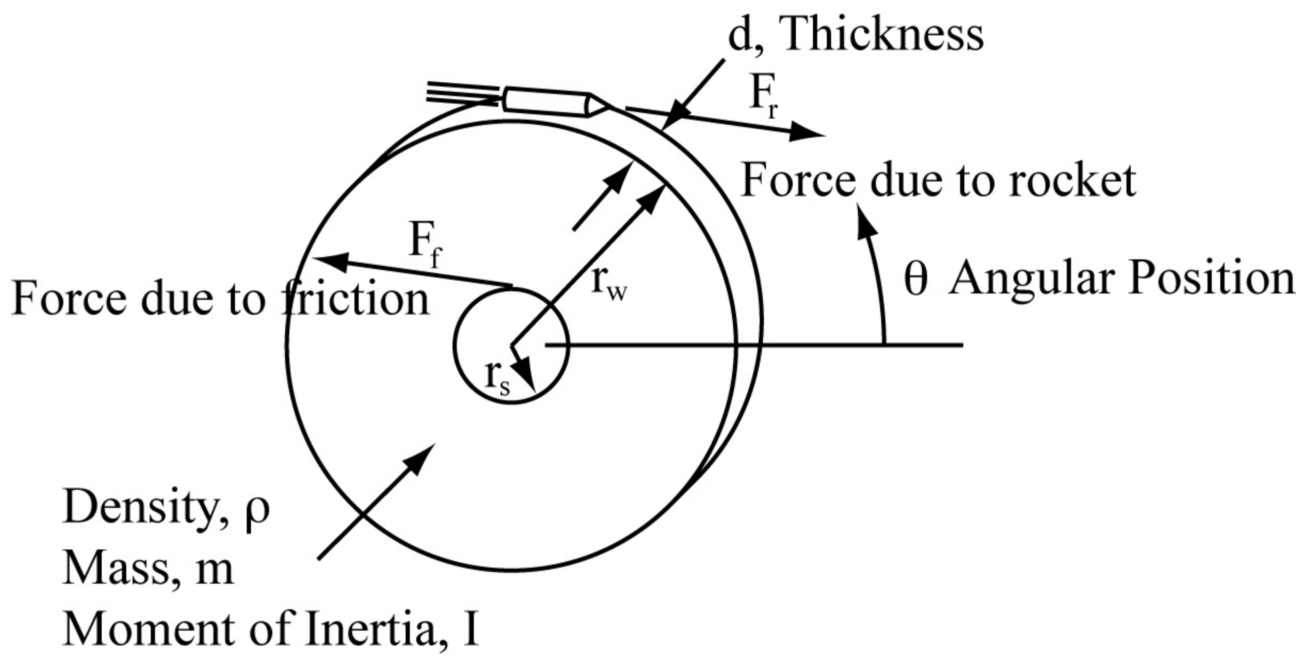
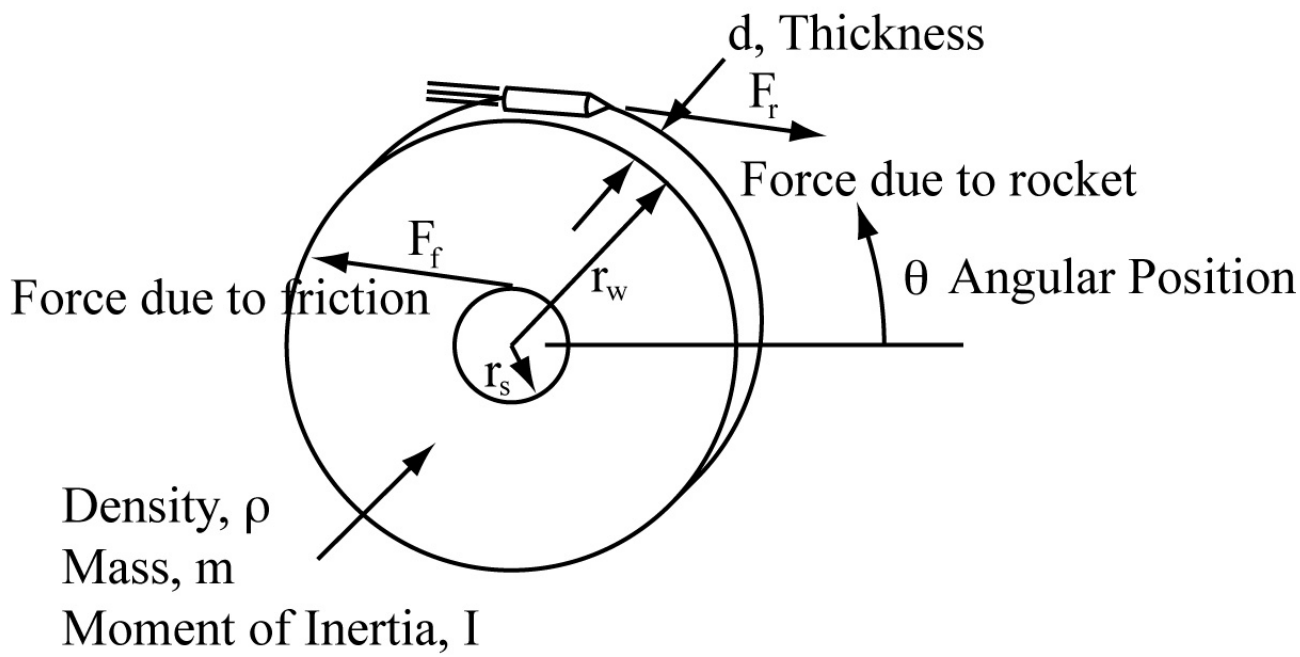


