

Twelve voice signals, each band-limited to 3 kHz, are frequency-multiplexed using 1 kHz guard bands between channels and between the main carrier and the first channel. The modulation of the main carrier is AM. Calculate the bandwidth of the composite channel if the subcarrier modulation is (a) DSB, (b) LSSB.

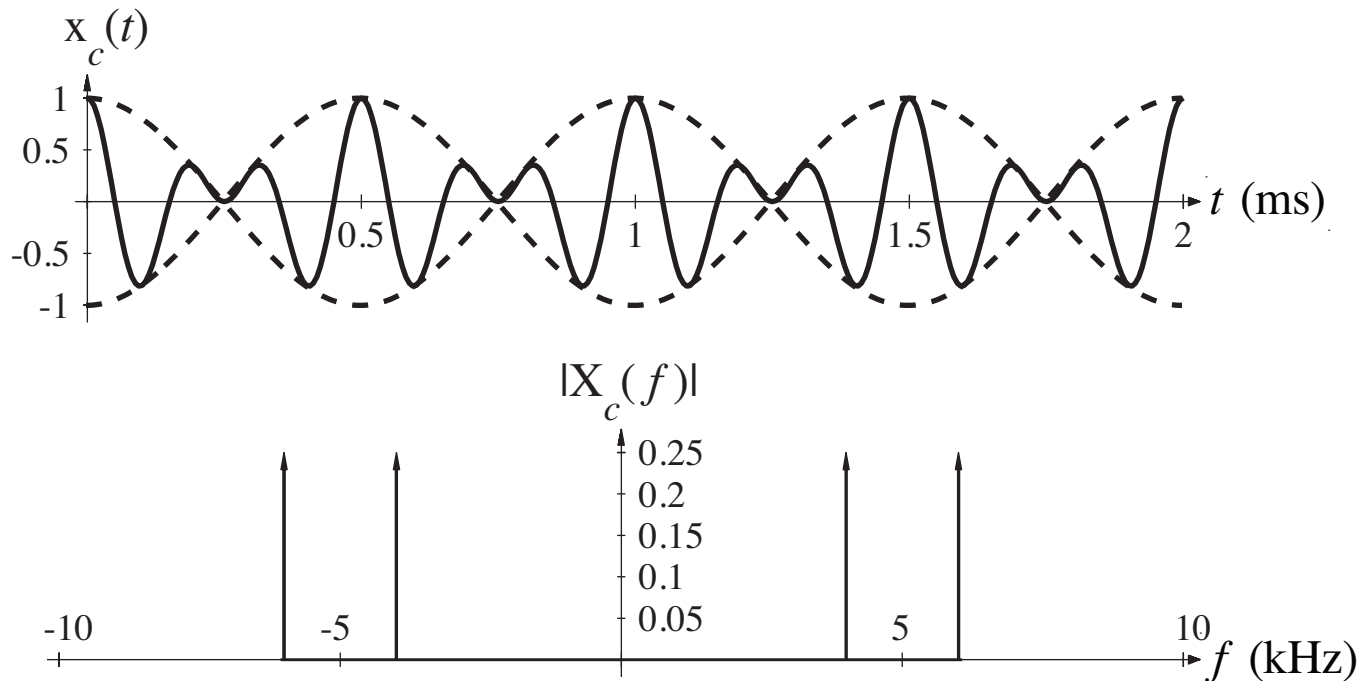
(a) With DSB each 3 kHz channel becomes a 6 kHz bandwidth when modulating a subcarrier. So each channel occupies 7 kHz for a total bandwidth of 84 kHz before modulating the main carrier. The main carrier is AM so the bandwidth requirement for that signal is double 84 kHz or 168 kHz.

(b) With LSSB each 3 kHz channel becomes a 3 kHz bandwidth when modulating a subcarrier. So each channel occupies 4 kHz for a total bandwidth of 48 kHz before modulating the main carrier. The main carrier is AM so the bandwidth requirement for that signal is double 48 kHz or 96 kHz.

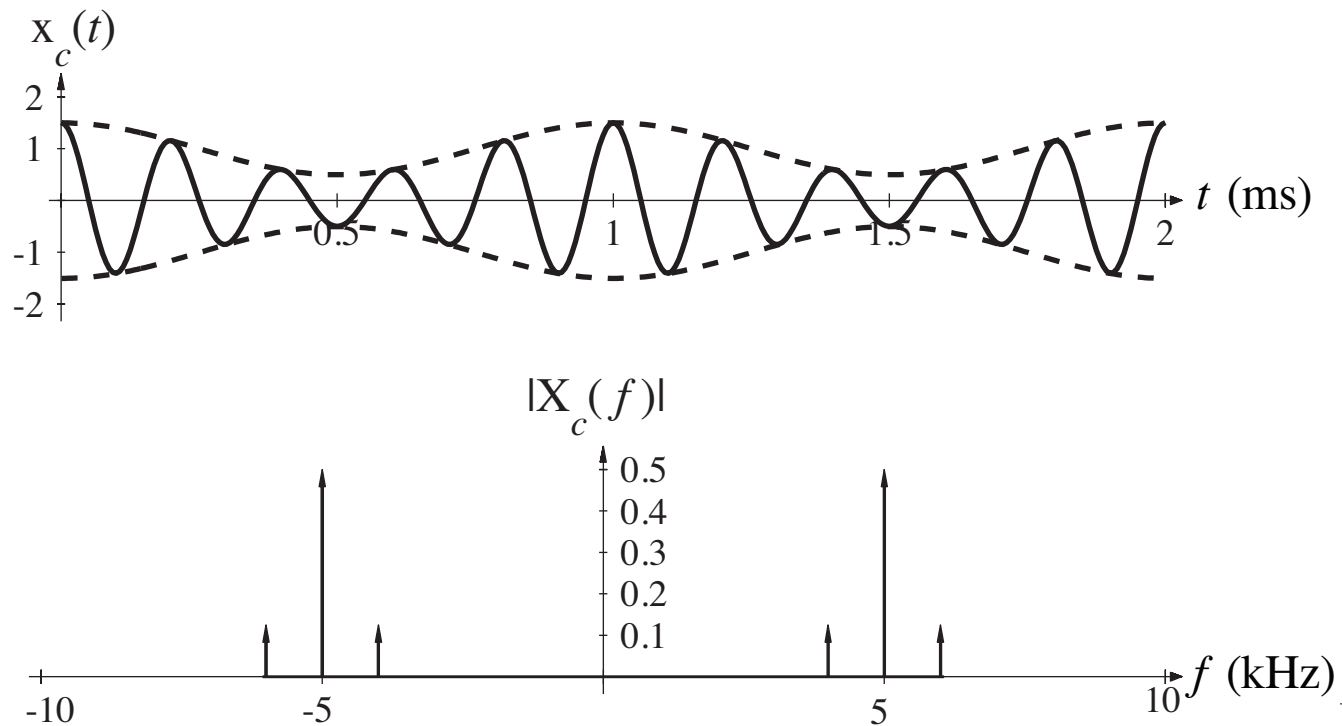
The signal $x(t) = \cos(2000\pi t)$ is used to modulate a 5 kHz carrier. Sketch the time waveforms and line spectra if the modulation used is (a) DSB, (b) AM with $\mu = 0.5$, (c) USSB, (d) LSSB.

