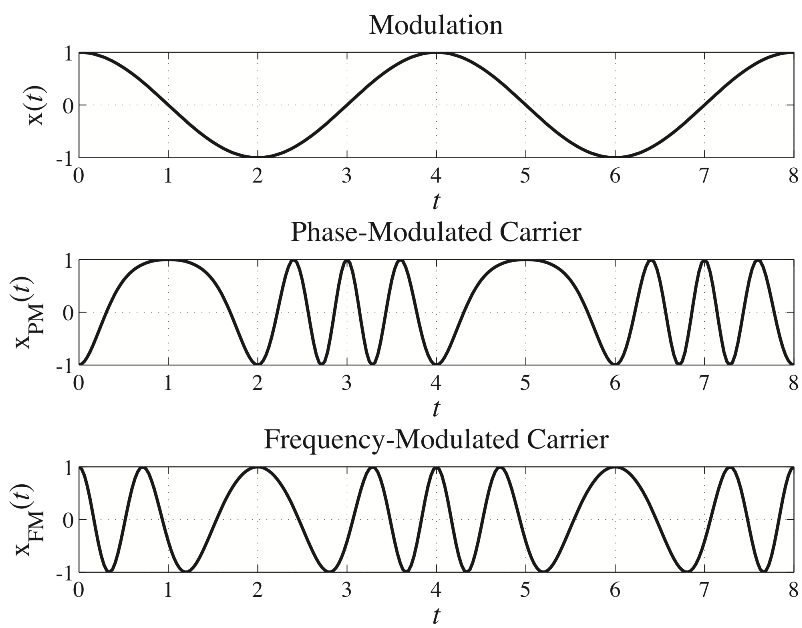
A sinusoidal signal $x(t) = \cos(2\pi f_m t)$ is the input to an angle-modulated transmitter with carrier frequency $f_c = 1$ Hz and modulation frequency $f_m = f_c / 4$.

(a) Graph $x(t)$ and the corresponding PM signal where $\phi_\Delta = \pi$.

(b) Graph $x(t)$ and the corresponding FM signal where $f_\Delta = 1/2$.

$$(a) \quad x_{PM}(t) = \cos(2\pi f_c t + \pi\cos(2\pi f_m t)) = \cos(2\pi t + \pi\cos(\pi t/2))$$

$$(b) \quad x_{FM}(t) = \cos\left(2\pi f_c t + 2\pi(1/2)\frac{\sin(2\pi f_m t)}{2\pi f_m}\right) = \cos(2\pi t + 2\sin(\pi t/2))$$



An FM signal has a sinusoidal modulation with a frequency of $f_m = 15$ kHz and a modulation index of $\beta = 2$.

(a) Find the transmission bandwidth using Carson's rule.

(b) What percentage of the total FM signal power lies within the Carson rule bandwidth?

Carson's Rule for $2 < D < 10$ is $B_T \approx 2(f_\Delta + 2W)$ where, for sinusoidal modulation, $W = f_m$ and $D = \beta = A_m f_\Delta / f_m$ (and A_m is assumed to be 1). Therefore $f_\Delta = \beta f_m$ and $B_T \approx 2(\beta + 2) f_m$ $= 120$ kHz. Since $\beta = 2$, this approximation is borderline, but others are also.

For tone modulation, $x_c(t) = A_c \sum_{k=-\infty}^{\infty} J_k(\beta)\cos((\omega_c + k\omega_m)t)$, Eqn. 18b, page 216

k	-5	-4	-3	-2	-1	0	1	2	3	4	5
$J_k(2)$	-0.007	0.034	-0.1289	0.3528	-0.5767	0.2239	0.5767	0.3528	0.1289	0.034	0.007
$[J_k(2)]^2$	4.9×10^{-5}	0.0012	0.0166	0.1245	0.3326	0.0501	0.3326	0.1245	0.0166	0.0012	4.9×10^{-5}

