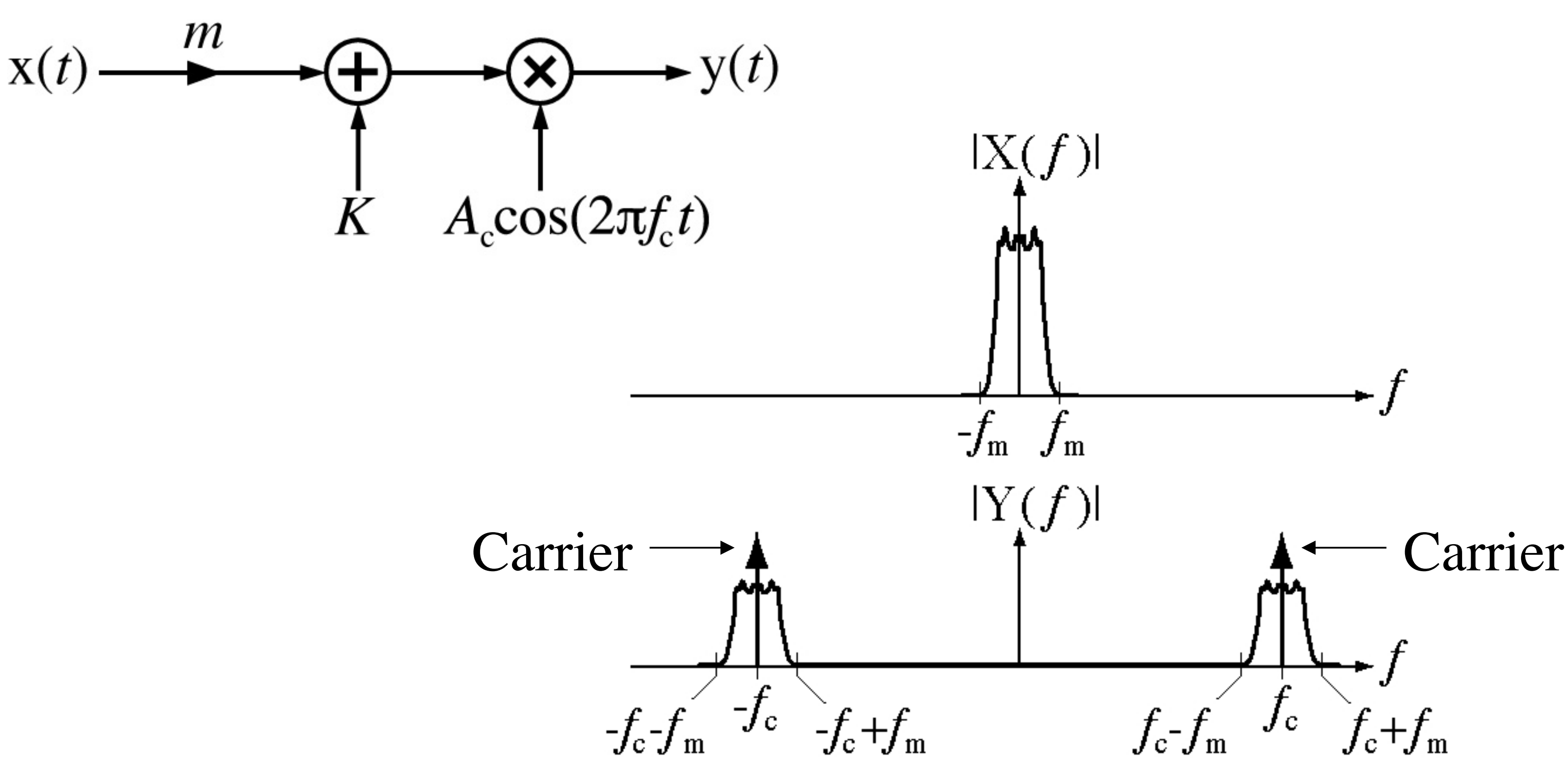
Double-Sideband Transmitted-Carrier (DSBTC) Modulation



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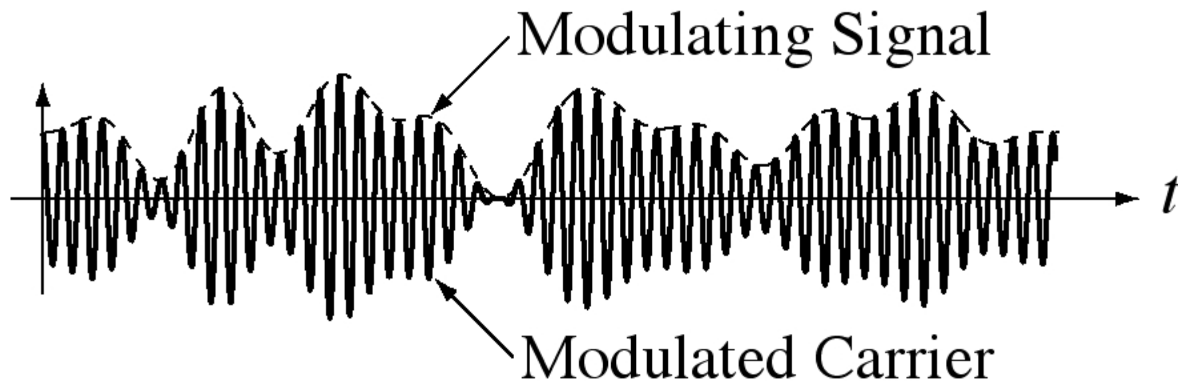
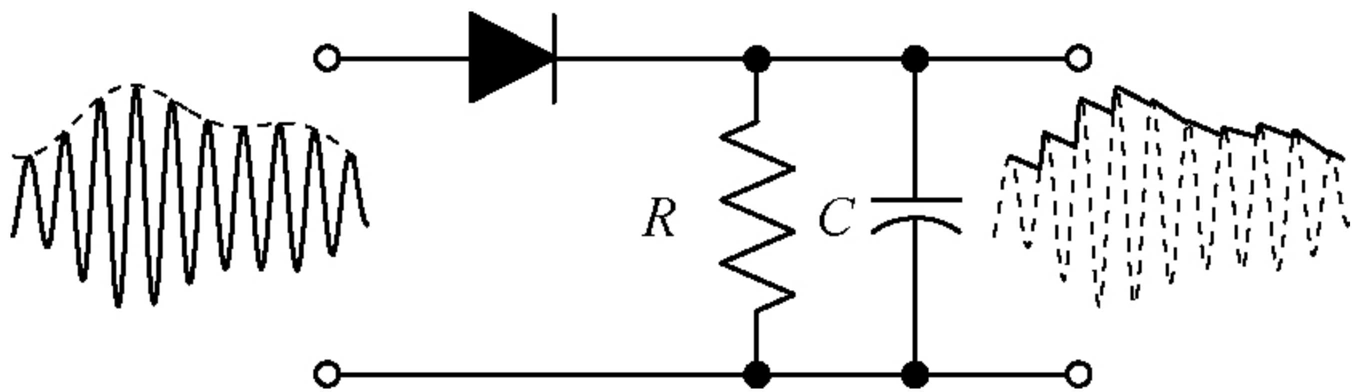
Double-Sideband Transmitted-Carrier (DSBTC) Modulation