(a)
$$x_c(t) = A_c \cos(2000\pi t) \cos(10000\pi t)$$

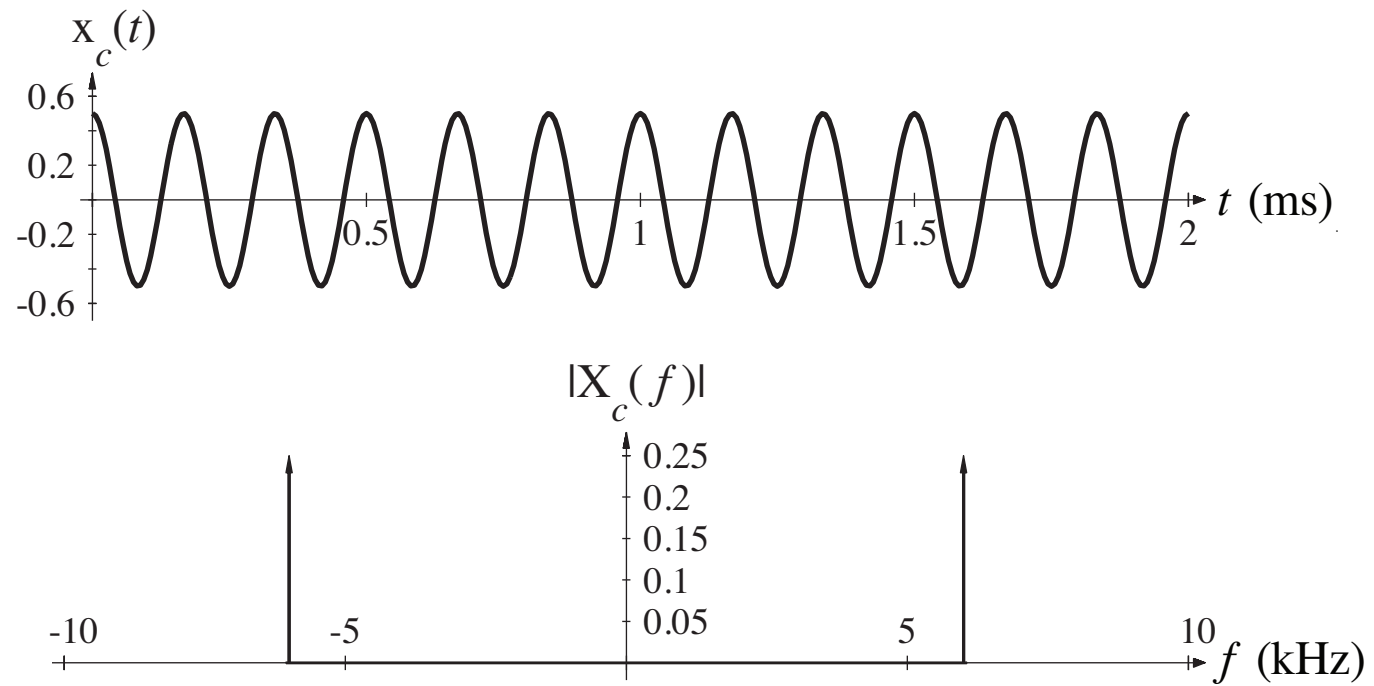
$$= (A_c / 2) [\cos(8000\pi t) + \cos(12000\pi t)]$$



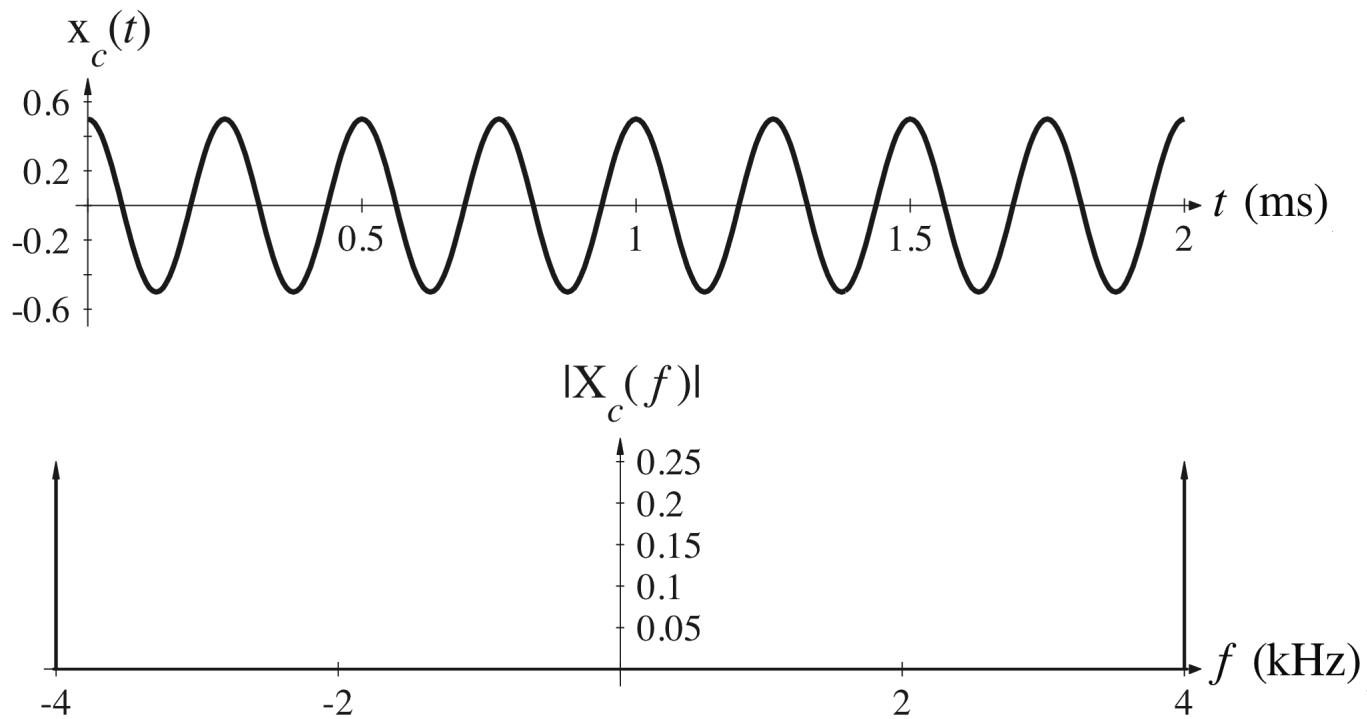
$$\begin{aligned}
 \text{(b) } x_c(t) &= A_c [1 + 0.5 \cos(2000\pi t)] \cos(10000\pi t) \\
 &= A_c \cos(10000\pi t) + (A_c / 4) [\cos(8000\pi t) + \cos(12000\pi t)]
 \end{aligned}$$