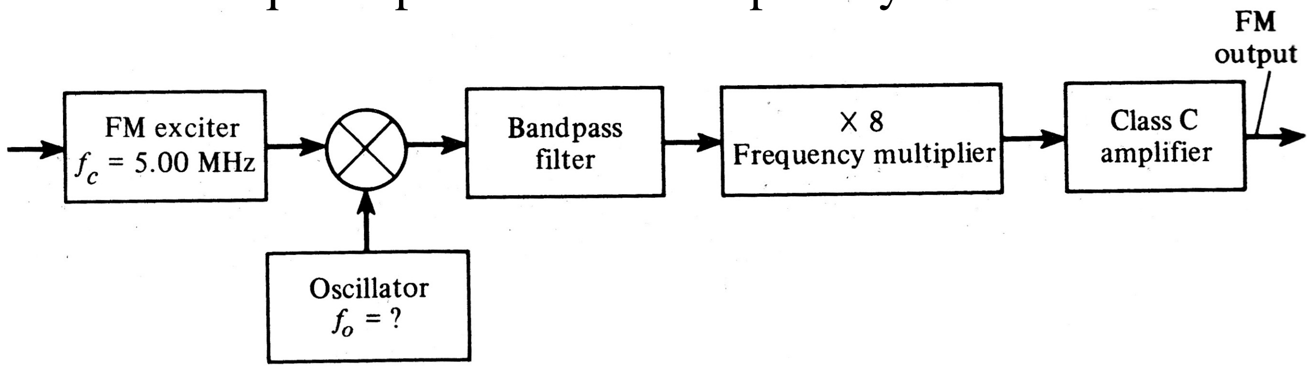
The total signal power of the modulated carrier is $A_c^2/2$. The signal power within the Carson's Rule bandwidth is half the sum of the squares of the Bessel function values for $-4 \le k \le 4$ which is $0.4999A_c^2$ (99.98% of $A_c^2/2$). The signal power for $-3 \le k \le 3$ is $0.4988A_c^2$ (99.76% of $A_c^2/2$) and for $-2 \le k \le 2$ is $0.4822A_c^2$ (96.44% of $A_c^2/2$). So it would seemthat Carson's rule, as construed here, is a little conservative.

An FM transmitter has a block diagram as shown below. The audio-frequency response is flat over the 20 Hz to 15 kHz audio band. The FM output signal is to have a carrier frequency of 103.7 MHz and a peak instantaneous frequency deviation of 75 kHz.

(a) Find the bandwidth and center frequency required for the bandpass filter.

(b) Calculate the frequency f_0 of the oscillator.

(c) What is the required peak deviation capability of the FM exciter?

