An information signal $\mathrm{x}(t) = 5\cos(1000\pi t)$ LSSB modulates a carrier with amplitude $A_c = 1$. This signal is transmitted through a channel with 30 dB loss. It is demodulated using a synchronous demodulator. Noise with power spectral density $\mathrm{G}_n(f) = N_0/2 = 10^{-8}$ adds to the signal at the receiver input. Find the $(S/N)_D$, in dB.

For SSB, $(S/N)_D = \dfrac{S_R}{N_0 W}$, $S_R = \dfrac{A_c^2 S_\mathrm{x}}{4L} = \dfrac{1^2 \times 5^2/2}{4 \times 1000} = 0.003125$

$N_0 = 2 \times 10^{-8}$ and $W = 500$.

Therefore $(S/N)_D = \dfrac{0.003125}{2 \times 10^{-8} \times 500} = 312.5$ or 24.9485 dB

A signal $x(t) = 2\cos(1000\pi t) + \cos(2000\pi t)$ USSB modulates a carrier with amplitude $A_c = 10$. The modulated carrier is transmitted through a channel with a loss of 70 dB. Noise of power spectral density $G_n(f) = N_0/2 = 10^{-9}$ adds to the signal at the receiver input. (a) Find the signal-to-noise ratio $(S/N)_D$. (b) Suppose a bandpass filter with passband between 400 and 1100 Hz is added to the output of the receiver. Find the improvement in $(S/N)_D$ in dB.

For SSB, $(S/N)_D = \dfrac{S_R}{N_0 W}$, $S_R = \dfrac{A_c^2 S_x}{4L} = \dfrac{10^2 \times \left(2^2/2 + 1^2/2\right)}{4 \times 10^7} = 6.25 \times 10^{-6}$,

$N_0 = 2 \times 10^{-9}$ and $W = 1000$. Therefore $(S/N)_D = \dfrac{6.25 \times 10^{-6}}{2 \times 10^{-9} \times 1000} = 3.125$ or 4.9485 dB. If we bandpass filter the output signal between 400 and 1100 Hz we reduce the noise bandwidth by a factor of 700/1000 or 0.7. That increases the signal-to-noise ratio by the reciprocal of that factor making it 4.4643 or 6.4975 dB, an increase of 1.249 dB.

A signal $\mathrm{x}(t)$ DSB modulates a carrier of amplitude $A_c = 3$. The power spectral density of $\mathrm{x}(t)$ is shown in the figure below. White noise of spectral density $\mathrm{G}_n(f) = N_0/2 = 10^{-4}$ is added at the receiver input. The channel loss is 35 dB. Find the signal-to-noise ratio $(S/N)_D$.

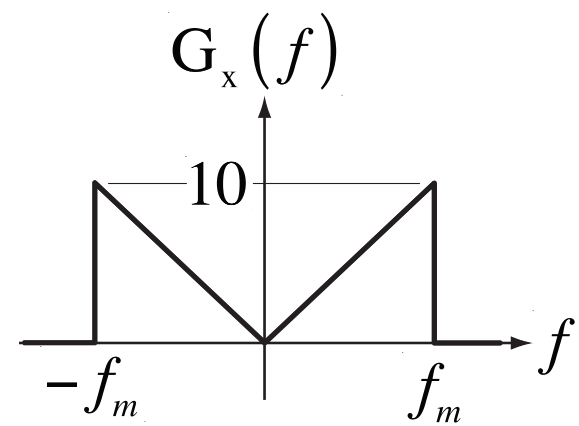
The received signal is $\mathrm{x}_c(t) = \frac{A_c}{\sqrt{L}}\mathrm{x}(t)\cos(\omega_c t)$. Its signal power is

$$S_R = \overline{\left[\frac{A_c}{\sqrt{L}}\mathrm{x}(t)\cos(\omega_c t)\right]^2} = \frac{A_c^2}{L}\overline{\mathrm{x}^2(t)} \times \underbrace{\overline{\cos^2(\omega_c t)}}_{=1/2}.$$

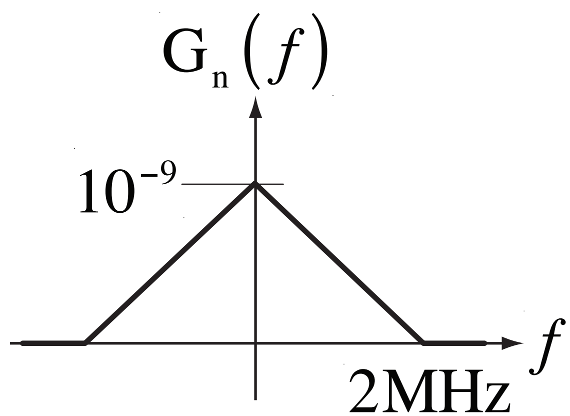
$\overline{\mathrm{x}^2(t)}$ is the area under its power spectral density which is $10f_m$. So the received signal power is $S_R = \frac{3^2}{3162.3}\frac{10f_m}{2} = 0.0142 f_m$ and

for DSB, $(S/N)_D = \frac{S_R}{N_0 W} = \frac{0.0142 f_m}{2\times 10^{-4} f_m} = 71$

or 18.5126 dB.