(c) $x_c(t) = (A_c / 2) \cos(12000\pi t)$



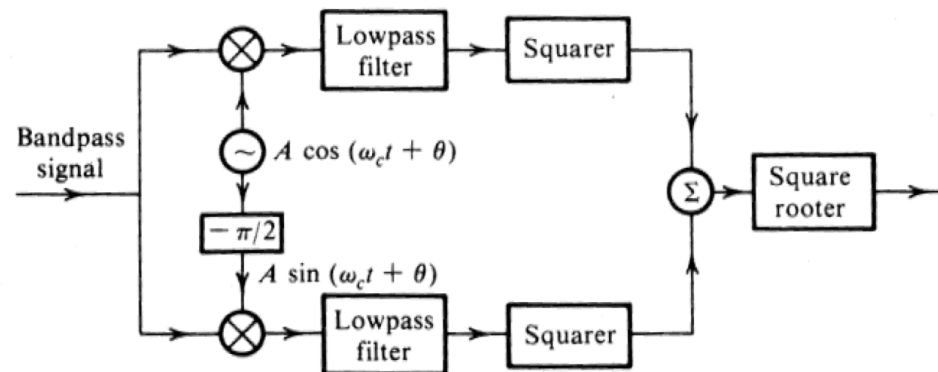
$$(d) \quad x_c(t) = (A_c / 2) \cos(8000\pi t)$$



Show that the system below acts as an envelope detector for a bandpass signal. Verify that the system can indeed demodulate an AM wave. (Hint: Consider a general bandpass signal

$$x_c(t) \cos(\omega_c t) + x_s(t) \sin(\omega_c t).$$

Show that the output is the envelope $(A/2) \sqrt{x_c^2(t) + x_s^2(t)}$. Assume a narrowband signal.)



$$x(t) = x_c(t) \cos(\omega_c t) + x_s(t) \sin(\omega_c t)$$

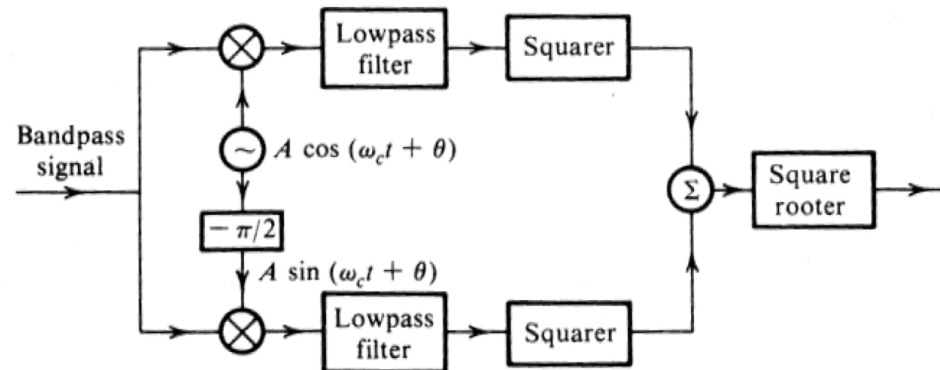
On the top path:

$$x_c(t) \cos(\omega_c t) + x_s(t) \sin(\omega_c t) \xrightarrow{\text{Mixer}} A \begin{bmatrix} x_c(t) \cos(\omega_c t) \\ + x_s(t) \sin(\omega_c t) \end{bmatrix} \cos(\omega_c t + \theta)$$

$$A \left[x_c(t) \cos(\omega_c t) \cos(\omega_c t + \theta) + x_s(t) \sin(\omega_c t) \cos(\omega_c t + \theta) \right] =$$

$$(A/2) \left\{ x_c(t) [\cos(\theta) + \cos(2\omega_c t + \theta)] + x_s(t) [\sin(-\theta) + \sin(2\omega_c t + \theta)] \right\} \xrightarrow{\text{LPF}}$$

$$(A/2) \begin{Bmatrix} x_c(t) \cos(\theta) \\ -x_s(t) \sin(\theta) \end{Bmatrix} \xrightarrow{\text{Squarer}} (A^2/4) \begin{Bmatrix} x_c^2(t) \cos^2(\theta) + x_s^2(t) \sin^2(\theta) \\ -2x_c(t)x_s(t) \cos(\theta) \sin(\theta) \end{Bmatrix}$$



$$x(t) = x_c(t) \cos(\omega_c t) + x_s(t) \sin(\omega_c t)$$

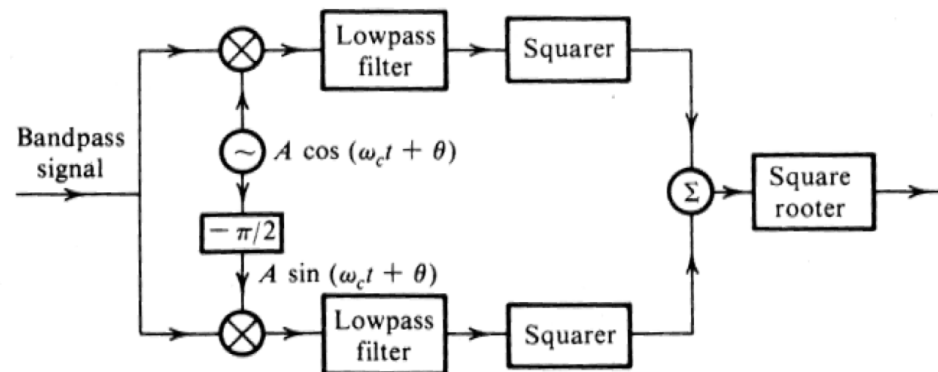
On the bottom path:

$$x_c(t) \cos(\omega_c t) + x_s(t) \sin(\omega_c t) \xrightarrow{\text{Mixer}} A \begin{bmatrix} x_c(t) \cos(\omega_c t) \\ + x_s(t) \sin(\omega_c t) \end{bmatrix} \sin(\omega_c t + \theta)$$