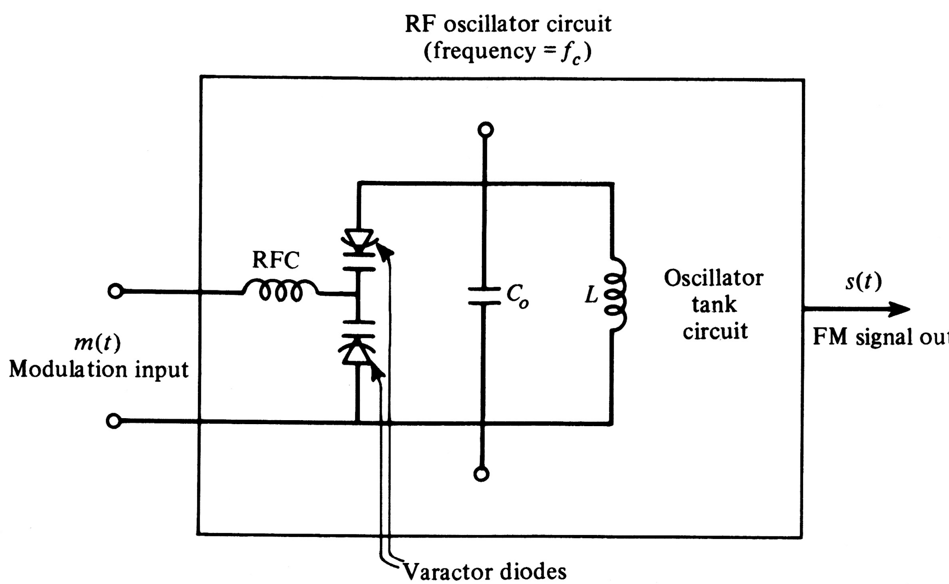


Under the usual assumption that the peak modulation amplitude is normalized to one, $f_\Delta = 75{,}000$ for the output FM signal and $D = f_\Delta / W = 5$. The bandwidth is approximately $B_T \approx 2\left(f_\Delta + 2W\right) = 2\left(75{,}000 + 30{,}000\right) = 210$ kHz . Prior to the $\times 8$ frequency multiplier that bandwidth would be approximately 26.5 kHz and the center frequency would be $103.7/8 = 12.9625$ MHz. So the oscillator frequency f_0 can be 7.9625 MHz or 17.9625 MHz. The peak frequency deviation capability of the FM exciter should be 75 kHz/8 = 9.375 kHz.

Analyze the performance of the FM circuit below. Assume that the voltage appearing across the reverse-biased varactor diodes is $\mathrm{v}(t) = 5 + 0.05\,\mathrm{x}(t)$ where $\mathrm{x}(t) = \cos(2000\pi t)$. The capacitance of each of the varactor diodes is $C_d = \dfrac{100}{\sqrt{1+2\,\mathrm{v}(t)}}\,\mathrm{pF}$. Assume that $C_0 = 180\,\mathrm{pF}$ and that L is chosen to resonate at 5 MHz.

(a) Find the value of L.

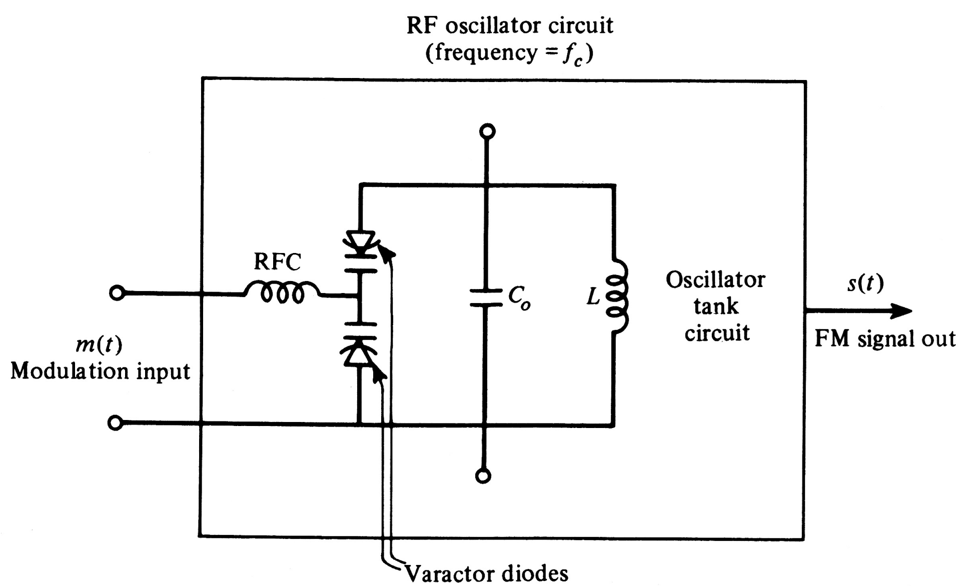
(b) Show that the resulting oscillator signal is an FM signal. Find the parameter f_Δ.



(a) Find the value of L.

With a modulating signal of zero, each varactor diode capacitance is 30.15pF. So the equivalent parallel capacitance in the LC resonator is $180\text{pF} + 30.1511\text{pF}/2 = 195.0756\text{pF}$. The resonant radian frequency is $10^7\pi$ rad/s. Therefore

$$10^7\pi = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\left(10^7\pi\right)^2 195.0756\times10^{-12}} = 5.1939\ \mu\text{H}$$



(b) Show that the resulting oscillator signal is an FM signal. Find the parameter f_Δ.