Envelope Detector



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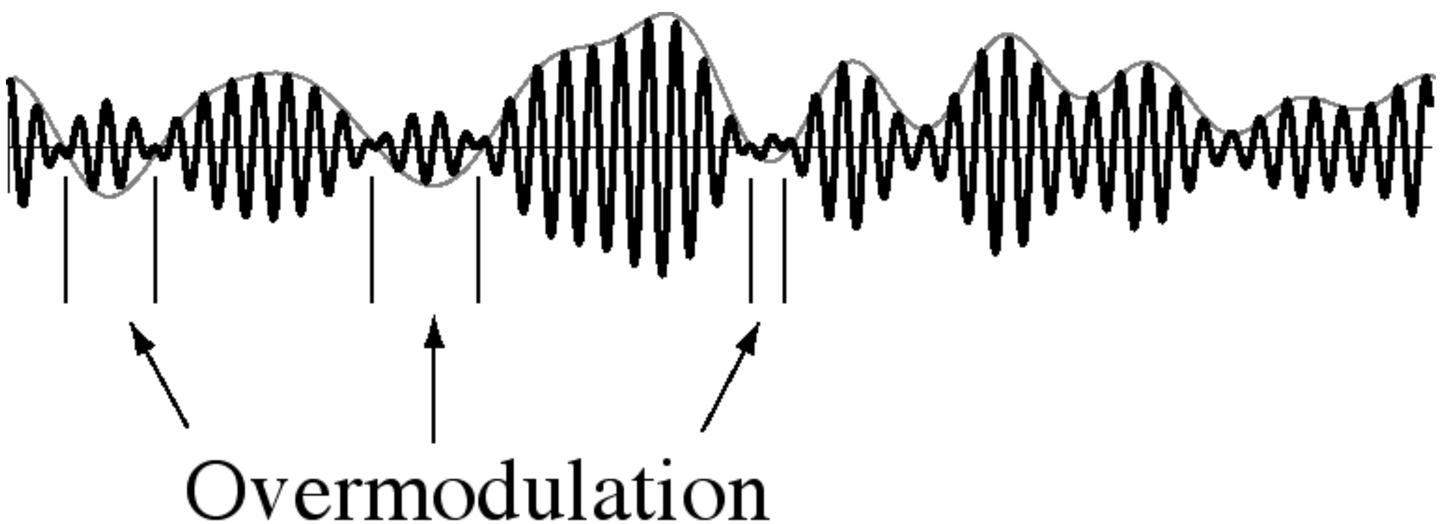
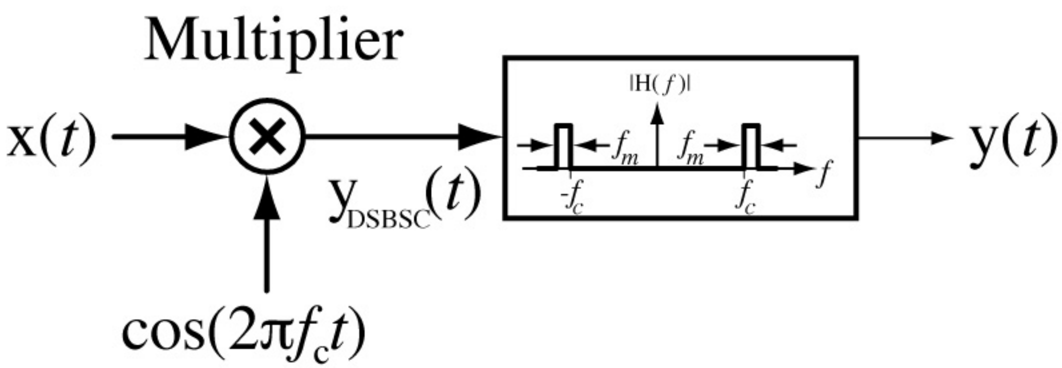
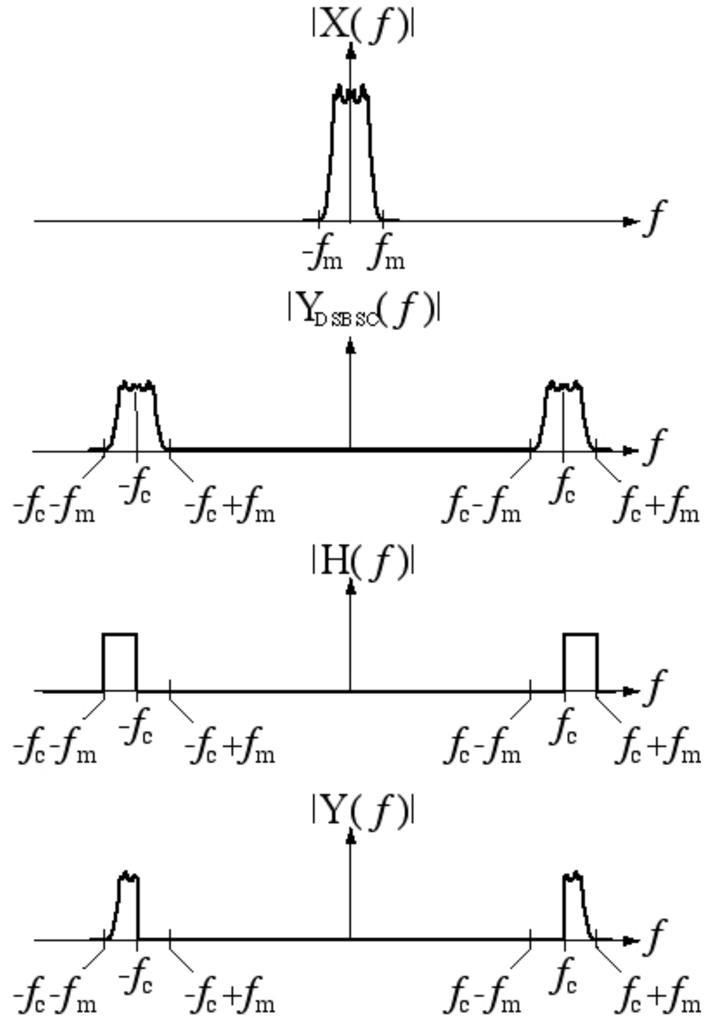
Double-Sideband Transmitted-Carrier (DSBTC) Modulation