$$A \left[x_c(t) \cos(\omega_c t) \sin(\omega_c t + \theta) + x_s(t) \sin(\omega_c t) \sin(\omega_c t + \theta) \right] =$$

$$(A/2) \left\{ x_c(t) \left[\sin(\theta) + \sin(2\omega_c t + \theta) \right] + x_s(t) \left[\cos(\theta) + \cos(2\omega_c t + \theta) \right] \right\} \xrightarrow{\text{LPF}}$$

$$(A/2) \begin{Bmatrix} x_c(t) \sin(\theta) \\ + x_s(t) \cos(\theta) \end{Bmatrix} \xrightarrow{\text{Squarer}} (A^2/4) \begin{Bmatrix} x_c^2(t) \sin^2(\theta) + x_s^2(t) \cos^2(\theta) \\ + 2x_c(t)x_s(t) \cos(\theta) \sin(\theta) \end{Bmatrix}$$



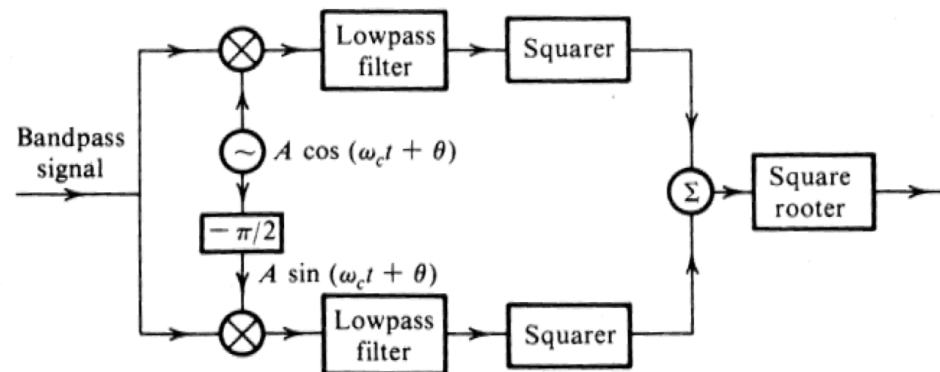
On the top path: $(A^2 / 4) \begin{Bmatrix} x_c^2(t) \cos^2(\theta) + x_s^2(t) \sin^2(\theta) \\ -2x_c(t)x_s(t) \cos(\theta) \sin(\theta) \end{Bmatrix}$

On the bottom path: $(A^2 / 4) \begin{Bmatrix} x_c^2(t) \sin^2(\theta) + x_s^2(t) \cos^2(\theta) \\ +2x_c(t)x_s(t) \cos(\theta) \sin(\theta) \end{Bmatrix}$

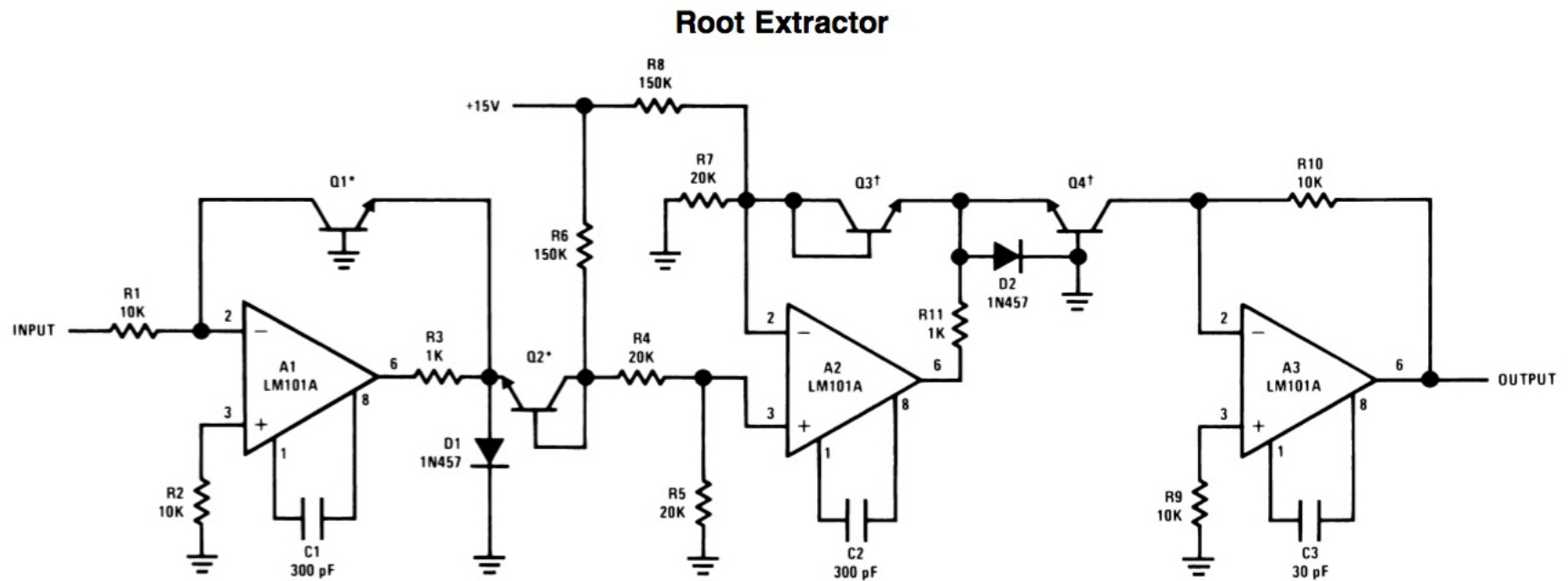
Adding the two signals we get

$$(A^2 / 4) \begin{Bmatrix} x_c^2(t) \cos^2(\theta) + x_s^2(t) \sin^2(\theta) \\ x_c^2(t) \sin^2(\theta) + x_s^2(t) \cos^2(\theta) \end{Bmatrix} = (A^2 / 4) [x_c^2(t) + x_s^2(t)]$$

→ $(A / 2) \sqrt{x_c^2(t) + x_s^2(t)}$



This is a circuit that performs the square-root function. It is taken from a National Semiconductor collection of op-amp circuits.