The time-varying instantaneous frequency of the oscillator is

$$\mathrm{f}(t) = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{L(C_0 + C_d/2)}} = \frac{1}{2\pi\sqrt{5.1939\,\mu\mathrm{H} \times \left(180 + \frac{50}{\sqrt{1 + 2\left[5 + 0.05\cos(2000\pi t)\right]}}\right)\mathrm{pF}}}$$

The maximum frequency occurs when the varactor diode capacitance is at a minimum.

That occurs when $\cos(2000\pi t) = 1$. Then

$$\mathrm{f}_{\max}(t) = \frac{1}{2\pi\sqrt{5.1939\,\mu\mathrm{H} \times \left(180 + \frac{50}{\sqrt{11.1}}\right)\mathrm{pF}}} = 5.0009\ \mathrm{MHz}$$

$$\mathrm{f}_{\min}(t) = \frac{1}{2\pi\sqrt{5.1939\,\mu\mathrm{H} \times \left(180 + \frac{50}{\sqrt{10.9}}\right)\mathrm{pF}}} = 4.9991\ \mathrm{MHz}$$

So the maximum instantaneous frequency deviation from the carrier frequency is 900 Hz and, with a modulation amplitude of one, $f_\Delta = 900$.

A modulated RF waveform is given by

$$x_c(t) = 500\cos\left(2\times10^8\pi t + 20\cos(2000\pi t)\right).$$

(a) If $\phi_\Delta = 100$, find the mathematical expression for the corresponding phase modulation voltage $x(t)$. What is its amplitude and its frequency?

(b) If $f_\Delta = 10^6/2\pi$ find the mathematical expression for the corresponding FM voltage $x(t)$. What is its amplitude and frequency?

(a) $x_c(t) = A_c\cos\left(\omega_c t + \phi_\Delta x(t)\right) \Rightarrow 20\cos(2000\pi t) = 100\,x(t)$

$\Rightarrow x(t) = 0.2\cos(2000\pi t)$, Amplitude is 0.2, frequency is 1 kHz.

(b) $x_c(t) = A_c\cos\left(\omega_c t + 2\pi f_\Delta \int x(\lambda)\,d\lambda\right) \Rightarrow 20\cos(2000\pi t) = 2\pi f_\Delta \int x(\lambda)\,d\lambda$

Differentiate both sides to yield

$$\frac{-40000\pi\sin(2000\pi t)}{2\pi 10^6/2\pi} = x(t) = -0.1257\sin(2000\pi t)$$

Amplitude is 0.1257, frequency is 1 kHz

Given the FM signal $x_c(t) = 10\cos\left(400\pi t + 100\int x(\lambda)d\lambda\right)$ where $x(t)$ is a square wave of period 1 second which is at 5V half the time and at -5V half the time,