Communication Systems

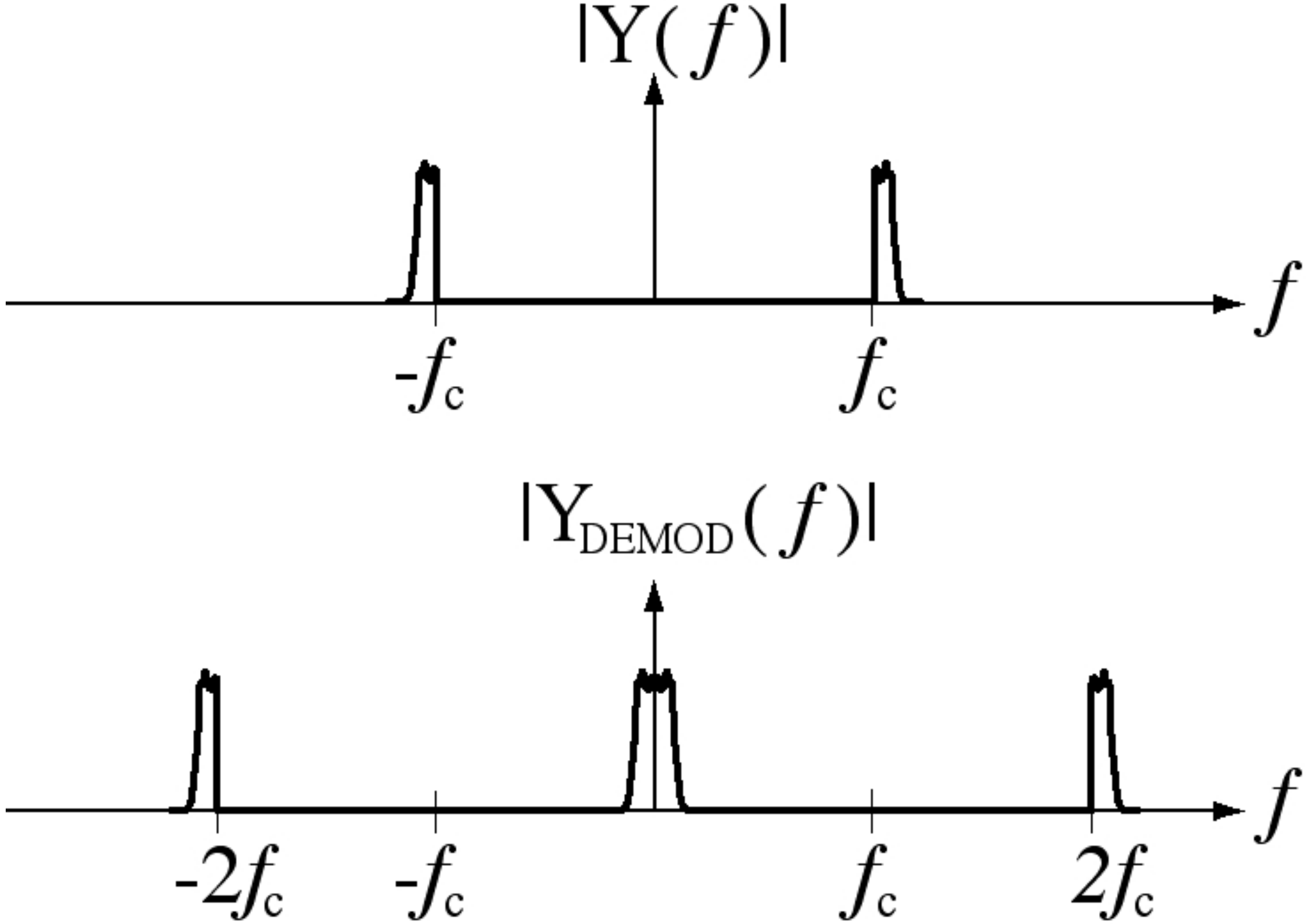
Single-Sideband Suppressed-Carrier (SSBSC) Modulation





Communication Systems

Single-Sideband Suppressed-Carrier (SSBSC) Modulation



Angle Modulation

Amplitude modulation varies the carrier amplitude in proportion to the information signal. Angle modulation varies the carrier phase angle in proportion to the information signal. Let the carrier be of the form $A_c \cos(\omega_c t)$ and let the modulated carrier be of the form $\mathrm{y}(t) = A_c \cos(\theta_c(t))$ or $\mathrm{y}(t) = A_c \cos(\omega_c t + \Delta\theta(t))$ where $\theta_c(t) = \omega_c t + \Delta\theta(t)$ and $\omega_c = 2\pi f_c$. If $\Delta\theta(t) = k_p \mathrm{x}(t)$ where $\mathrm{x}(t)$ is the information signal this kind of angle modulation is called **phase modulation** (**PM**).

Angle Modulation

In an unmodulated carrier the radian frequency is ω_c. If we differentiate the sinusoidal argument $\omega_c t$ of an unmodulated carrier with respect to time we get the constant ω_c. So one way of defining the radian frequency of a sinusoid is as the derivative of the argument of the sinusoid. We could similarly define cyclic frequency as the derivative of the argument divided by 2π. If we apply that definition to the modulated angle $\theta_c(t) = \omega_c t + \Delta\theta(t)$ we get a function of time that is defined as **instantaneous frequency**

$$\omega(t) = \frac{d}{dt}\left(\theta_c(t)\right) = \omega_c + \frac{d}{dt}\left(\Delta\theta(t)\right) \leftarrow \text{radian frequency}$$

or

$$\mathrm{f}(t) = \frac{1}{2\pi}\frac{d}{dt}\left(\theta_c(t)\right) = f_c + \frac{1}{2\pi}\frac{d}{dt}\left(\Delta\theta(t)\right) \leftarrow \text{cyclic frequency}$$

Angle Modulation

In phase modulation the instantaneous radian frequency as a function of time is $\omega(t) = \omega_c + k_p \frac{d}{dt}(\mathrm{x}(t))$. If we control the derivative of the phase with the information signal instead of controlling the phase directly with the information signal