*†2N3728 matched pairs

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Show that a squaring circuit followed by a lowpass filter followed by a square rooter acts as an envelope detector for an AM wave. Show that if a DSB signal $x(t)\cos(\omega_c t)$ is demodulated by this scheme the output will be $|x(t)|/\sqrt{2}$.

$$\text{AM: } x_c(t) = A_c [1 + \mu x(t)] \cos(\omega_c t)$$

$$x_c^2(t) = A_c^2 [1 + \mu x(t)]^2 \cos^2(\omega_c t) = (A_c^2 / 2) [1 + \mu x(t)]^2 [1 + \cos(2\omega_c t)]$$

$$(A_c^2 / 2) [1 + \mu x(t)]^2 [1 + \cos(2\omega_c t)] \xrightarrow{\text{LPF}} (A_c^2 / 2) [1 + \mu x(t)]^2$$

$$(A_c^2 / 2) [1 + \mu x(t)]^2 \xrightarrow{\text{Square Rooter}} (A_c / \sqrt{2}) [1 + \mu x(t)]$$

$$\text{DSB: } x_c(t) = x(t) \cos(\omega_c t)$$

$$x_c^2(t) = x^2(t) \cos^2(\omega_c t) = x^2(t) (1/2) [1 + \cos(2\omega_c t)]$$

$$x^2(t) (1/2) [1 + \cos(2\omega_c t)] \xrightarrow{\text{LPF}} x^2(t) / 2$$

$$x^2(t) / 2 \xrightarrow{\text{Square Rooter}} \sqrt{x^2(t) / 2} = |x(t)| / \sqrt{2}$$

Twenty-five radio stations are broadcasting in the band between 3 MHz and 3.5 MHz. You wish to modify an AM broadcast receiver to receive the broadcasts. Each audio signal has a maximum frequency $f_m = 10$ kHz. Describe in detail the changes you would have to make to the standard broadcast superheterodyne receiver in order to receive the broadcast.

For standard AM, each channel has a transmitted bandwidth of $B_T = 10$ kHz. After demodulation that becomes a baseband bandwidth of $W = 5$ kHz. We need here a bandwidth of 10 kHz for the demodulated baseband signal. So we must increase the IF bandwidth by a factor of two. The RF range of a standard AM receiver is 540 to 1700 kHz with an IF of 455 kHz. The local oscillator range is then 995 to 2155 kHz. We need to change that to a range of 3.455 to 3.955 MHz. We must also modify the RF amplifier to pass the signals in the 3 to 3.5 MHz range and reject the image frequencies in the 3.91 to 4.41 MHz range.

A superheterodyne receiver is tuned to a station at 20 MHz. The local oscillator frequency is 80 MHz and the IF is 100 MHz. (a) What is the image frequency? (b) If the LO has appreciable second-harmonic content, what two additional frequencies are received? (c) If the RF amplifier contains a single-tuned parallel resonant circuit with $Q = 50$ tuned to 20 MHz, what will be the image attenuation in dB?

(a) The image frequency is $f_{LO} + f_{IF} = 80 + 100 = 180$ MHz.

(b) The second harmonic of the local oscillator is at $2f_{LO} = 160$ MHz.

$2f_{LO} - f_{IF} = f_c = 60$ MHz and $2f_{LO} + f_{IF} = f_c = 260$ MHz

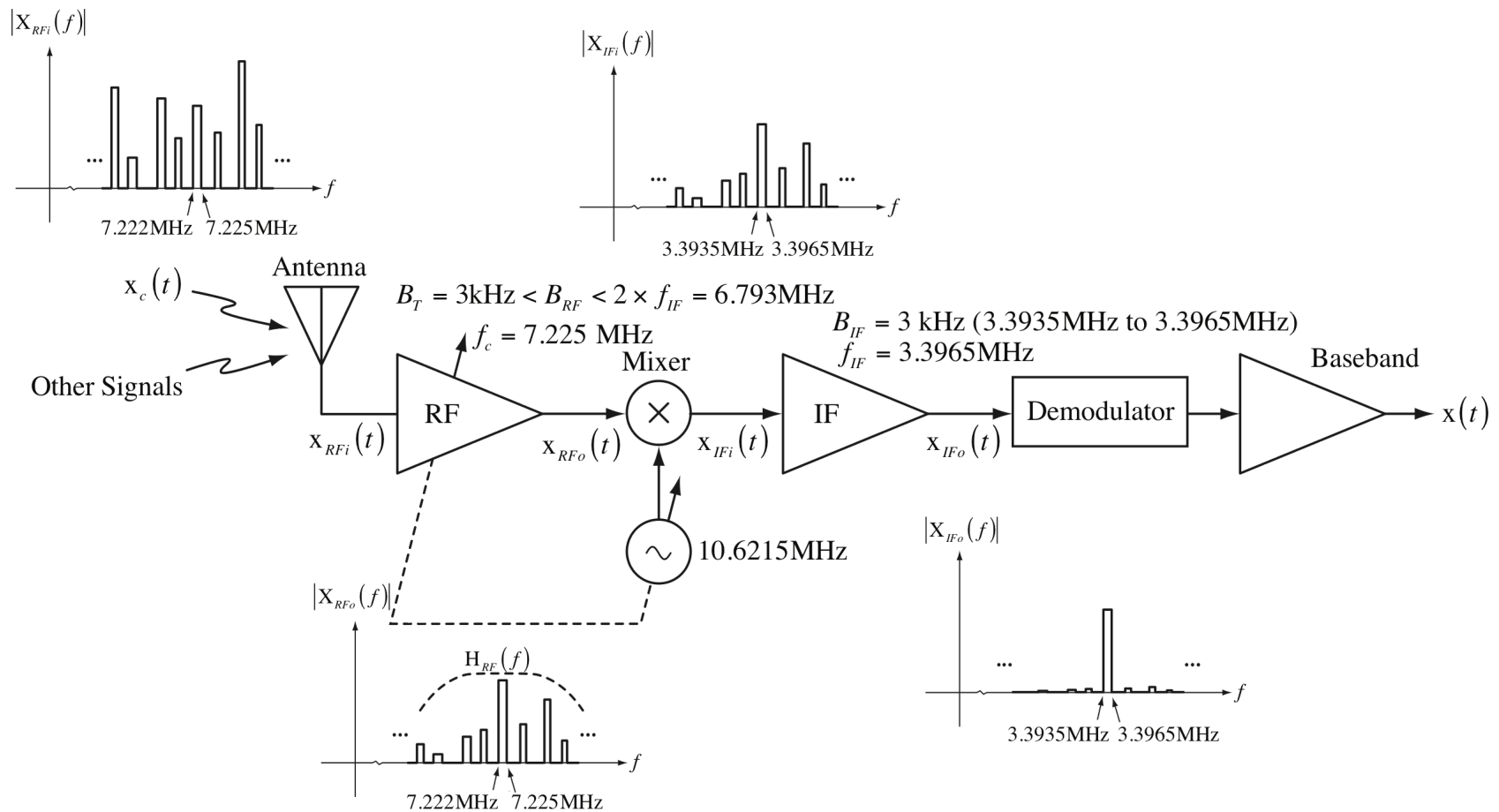
$$(c) \quad H(f) = \frac{1}{1 + jQ \left(\frac{f}{f_0} - \frac{f_0}{f} \right)}$$

