A temperature measurement signal is corrupted by electromagnetic interference (EMI) at 180 Hz from a large nearby induction motor. The temperature signal is sampled at a rate of 1 kHz. Design a digital filter that will reject the 180 Hz EMI and retain as much of the rest of the signal as possible.

Let the continuous-time signal be $\mathrm{x}(t)$. Then the discrete-time signal is $\mathrm{x}[n]=\mathrm{x}(nT_s)$ where $T_s = 1/f_s$ and f_s is 1000. Then any signal at 180 Hz translates into a signal in discrete-time of $F = 180/1000$ cycles/sample or $\Omega = 360\pi/1000 = 1.131$ radians/sample. A popular type of notch filter to reject one frequency Ω_0 and retain the rest of the signal is one with a transfer function

$$\mathrm{H}(z)=\frac{\left(z-e^{j\Omega_0}\right)\left(z-e^{-j\Omega_0}\right)}{\left(z-re^{j\Omega_0}\right)\left(z-re^{-j\Omega_0}\right)}$$

where $r<1$ but close to one. Then the filter response at $z=e^{\pm j\Omega_0}$ is zero.

$$\mathrm{H}(z) = A\frac{\left(z - e^{j\Omega_0}\right)\left(z - e^{-j\Omega_0}\right)}{\left(z - re^{j\Omega_0}\right)\left(z - re^{-j\Omega_0}\right)}$$

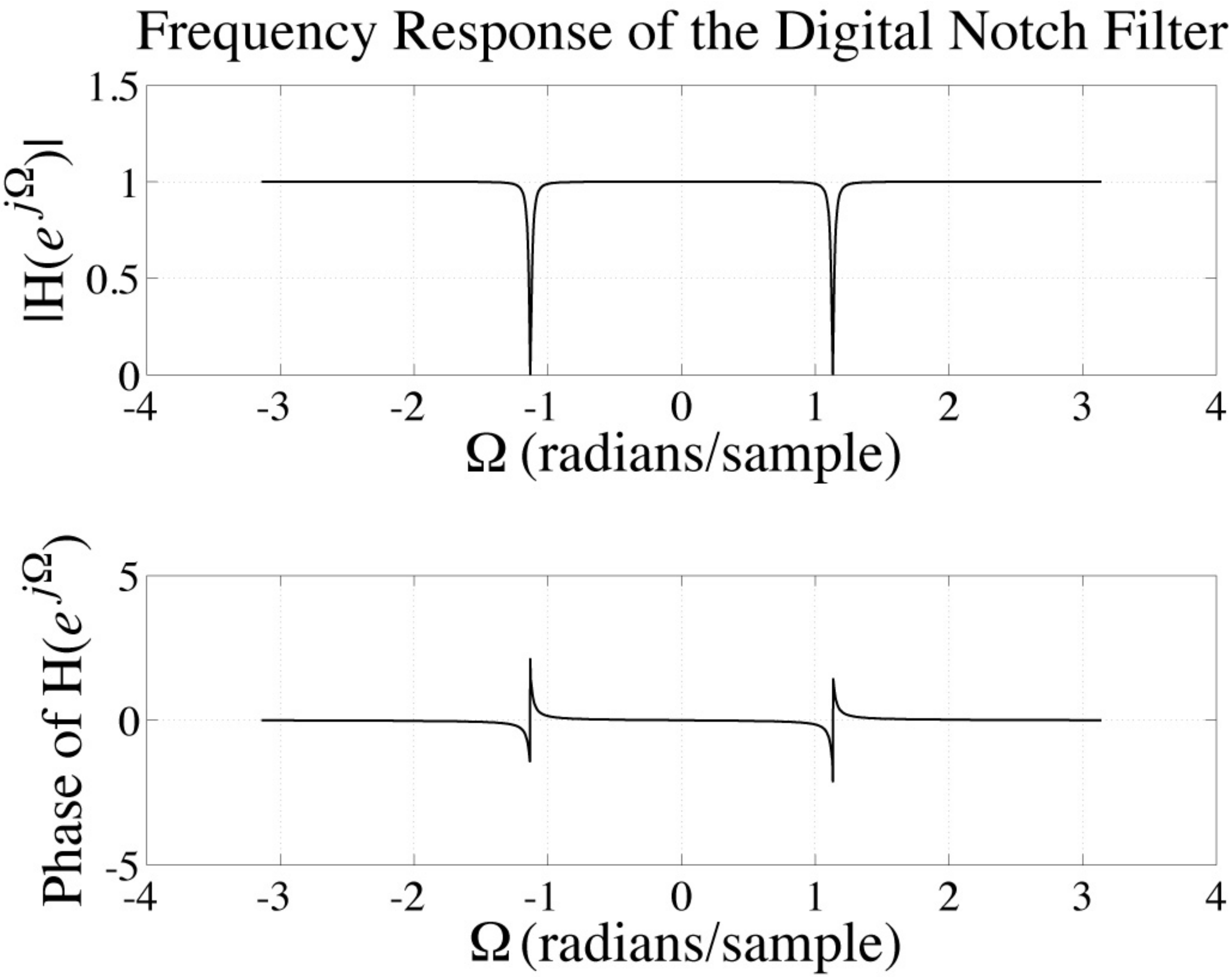
r should be close to one but if it is too close that can cause numerical round-off errors in the digital filter computations. Let $r = 0.98$ and choose A to make the frequency response at zero frequency have a magnitude of one. Then

$$\mathrm{H}(z) = A\frac{\left(z - e^{j\Omega_0}\right)\left(z - e^{-j\Omega_0}\right)}{\left(z - 0.98e^{j\Omega_0}\right)\left(z - 0.98e^{-j\Omega_0}\right)} = A\frac{z^2 - 0.8516z + 1}{z^2 - 0.8345z + 0.9604}$$

The frequency response is $\mathrm{H}\left(e^{j\Omega}\right) = A\dfrac{e^{j2\Omega} - 0.8516e^{j\Omega} + 1}{e^{j2\Omega} - 0.8345e^{j\Omega} + 0.9604}$.

At zero frequency, $\mathrm{H}\left(e^{j0}\right) = A\dfrac{1 - 0.8516 + 1}{1 - 0.8345 + 0.9604} = 1 \Rightarrow A = 0.9803$.

$$H(z) = 0.9803 \frac{z^2 - 0.8516z + 1}{z^2 - 0.8345z + 0.9604}$$



Suppose the actual temperature suddenly changes from -10 °C to +10°C (a step change) at time $t = 0$. Then the indicated temperature is

$$\mathrm{T}(t) = -10 + 20\,\mathrm{u}(t) + V_0 \cos(360\pi t)$$

where V_0 is the amplitude of the EMI corrupting the temperature measurement. Let V_0 be 10. Graph the continuous-time signal from the temperature measurement system and the response of the digital notch filter.