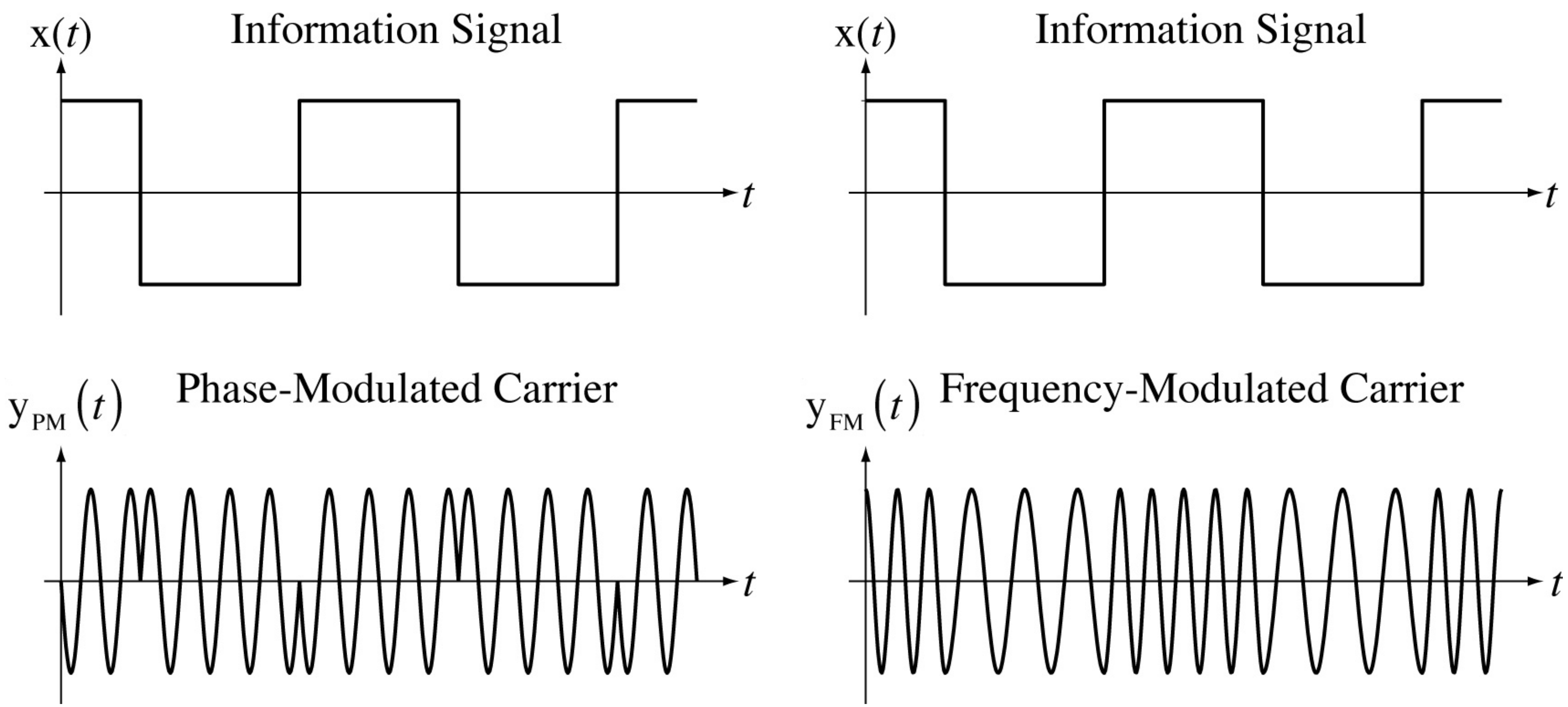
$$\frac{d}{dt}(\Delta\theta(t)) = k_f \, \mathrm{x}(t)$$

and

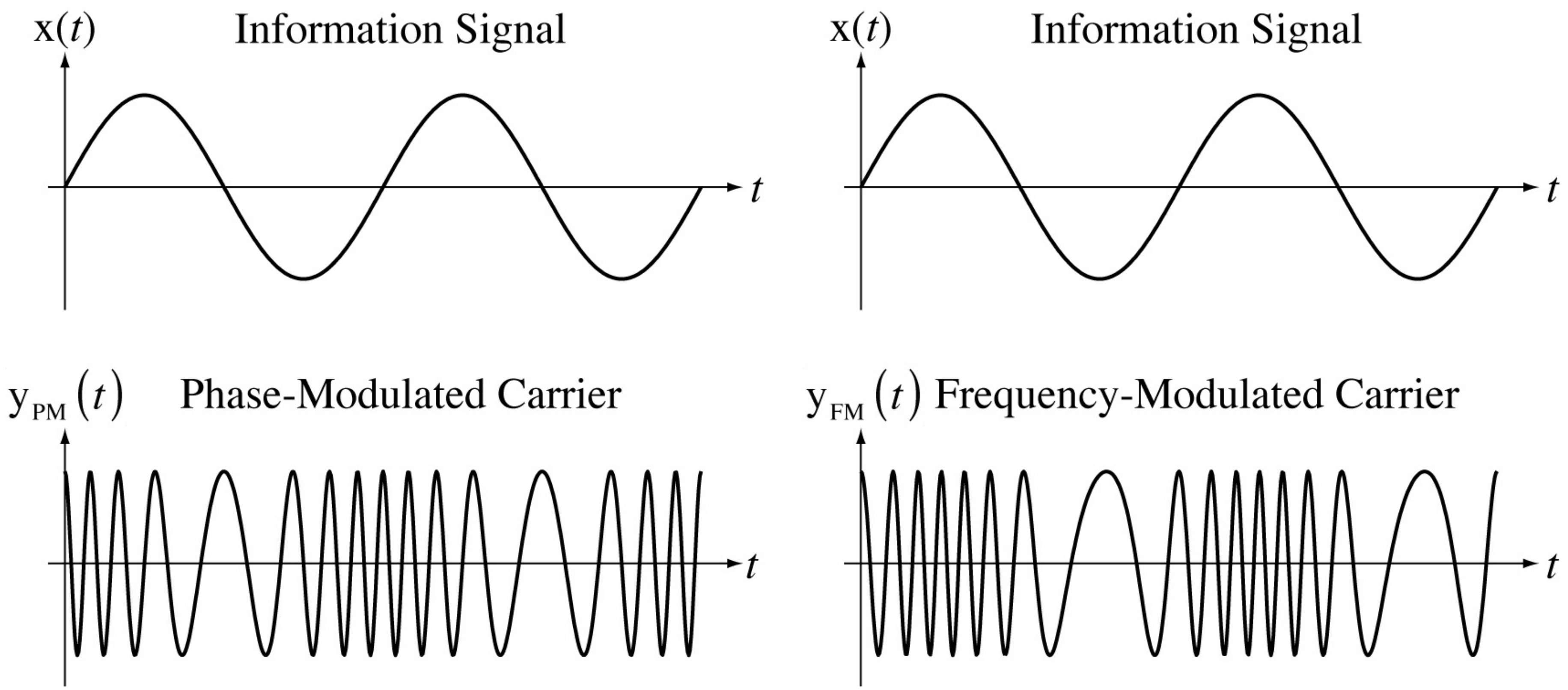
$$\omega(t) = \omega_c + k_f \, \mathrm{x}(t) \quad \text{and} \quad \mathrm{f}(t) = f_c + \frac{k_f}{2\pi} \mathrm{x}(t)$$

This type of angle modulation is called **frequency modulation** (**FM**).