$$I = m\left(r_w^2 + r_s^2\right)/2 \quad , \; m = d\rho\pi\left(r_w^2 - r_s^2\right) \quad \Rightarrow I = d\rho\pi\left(r_w^2 - r_s^2\right)\left(r_w^2 + r_s^2\right)/2$$

$$\rho = 19.1 \text{ g/cm}^3 \times \left(100\text{cm/m}\right)^3 \times 1\text{kg/1000g} = 19{,}100 \text{ kg/m}^3$$

Let $r_w = 0.5$ m, $r_s = 0.1$ m, $d = 0.1$ m. Then

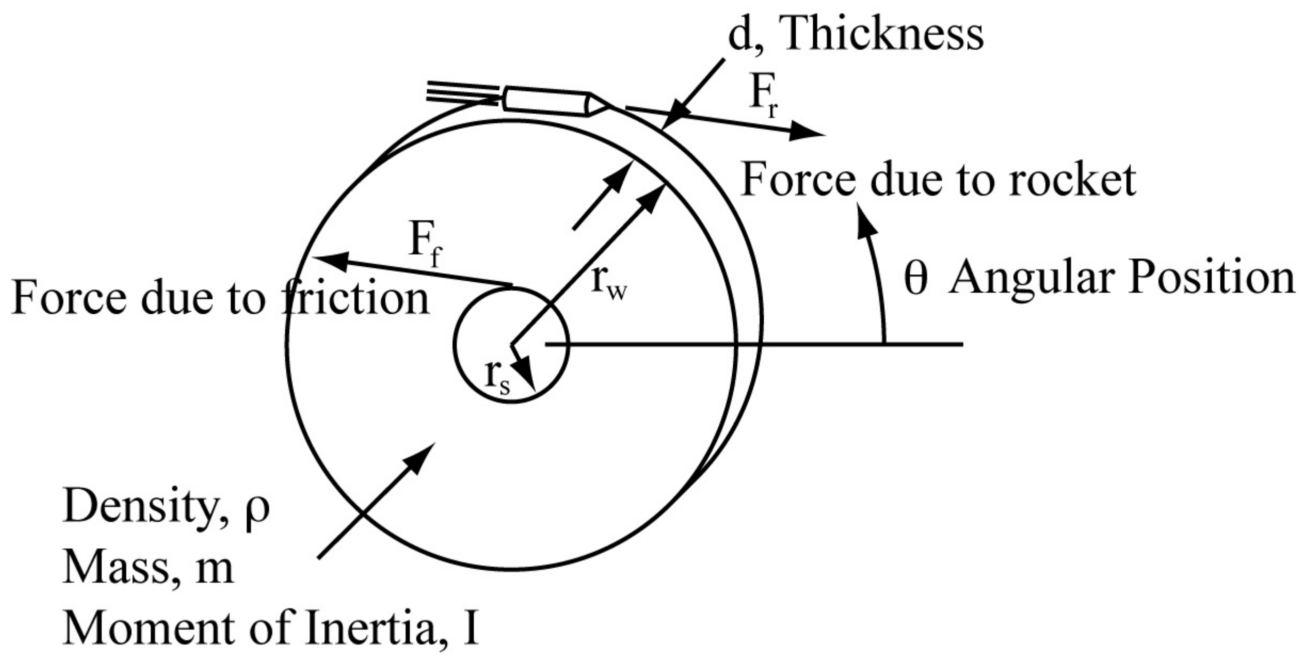
$$I = \pi\left(0.1\text{m}\right)\left(19{,}100\text{kg/m}^3\right)\left(0.24\text{m}^2\right)\left(0.26\text{m}^2\right)/2 = 187.21 \text{ kg}\cdot\text{m}^2$$



Let $F_r = 1000$ N and let the coefficient of friction between spindle and wheel be $k_f = 50\ \text{N}\cdot\text{s}$.