$$H(180\text{MHz}) = \frac{1}{1 + j50 \left(\frac{180}{20} - \frac{20}{180} \right)} = 0.00225 e^{-j1.5685}$$

$$|H(180\text{MHz})|_{\text{dB}} = -52.96 \text{ dB}$$

A receiver is tuned to receive a 7.225 MHz LSSB signal. The LSSB signal is modulated by an audio signal that has a 3 kHz bandwidth. Assume that the receiver uses a superheterodyne circuit with an SSB IF filter. The IF filter is centered on 3.395 MHz. The LO frequency is on the high side of the input LSSB signal. (a) Draw a block diagram of the single-conversion superheterodyne receiver, indicating frequencies present and typical spectra of the signals at various points within the receiver. (b) Determine the required RF and IF filter specifications, assuming that the image frequency is to be attenuated by 40 dB.

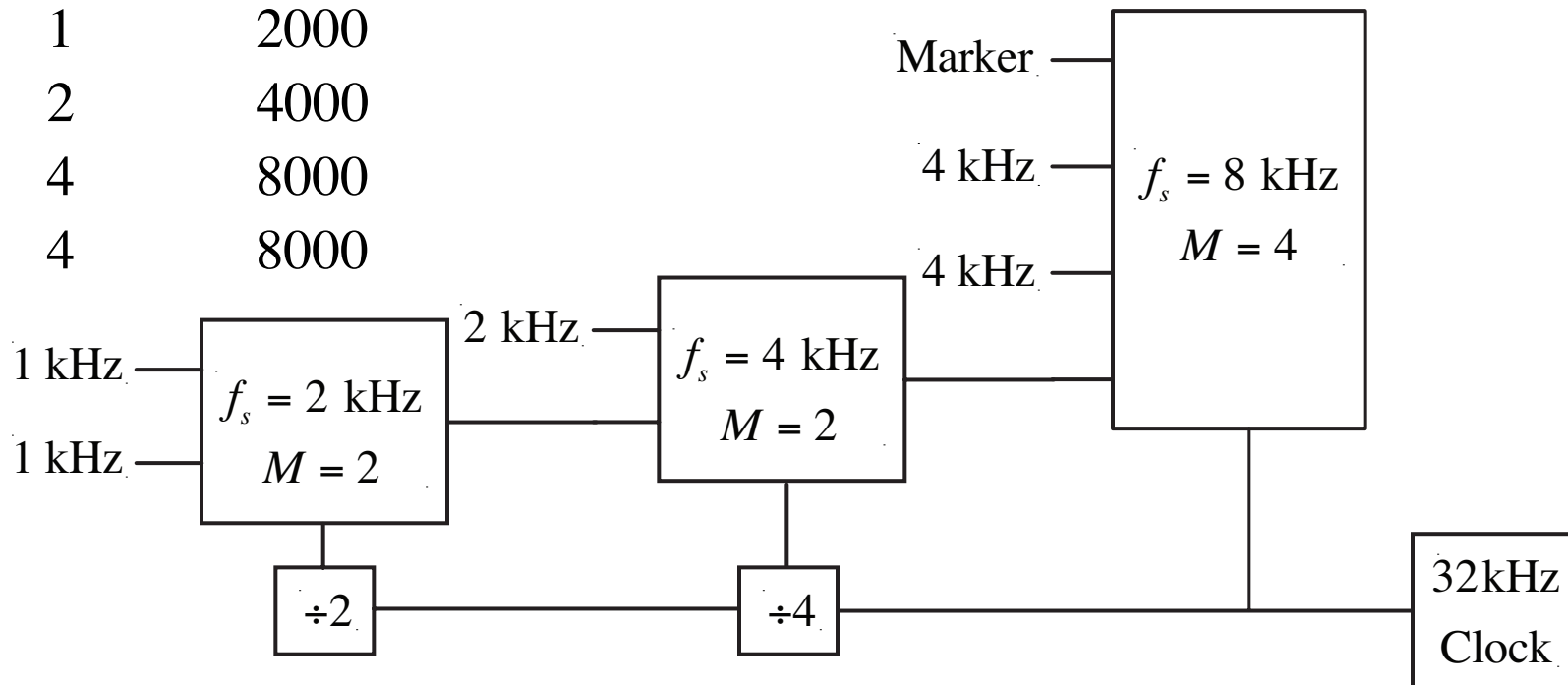
$f_c = 7.225$ MHz. The IF filter is centered at 3.395 MHz and, since the signal is LSSB, the upper edge of its passband at $f_{IF} = 3.3965$ MHz is $f_{LO} - f_c$. Therefore, $f_{LO} = 7.225 \text{ MHz} + 3.3965 \text{ MHz} = 10.6215 \text{ MHz}$. The image frequency is $f_{LO} + f_{IF} = 14.018 \text{ MHz}$.



The RF amplifier/filter should be more than 40 dB down at 14.018MHz and the IF filter should have a passband from 3.3935MHz to 3.3965MHz and very steep skirts outside that range.

Five messages with bandwidths 1 kHz, 1 kHz, 2 kHz, 4 kHz and 4 kHz respectively are to be time-division multiplexed. You have a 4 input multiplexer with a maximum sampling rate of 8 kHz and a 32 kHz clock. Design a system, in block diagram form, that will multiplex these signals plus an 8 kHz marker.

BW (kHz)	Min. f_s
1	2000
1	2000
2	4000
4	8000
4	8000



In an FDM communication system, the transmitted baseband signal is $x(t) = m_1(t)\cos(\omega_1 t) + m_2(t)\cos(\omega_2 t)$. The system under study has a second-order nonlinearity between transmitter input and receiver output.