Angle Modulation



Angle Modulation



Angle Modulation

For phase modulation $\mathrm{y}_{\mathrm{PM}}(t) = A_c \cos\left(\omega_c t + k_p\, \mathrm{x}(t)\right)$

For frequency modulation $\mathrm{y}_{\mathrm{FM}}(t) = A_c \cos\left(\omega_c t + k_f \int_{t_0}^{t} \mathrm{x}(\tau)\,d\tau\right)$

There is no simple expression for the CTFT's of these signals in the general case. Using $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ we can write $\mathrm{y}_{\mathrm{PM}}(t) = A_c\left[\cos(\omega_c t)\cos\left(k_p\, \mathrm{x}(t)\right) - \sin(\omega_c t)\sin\left(k_p\, \mathrm{x}(t)\right)\right]$

and $\mathrm{y}_{\mathrm{FM}}(t) = A_c\left[\cos(\omega_c t)\cos\left(k_f \int_{t_0}^{t} \mathrm{x}(\tau)\,d\tau\right) - \sin(\omega_c t)\sin\left(k_f \int_{t_0}^{t} \mathrm{x}(\tau)\,d\tau\right)\right]$

Angle Modulation

If k_p and k_f are small enough $\cos\left(k_p \, \mathrm{x}(t)\right) \cong 1$ and $\sin\left(k_p \, \mathrm{x}(t)\right) \cong k_p \, \mathrm{x}(t)$

and $\cos\left(k_f \int_{t_0}^{t} \mathrm{x}(\tau)\, d\tau\right) \cong 1$ and $\sin\left(k_f \int_{t_0}^{t} \mathrm{x}(\tau)\, d\tau\right) \cong k_f \int_{t_0}^{t} \mathrm{x}(\tau)\, d\tau.$

Then $\mathrm{y}_{\mathrm{PM}}(t) \cong A_c\left[\cos(\omega_c t) - k_p \, \mathrm{x}(t)\sin(\omega_c t)\right]$

and $\mathrm{y}_{\mathrm{FM}}(t) \cong A_c\left[\cos(\omega_c t) - \sin(\omega_c t)\, k_f \int_{t_0}^{t} \mathrm{x}(\tau)\, d\tau\right]$

These approximations are called **narrowband PM** and **narrowband FM** and we can find their CTFT's.

Angle Modulation

$$Y_{PM}(\omega) \cong (A_c/2)\left\{2\pi\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]-jk_p\left[X(\omega+\omega_c)-X(\omega-\omega_c)\right]\right\}$$

$$Y_{FM}(\omega) \cong (A_c/2)\left\{2\pi\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]-k_f\left[\frac{X(\omega+\omega_c)}{\omega+\omega_c}-\frac{X(\omega-\omega_c)}{\omega-\omega_c}\right]\right\}$$

or

$$Y_{PM}(f) \cong (A_c/2)\left\{\left[\delta(f-f_c)+\delta(f+f_c)\right]-jk_p\left[X(f+f_c)-X(f-f_c)\right]\right\}$$

$$Y_{FM}(f) \cong (A_c/2)\left\{\left[\delta(f-f_c)+\delta(f+f_c)\right]-\frac{k_f}{2\pi}\left[\frac{X(f+f_c)}{f+f_c}-\frac{X(f-f_c)}{f-f_c}\right]\right\}$$

(on the assumption that the average value of $x(t)$ is zero)

Angle Modulation

If the information signal is a sinusoid $\mathrm{x}(t) = A_m \cos(\omega_m t) = A_m \cos(2\pi f_m t)$

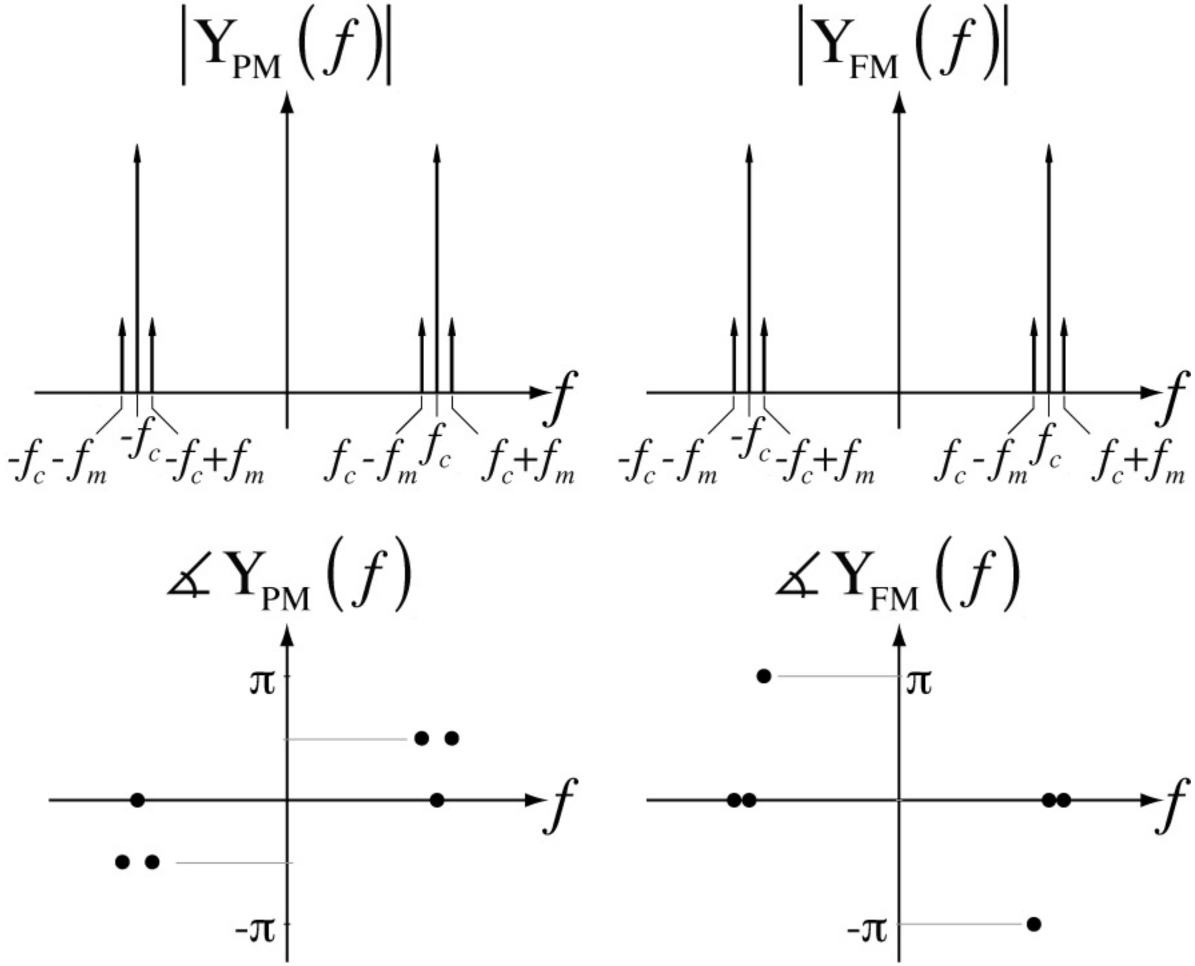
then $\mathrm{X}(f) = (A_m / 2)\left[\delta(f - f_m) + \delta(f + f_m)\right]$ and, in the narrowband approximation,