A message $x(t)$, with $W = 5\text{kHz}$, DSB modulates a carrier of frequency $f_c = 1\text{MHz}$ and amplitude $A_c = 1$. The channel loss is 20 dB. Nonwhite noise with power spectral density as shown in the figure below adds to the signal prior to detection with a synchronous demodulator. Find the signal-to-noise ratio $(S/N)_D$ assuming that the power of the signal is 1 W.



The received signal is $x_c(t) = \frac{A_c}{\sqrt{L}} x(t)\cos(\omega_c t)$. The signal plus noise is

$$v(t) = \left[\frac{A_c}{\sqrt{L}} x(t) + n_i(t)\right]\cos(\omega_c t) - n_q(t)\sin(\omega_c t)$$

The synchronously demodulated signal plus noise is $y(t) = y_D(t) = \frac{A_c}{2\sqrt{L}} x(t) + \frac{n_i(t)}{2}$

$$G_n(f) = 10^{-9}\Lambda(f/2\times 10^6) \Rightarrow G_{n_i}(f) = G_{n_q}(f) = \begin{bmatrix} G_n(f+f_c)u(f+f_c) \\ +G_n(f-f_c)u(f_c-f) \end{bmatrix}$$

$$G_{n_i}(f) = 10^{-9}\left[\Lambda\left(\left(f+10^6\right)/2\times 10^6\right)u(f+f_c) + \Lambda\left(\left(f-10^6\right)/2\times 10^6\right)u(f_c-f)\right]$$

$$N_D = \overline{\left[\frac{n_i(t)}{2}\right]^2} = (1/4)\int_{-W}^{W} G_{n_i}(f)\,df = (1/4)\int_{-W}^{W} 10^{-9}\begin{bmatrix} \Lambda\left(\left(f+10^6\right)/2\times 10^6\right)u(f+f_c) \\ +\Lambda\left(\left(f-10^6\right)/2\times 10^6\right)u(f_c-f) \end{bmatrix} df$$

$$N_D = (1/2)\times 10^{-9}\int_0^W \begin{bmatrix} \Lambda\left(\left(f+10^6\right)/2\times 10^6\right) \\ +\Lambda\left(\left(f-10^6\right)/2\times 10^6\right) \end{bmatrix} df = (1/2)\times 10^{-9}\times 1\times W = 2.5\times 10^{-6}$$

$$S_D = \frac{A_c^2}{4L}\overline{x^2(t)} = \frac{1^2}{4\times 100}\times 1 = 0.0025 \Rightarrow (S/N)_D = \frac{S_D}{N_D} = 1{,}000 \text{ or } 30 \text{ dB}$$

An information signal $x(t)=\cos(2000\pi t)$ frequency modulates a carrier of frequency 1MHz. $A_c=2$ and $f_\Delta=100\text{Hz/V}$. The channel loss is 90 dB. The receiver noise temperature is $\mathcal{T}=2000\text{K}$. (a) Find the signal-to-noise ratio $(S/N)_D$ in dB. (b) If the output signal is passed through a bandpass filter with passband from 900 to 1100 Hz find the improvement in $(S/N)_D$ in dB. (c) Repeat parts (a) and (b) for AM with synchronous demodulation and compare results.

(a)

$$S_R = \frac{A_c^2}{2L} = \frac{2^2}{2\times10^9} = 2\times10^{-9} \text{ and}$$

$$N_0 = k\mathrm{T} = k\mathrm{T}_0\left(\mathrm{T}/\mathrm{T}_0\right) = 4\times10^{-21}\frac{2000}{290} = 2.76\times10^{-20}$$

$$W = 1000 \Rightarrow D = \frac{100}{1000} = 0.1 \text{ and } B_T \cong 2\left(D+1\right)W \Rightarrow N_0 B_T = 6.072\times10^{-17}$$

$S_R >> N_0 B_T$ so the approximations for high signal-to-noise ratio apply.

$$\left(S/N\right)_D = 3D^2\frac{S_x S_R}{N_0 W} = 3\left(0.1\right)^2\frac{1/2\times2\times10^{-9}}{2.76\times10^{-20}\times1000} = 1.09\times10^6$$

or 60.36 dB

(b)

If the output signal is passed through a bandpass filter with passband from 900 to 1100 Hz the noise bandwidth is reduced by a factor of 200/1000 or 0.2. This increases the signal-to-noise ratio by a factor of 1/0.2 or 5, making it 5.45×10^6 or 67.36 dB, an increase of 6.99 dB.