Thus, the received baseband signal can be expressed as

$y(t) = a_1 x(t) + a_2 x^2(t)$. Assuming that the two message signals $m_1(t)$ and $m_2(t)$ have spectra $M_1(f) = M_2(f) = \Pi(f/W)$ sketch the spectrum of $y(t)$. Discuss the difficulties encountered in demodulating the received baseband signal. In many FDM systems the subcarrier frequencies ω_1 and ω_2 are harmonically related. Describe any additional problems this presents.

$$y(t) = a_1 [m_1(t)\cos(\omega_1 t) + m_2(t)\cos(\omega_2 t)] + a_2 [m_1(t)\cos(\omega_1 t) + m_2(t)\cos(\omega_2 t)]^2$$

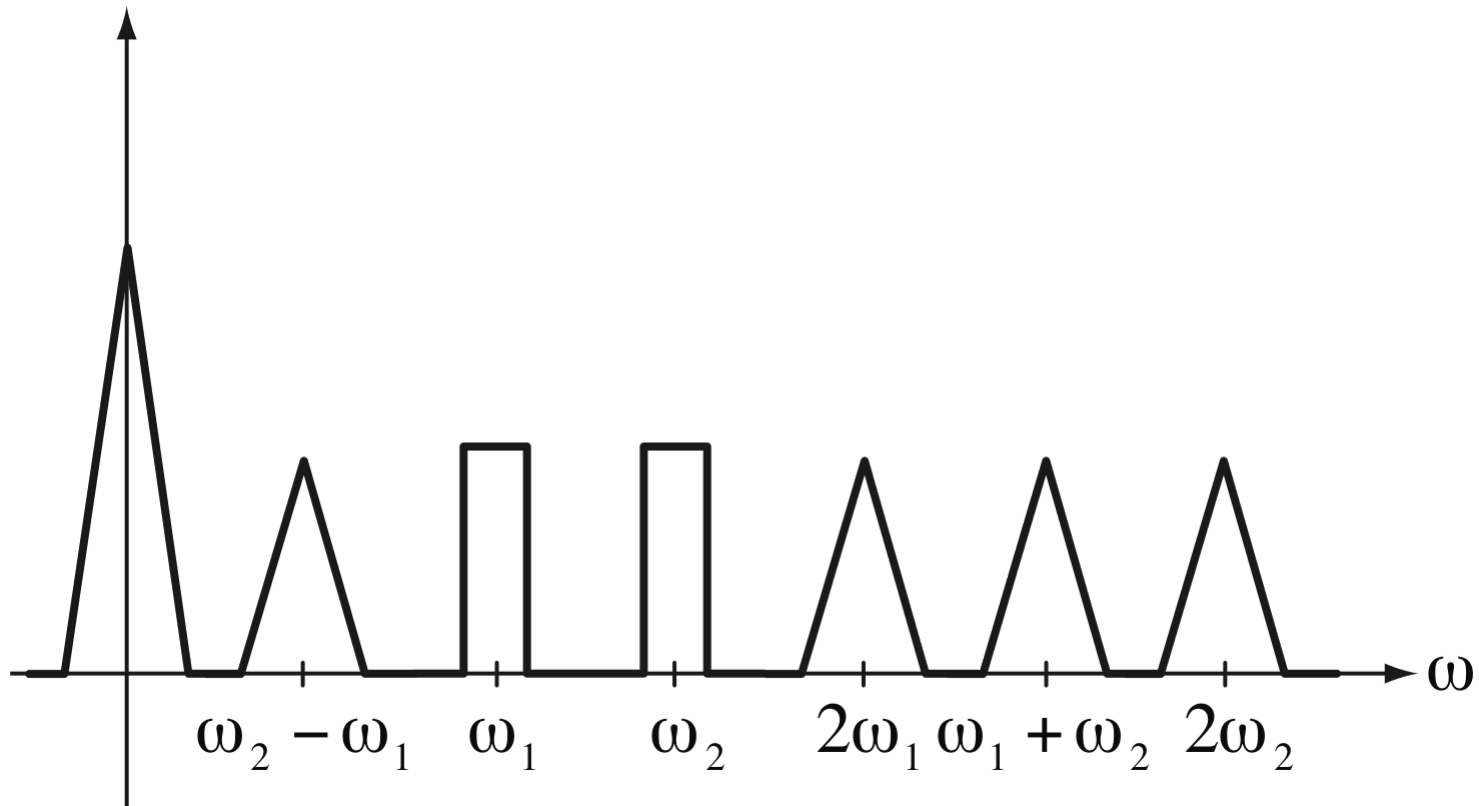
$$y(t) = a_1 m_1(t)\cos(\omega_1 t) + a_1 m_2(t)\cos(\omega_2 t)$$

$$+ a_2 [m_1^2(t)\cos^2(\omega_1 t) + m_2^2(t)\cos^2(\omega_2 t) + 2m_1(t)m_2(t)\cos(\omega_1 t)\cos(\omega_2 t)]$$

$$y(t) = a_1 m_1(t)\cos(\omega_1 t) + a_1 m_2(t)\cos(\omega_2 t)$$

$$+ (a_2 / 2) \left[\begin{array}{l} m_1^2(t) + m_1^2(t)\cos(2\omega_1 t) + m_2^2(t) + m_2^2(t)\cos(2\omega_2 t) \\ + 2m_1(t)m_2(t)\cos((\omega_1 - \omega_2)t) + 2m_1(t)m_2(t)\cos((\omega_1 + \omega_2)t) \end{array} \right]$$

$$y(t) = a_1 m_1(t) \cos(\omega_1 t) + a_1 m_2(t) \cos(\omega_2 t) \\ + (a_2 / 2) \left[m_1^2(t) + m_1^2(t) \cos(2\omega_1 t) + m_2^2(t) + m_2^2(t) \cos(2\omega_2 t) \right. \\ \left. + 2m_1(t)m_2(t) \cos((\omega_1 - \omega_2)t) + 2m_1(t)m_2(t) \cos((\omega_1 + \omega_2)t) \right]$$



A superheterodyne receiver is designed to cover the RF frequency range of 45 to 860 MHz, with channel spacings of 8 MHz and an IF of 40 MHz.

Assume high-side injection. (a) If the receiver down-converts the RF signals to an IF of 40 MHz, calculate the range of frequencies for the LO. (b) Calculate the range of image frequencies. (Note that the band of image frequencies and the signal band overlap. This is undesirable. So we up-convert to a higher IF of 1.2 GHz.) (c) Calculate the new range of frequencies for the LO. (d) Determine the range of image frequencies.