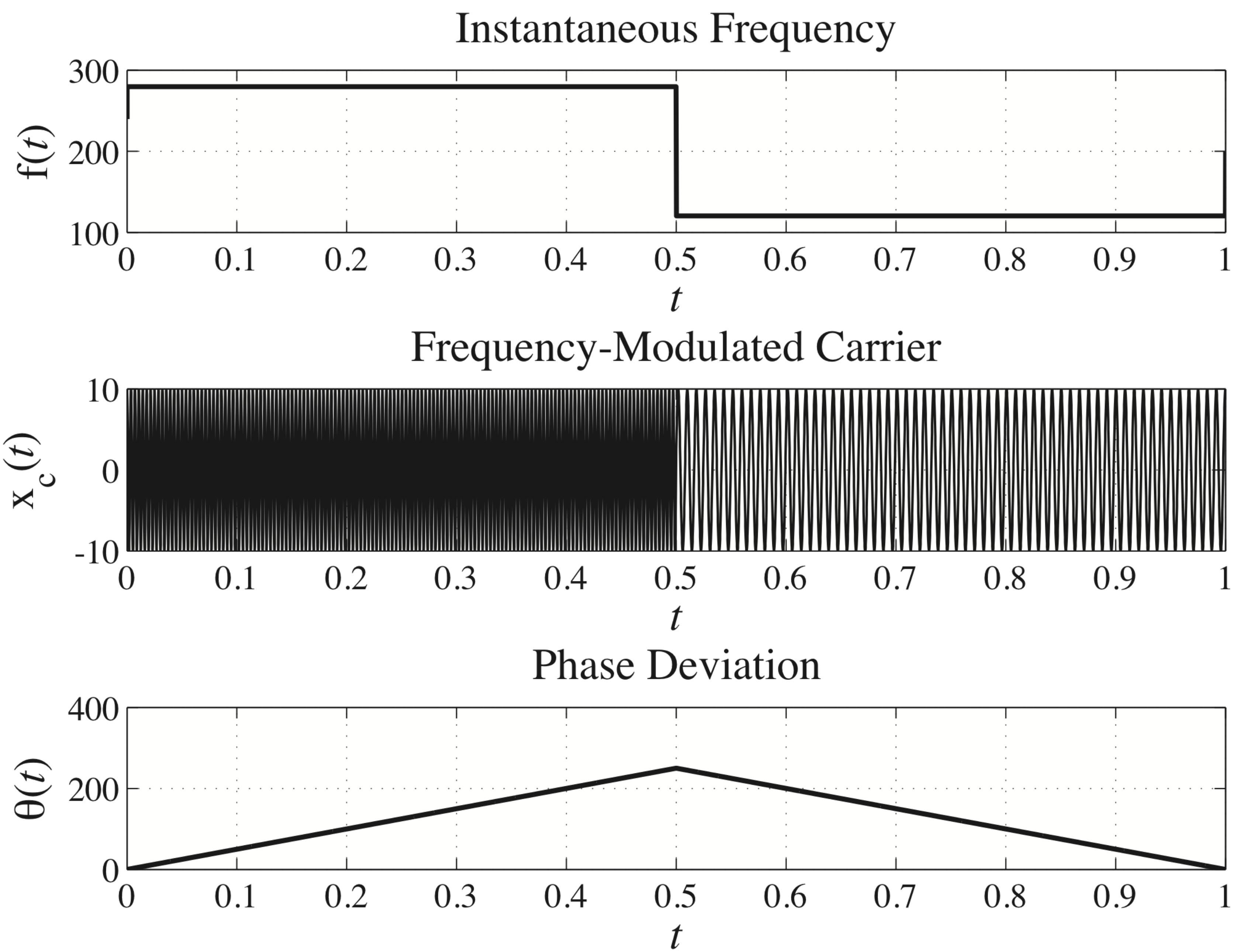
(a) Graph the instantaneous frequency waveform and the waveform of the corresponding FM signal.

(b) Graph the phase deviation as a function of time.

(c) Evaluate the peak frequency deviation.

The exact waveforms depend on the phase of the square wave. Assume it starts at time $t = 0$ at 5V for 0.5 seconds and then at -5V for 0.5 seconds, etc... Then the integral of the square wave is a triangle wave whose first period goes linearly from 0 to 2.5 and back to 0 in the time interval $0 < t < 1$.

From $x_c(t) = A_c\cos\left(\omega_c t + 2\pi f_\Delta \int x(\lambda)d\lambda\right)$, $2\pi f_\Delta = 100 \Rightarrow f_\Delta = 100/2\pi$ and the instantaneous frequency, which is $f_c + f_\Delta x(t)$, switches back and forth between $200 \pm 500/2\pi = 200 \pm 79.5775$ Hz. The peak frequency deviation is therefore 70.5775 Hz. (Graphs on next slide.)



Instantaneous Frequency
f(t)
300
200
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
Frequency-Modulated Carrier
x_c(t)
10
0
-10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
Phase Deviation
θ(t)
400
200
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t