$$\tau = I\frac{d^2}{dt^2}\left(\theta(t)\right) = -F_r \times r_w + F_f \times r_s \leftarrow \text{Torque} = \text{Moment of Inertia} \times \text{Angular Acceleration}$$

$$\left(187.21\ \text{kg}\cdot\text{m}^2\right)\frac{d^2}{dt^2}\left(\theta(t)\right) = -(1000\text{N})(0.5\text{m}) - (50\ \text{N}\cdot\text{s})(0.1\text{m})\frac{d}{dt}\left(\theta(t)\right)$$



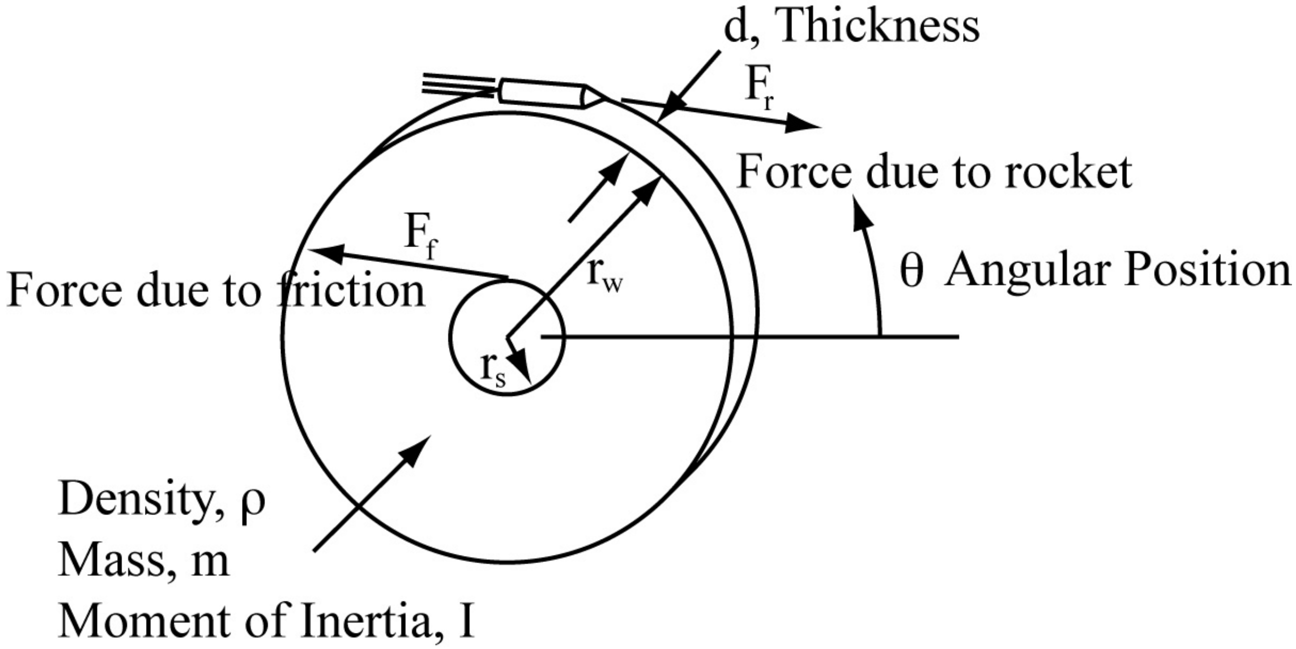
$$187.21\theta''(t)+5\theta'(t)=-500\,\mathrm{u}(t)$$

$$187.21\left[s^2\Theta(s)-s\theta(0)-\left(\frac{d}{dt}\theta(t)\right)_{t=0}\right]+5\left[s\Theta(s)-\theta(0)\right]=-500/s$$

Let $\theta(0)=0$ and let $\left(\frac{d}{dt}\theta(t)\right)_{t=0}=0$. Then

$$187.21s^2\Theta(s)+5s\Theta(s)=-500/s$$

$$\Theta(s)=-\frac{500/s}{187.21s^2+5s}=-\frac{500}{187.21}\frac{1}{s^2(s+0.0267)}=-2.671\left[\frac{37.45}{s^2}-\frac{1403}{s}+\frac{1403}{s+0.0267}\right]$$



$$\Theta(s) = -2.671\left[\frac{37.45}{s^2} - \frac{1403}{s} + \frac{1403}{s+0.0267}\right]$$

$$\Theta(s) = 100\left[-\frac{1}{s^2} + 37.46\left(\frac{1}{s} - \frac{1}{s+0.0267}\right)\right]$$

$$\theta(t) = 100\left[-\mathrm{ramp}(t) + 37.46\left(1 - e^{-0.0267t}\right)\right]\mathrm{u}(t) \text{ radians}$$

$$\theta'(t) = 100\left(-1 + e^{-0.0267t}\right)\mathrm{u}(t) \text{ rad/s}$$