(c)

For AM modulation (with μ=1)

$$\left(S/N\right)_D = \frac{S_R S_x}{N_0 W\left(1+S_x\right)}$$

$$S_R = A_c^2\left(1+S_x\right)/2L = 2^2\times\left(1+1/2\right)/2\times10^9 = 3\times10^{-9}$$

$$\left(S/N\right)_D = \frac{3\times10^{-9}\times1/2}{2.76\times10^{-20}\times1000\times\left(1+1/2\right)} = 3.623\times10^7 \text{ or } 75.59 \text{ dB}$$

If the output signal is passed through a bandpass filter with passband from 900 to 1100 Hz the noise bandwidth is reduced by a factor of 200/1000 or 0.2. This increases the signal-to-noise ratio by a factor of 1/0.2 or 5, making it 1.811×10^8 or 82.58 dB, an increase of 6.99 dB.

A signal $x(t)$ with power spectral density $G_x(f) = K\Pi(f/8000)$ is transmitted over a telephone channel with transfer function

$$H_c(f) = \frac{4}{jf + 4000}.$$ The noise power spectral density at the receiver input is $G_n(f) = 10^{-10}$. To compensate for the channel distortion, the receiver filter transfer function is chosen to be

$$H_D(f) = \frac{jf + 4000}{4000}\Pi(f/8000).$$ The receiver's $(S/N)_D$ is required to be at least 35 dB. Determine the minimum required value of K and the corresponding transmitted power S_T and the received power S_R.

$$G_{x_c}(f) = G_x(f)\left|H_c(f)\right|^2 = K\Pi(f/8000)\frac{4^2}{f^2+4000^2}$$

$$G_{y_D}(f) = G_{x_c}(f)\left|H_D(f)\right|^2 = K\Pi(f/8000)\frac{4^2}{f^2+4000^2}\times\frac{f^2+4000^2}{4000^2}\Pi^2(f/8000)$$

$$G_{y_D}(f) = K\Pi(f/8000)\frac{4^2}{4000^2}$$

$$S_D = \int_{-\infty}^{\infty} K\Pi(f/8000)\frac{4^2\,df}{4000^2} = \frac{16K}{1.6\times10^7}\int_{-4000}^{4000} df = 0.008K$$

$$G_n(f) = 10^{-10} \Rightarrow N_D = \int_{-4000}^{4000} 10^{-10}\,\frac{f^2+4000^2}{4000^2}df = 1.25\times10^{-17}\left[f^3/3+4000^2 f\right]_0^{4000} = 1.0667\times10^{-6}$$

$$(S/N)_D = \frac{S_D}{N_D} = \frac{0.008K}{1.0667\times10^{-6}} = 7500K$$

35 dB is a $(S/N)_D$ of $3{,}162.3 \Rightarrow K = 3{,}162.3/7500 = 0.4216$

$$S_T = 0.4216\times8000 = 3373.1$$

$$S_R = \int_{-\infty}^{\infty} 0.4216\Pi(f/8000)\frac{4^2\,df}{f^2+4000^2} = 6.7456\int_{-4000}^{4000}\frac{df}{f^2+4000^2} = 6.7456\left[\frac{\tan^{-1}(f/4000)}{4000}\right]_{-4000}^{4000}$$

$$S_R = \frac{6.7456}{4000}\left[\tan^{-1}(1)-\tan^{-1}(-1)\right] = 0.00265$$

A message $\mathrm{x}(t)$ is a zero-mean Gaussian random signal with $W = 40$ kHz and $\sigma_{\mathrm{x}} = 0.3$. It AM modulates a carrier of amplitude $A_c = 1$ (with $\mu = 1$). The channel loss is 70 dB. The noise temperature of the receiver is $\mathscr{T} = 10{,}000\mathrm{K}$. If the demodulation is synchronous, find the signal-to-noise ratio at the destination.

$$S_R = \frac{A_c^2}{2L}\left(1 + S_{\mathrm{x}}\right) = \frac{1^2 \times 1.09}{2 \times 10^7} = 5.45 \times 10^{-8}$$

$$N_0 = k\mathscr{T} = 1.379 \times 10^{-19}$$

$$(S/N)_D = \frac{S_R S_{\mathrm{x}}}{N_0 W\left(1 + S_{\mathrm{x}}\right)} = \frac{5.45 \times 10^{-8} \times 0.09}{1.379 \times 10^{-19} \times 40000 \times 1.09}$$

$$(S/N)_D = 8.158 \times 10^5 \text{ or } 59.12 \text{ dB}$$