$$\mathrm{Y}_{\mathrm{PM}}(f) \cong (A_c / 2)\left\{\left[\delta(f - f_c) + \delta(f + f_c)\right] - \frac{jA_m k_p}{2}\begin{bmatrix}\delta(f + f_c - f_m) + \delta(f + f_c + f_m) \\ -\delta(f - f_c - f_m) - \delta(f - f_c + f_m)\end{bmatrix}\right\}$$

$$\mathrm{Y}_{\mathrm{FM}}(f) \cong (A_c / 2)\left\{\left[\delta(f - f_c) + \delta(f + f_c)\right] - \frac{A_m k_f}{4\pi f_m}\begin{bmatrix}\delta(f + f_c - f_m) - \delta(f + f_c + f_m) \\ -\delta(f - f_c - f_m) + \delta(f - f_c + f_m)\end{bmatrix}\right\}$$

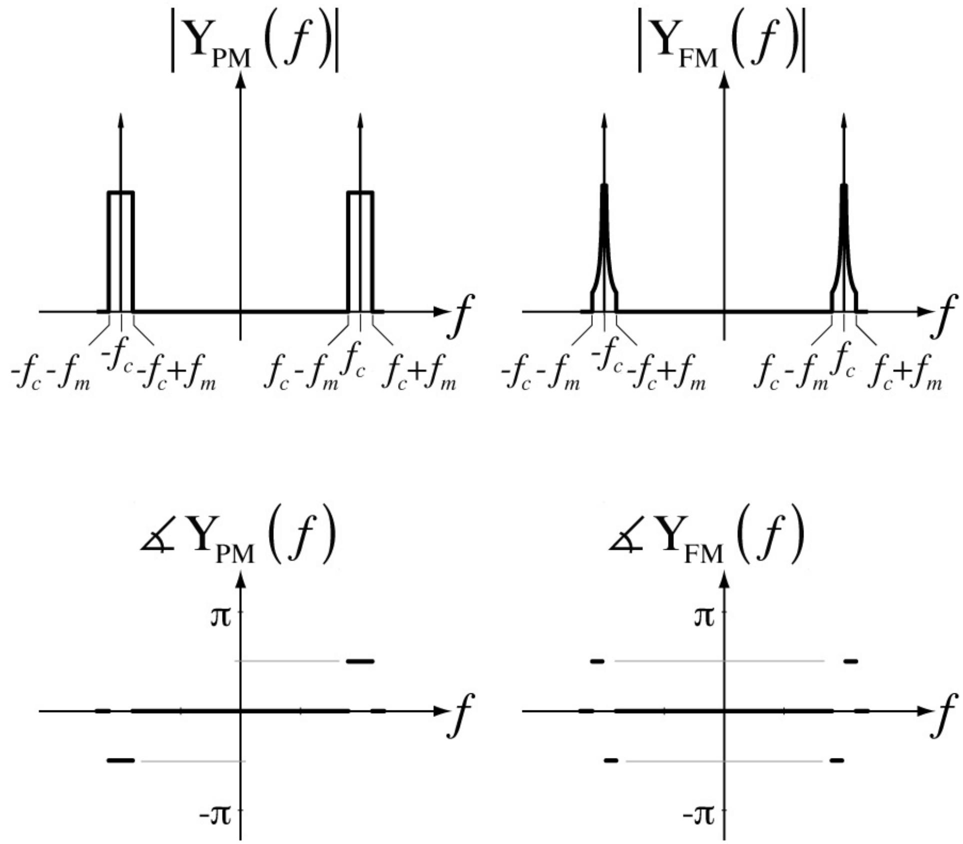
Angle Modulation

Narrowband PM and FM Spectra for a Sinusoidal Information Signal



Angle Modulation

Narrowband PM and FM Spectra for a Sinc Information Signal



Angle Modulation

If the narrowband approximation is not adequate we must deal with the more complicated wideband case. For FM

$$\mathrm{y}_{\mathrm{FM}}(t) = A_c\left[\cos(\omega_c t)\cos\left(k_f\int_{t_0}^{t}\mathrm{x}(\tau)\,d\tau\right) - \sin(\omega_c t)\sin\left(k_f\int_{t_0}^{t}\mathrm{x}(\tau)\,d\tau\right)\right]$$

If the modulation is $\mathrm{x}(t) = A_m\cos(\omega_m t)$,

$$\mathrm{y}_{\mathrm{FM}}(t) = A_c\left[\cos(\omega_c t)\cos\left(\frac{k_f A_m}{\omega_m}\sin(\omega_m t)\right) - \sin(\omega_c t)\sin\left(\frac{k_f A_m}{\omega_m}\sin(\omega_m t)\right)\right]$$

Let $m = k_f A_m / \omega_m$, the modulation index.

Then $\mathrm{y}_{\mathrm{FM}}(t) = A_c\left[\cos(\omega_c t)\cos(m\sin(\omega_m t)) - \sin(\omega_c t)\sin(m\sin(\omega_m t))\right]$

Angle Modulation

In $y_{FM}(t) = A_c\left[\cos(\omega_c t)\cos(m\sin(\omega_m t)) - \sin(\omega_c t)\sin(m\sin(\omega_m t))\right]$

$\cos(m\sin(\omega_m t))$ and $\sin(m\sin(\omega_m t))$ are periodic with fundamental period $2\pi/\omega_m$. Therefore they can each be expressed as a Fourier series. For example, $\cos(m\sin(\omega_m t)) = \sum_{k=-\infty}^{\infty} c_c[k] e^{jk\omega_m t}$ .with

$c_c[k] = \frac{\omega_m}{2\pi}\int_{2\pi/\omega_m} \cos(m\sin(\omega_m t)) e^{-jk\omega_m t}\,dt$. It then follows

that $\cos(\omega_c t)\cos(m\sin(\omega_m t)) = \frac{1}{2}\sum_{k=-\infty}^{\infty} c_c[k]\left[e^{j(k\omega_m+\omega_c)t} + e^{j(k\omega_m-\omega_c)t}\right]$.

The CTFS harmonic function can be written in the form

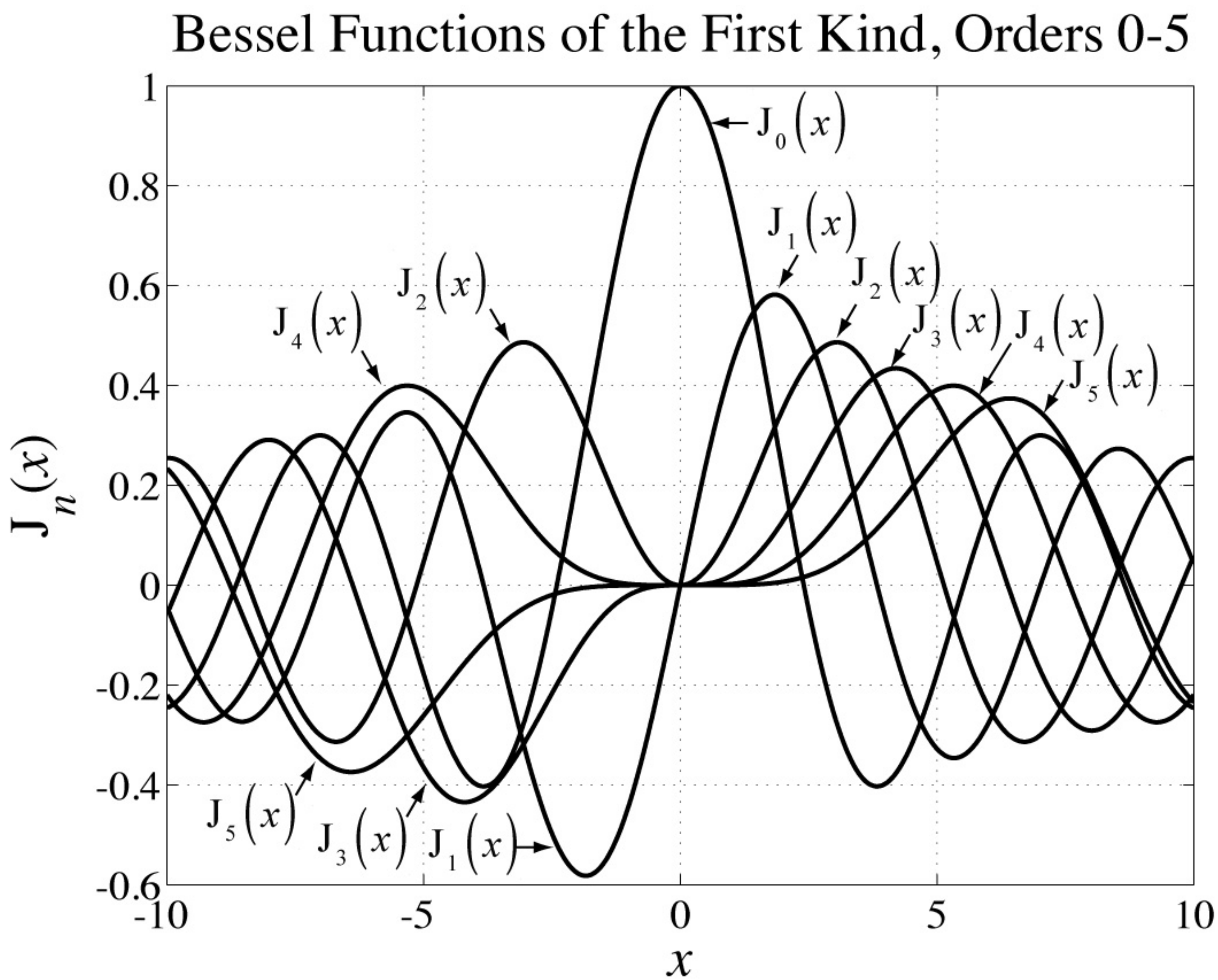
$$c_c[k] = \frac{\omega_m}{4\pi}\int_{-\pi/\omega_m}^{\pi/\omega_m}\left[e^{j[m\sin(\omega_m t)-k\omega_m t]} + e^{j[-m\sin(\omega_m t)-k\omega_m t]}\right]dt$$

Angle Modulation

The integral $c_c[k] = \frac{\omega_m}{4\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} \left[e^{j[m\sin(\omega_m t) - k\omega_m t]} + e^{j[-m\sin(\omega_m t) - k\omega_m t]} \right] dt$

can be evaluated using $J_k(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(z\sin(\lambda) - k\lambda)} d\lambda$ where $J_k(\cdot)$ is the Bessel function of the first kind of order k. One useful property of this Bessel function is $J_k(z) = J_{-k}(-z)$.

Angle Modulation



Angle Modulation

It can be shown (and is in the text) that, for cosine-wave frequency modulation,

$$Y_{FM}(f) = \frac{A_c}{2} \sum_{k=-\infty}^{\infty} \left[J_k(m)\delta\left(f - (kf_m + f_c)\right) + J_{-k}(m)\delta\left(f - (kf_m - f_c)\right) \right]$$

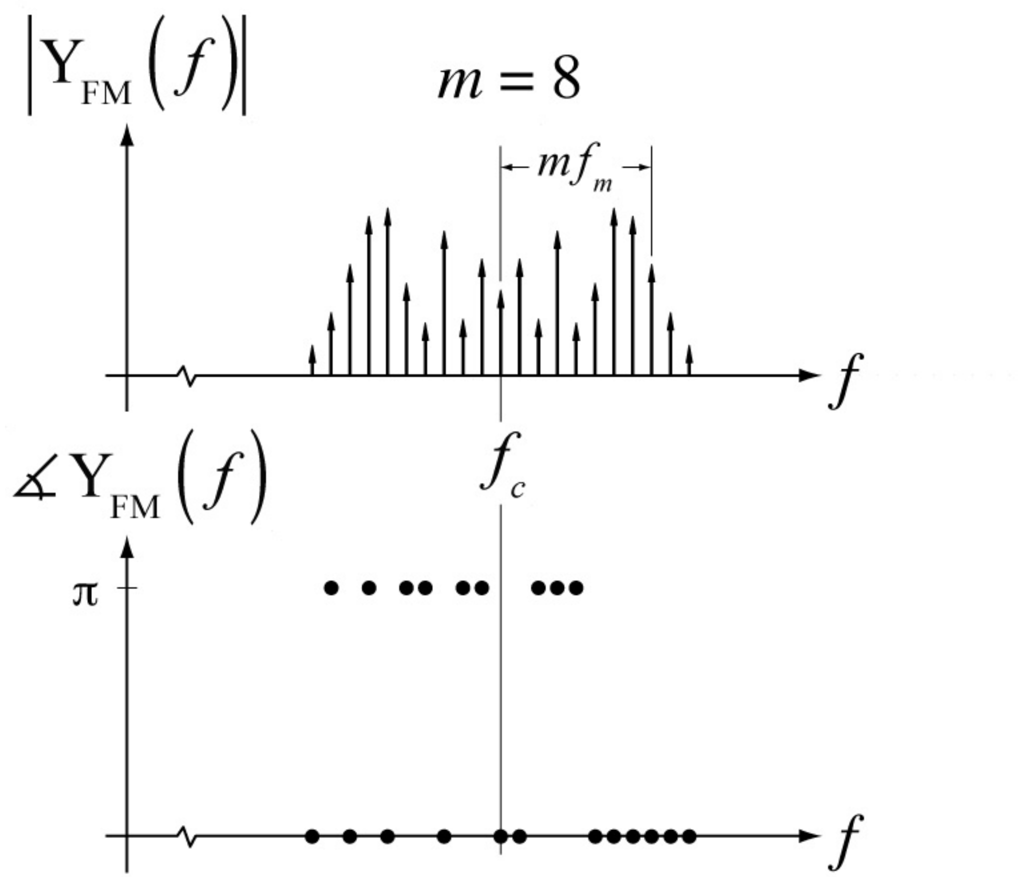
or

$$Y_{FM}(f) = \frac{A_c}{2} \left\{ \begin{array}{l} J_0(m)\left[\delta(f - f_c) + \delta(f + f_c)\right] \\ +\sum_{k=1}^{\infty} \left[\begin{array}{l} J_k(m)\delta\left(f - (kf_m + f_c)\right) + J_{-k}(m)\delta\left(f - (kf_m - f_c)\right) \\ + J_{-k}(m)\delta\left(f - (-kf_m + f_c)\right) + J_k(m)\delta\left(f - (-kf_m - f_c)\right) \end{array} \right] \end{array} \right\}$$

The impulses in the FM spectrum extend in frequency all the way to infinity. But beyond mf_m (where m is the modulation index and f_m is the cyclic frequency of the modulating cosine) the impulse strengths die rapidly. For practical purposes the bandwidth is approximately $2mf_m$.

Angle Modulation

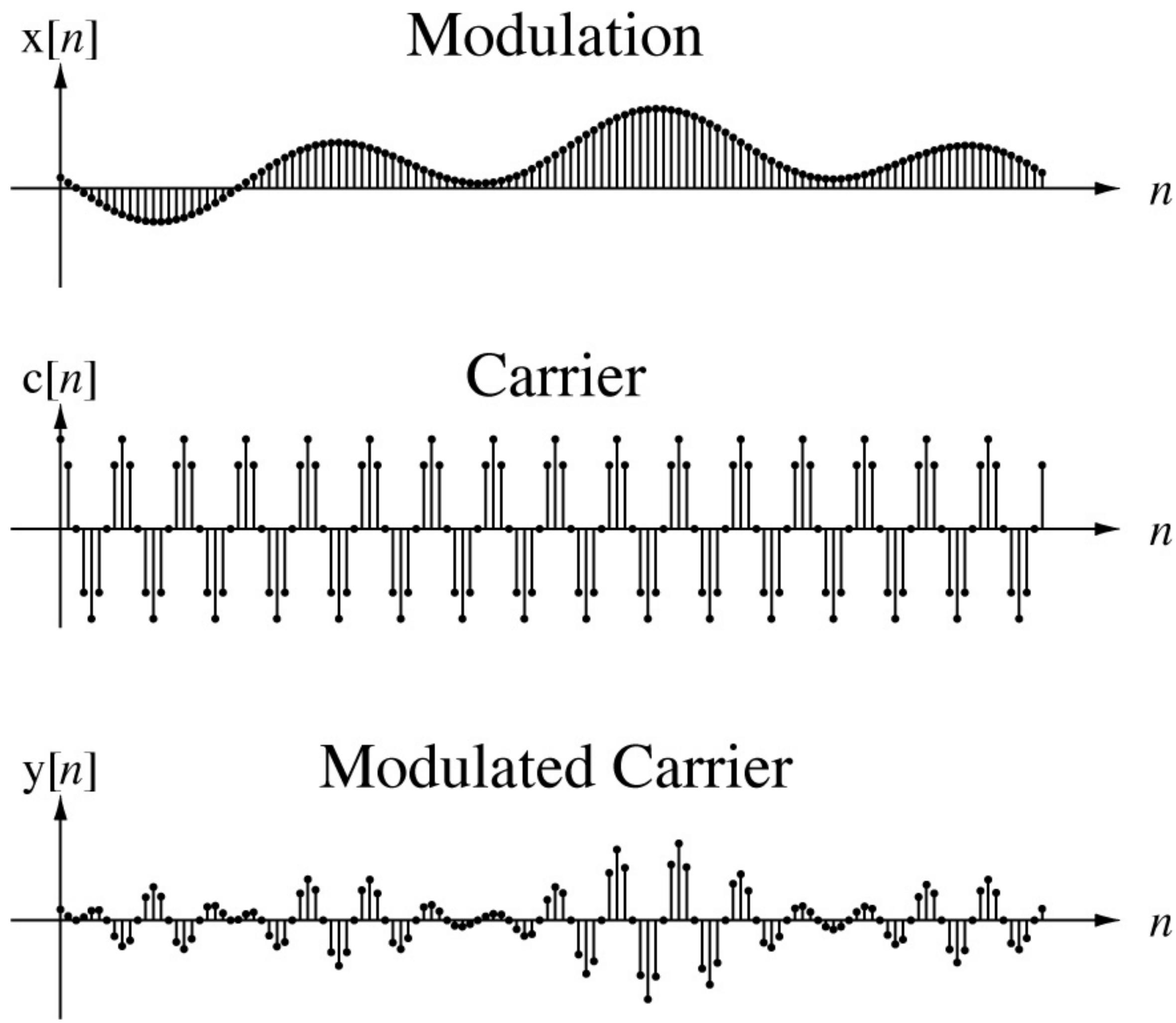
Wideband FM Spectrum for Cosine-Wave Modulation



Discrete-Time Modulation

Discrete-time DSBSC modulation of a sinusoidal carrier $c[n] = \cos(2\pi F_0 n)$

$$y[n] = x[n]c[n]$$
$$= x[n]\cos(2\pi F_0 n)$$



Discrete-Time Modulation

$$\mathrm{Y}(F) = (1/2)\left[\mathrm{X}(F - F_0) + \mathrm{X}(F + F_0)\right]$$