(a) $f_{LO} = f_c + f_{IF} \Rightarrow$ Range of $f_{LO} = 45 + 40$ to $860 + 40$ or 85 to 900 MHz

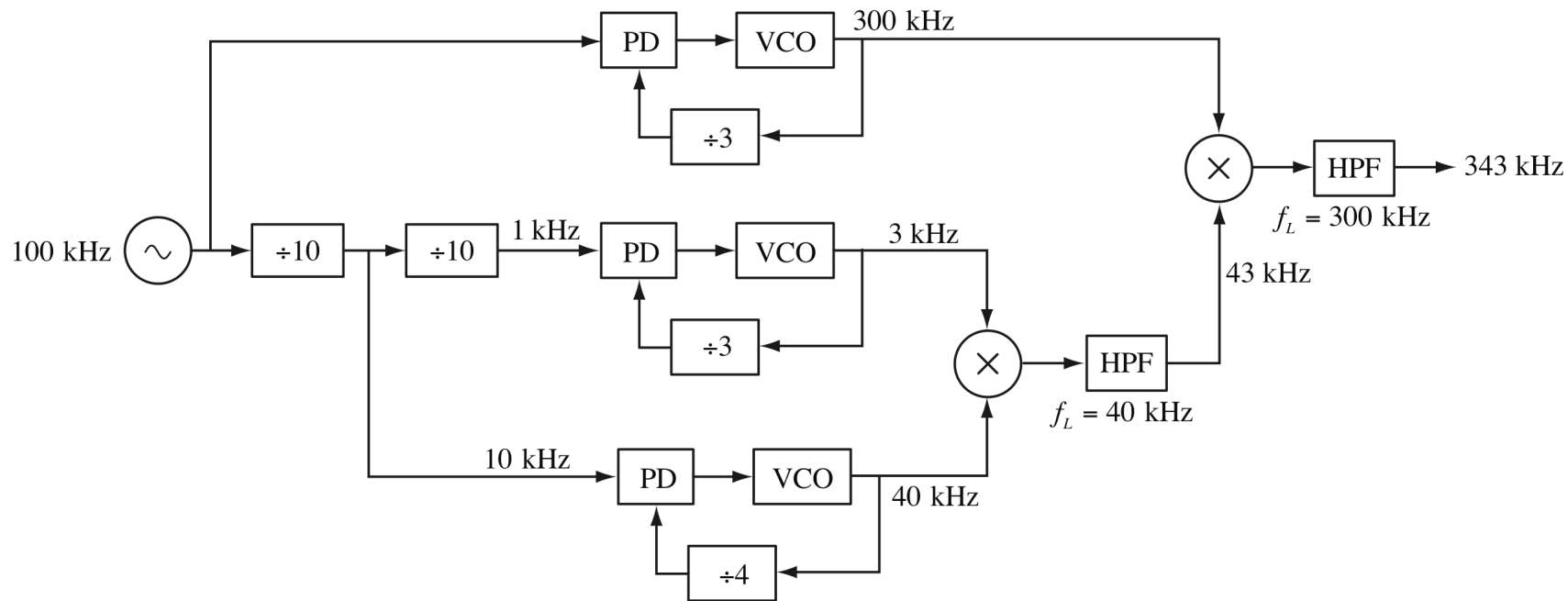
(b) The image frequencies are the carrier frequencies plus twice the IF frequency or 125 to 940 MHz.

(c) New range of f_{LO} is 1.245 GHz to 2.06 GHz.

(d) New range of image frequencies is 2.445 GHz to 3.26 GHz

You have a crystal-controlled 100 kHz oscillator and as many divide-by- n counters ($n \leq 10$), voltage-controlled oscillators, phase detectors and highpass filters as needed. Design a frequency synthesizer that will generate a 343 kHz signal.

We will need a frequency resolution of 1 kHz so we will need to divide the 100 kHz down to 1 kHz initially. We can do that with two stages of $\div 10$ each. We can multiply the 100 kHz by 3 to get 300 kHz and we can multiply the 10 kHz signal already obtained by 4 to get 40 kHz. Then we can multiply the 1 kHz signal by 3 to get the needed 3 kHz. We can combine these in two steps to get the 343 kHz signal. It is best to combine the 40 and 3 first to get 37 and 43 and then filter out the 37 with a highpass filter. Then combine the 43 with the 300 to get 343 and 257 and filter out the 257 with a highpass filter.



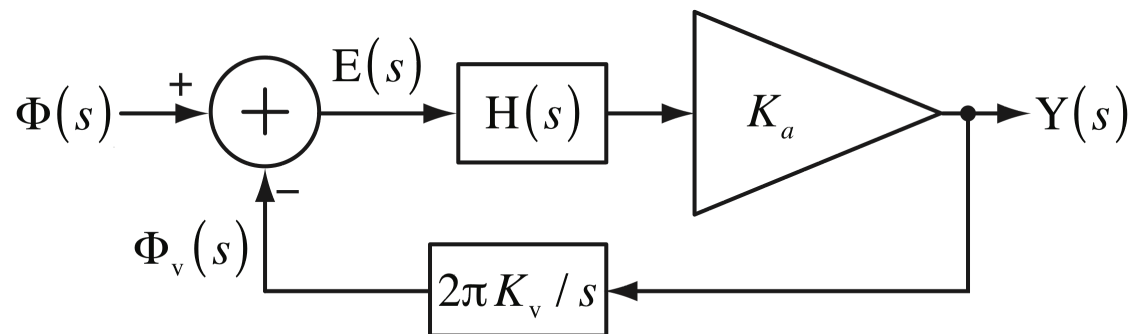
An analog multiplier, a filter with transfer function $H(s) = \frac{0.1s + 100}{s}$, an amplifier with gain $K_a = 10$ and a VCO with $K_v = 2$ MHz/volt are used to make a phase-locked loop. Using the linearized model of a phase-locked loop find the closed-loop system poles. Is this phase-locked loop stable?

$$\frac{Y(s)}{\Phi(s)} = \frac{sK_a H(s)}{s + 2\pi K_a K_v H(s)} = \frac{10s \frac{0.1s + 100}{s}}{s + 2\pi \times 10 \times 2 \times 10^6 \frac{0.1s + 100}{s}}$$

$$\frac{Y(s)}{\Phi(s)} = \frac{s(s + 1000)}{s^2 + 2\pi \times 10 \times 2 \times 10^6 (0.1s + 100)} = \frac{s(s + 1000)}{s^2 + 4\pi \times 10^6 s + 4\pi \times 10^9}$$

Poles at -1.257×10^7 and -1000 .

Both poles are in the open left half-plane, therefore the system is stable.



A stable phase-locked loop is locked. The input signal's phase suddenly jumps up by a small amount. Describe the sequence of events in the system that makes it settle to a new locked state with the new input signal phase.

The first thing that happens is that the phase of the input signal lags the VCO output signal less than previously. This causes the lowpass filter output to move to a more positive voltage. That more positive voltage drives the VCO to raise its output signal frequency. That tends to make the phase difference return to quadrature (or near quadrature). That lowers the output of the lowpass filter reducing the VCO output signal's frequency. Eventually the lowpass filter's output signal returns to its original value with the VCO output again leading the input signal by approximately 90° .

A stable phase-locked loop is locked. The input signal's frequency suddenly drops by a small amount. Describe the sequence of events in the system that makes it settle to a new locked state with the new input signal frequency.

The first thing that happens is that the phase of the input signal starts lagging the VCO output signal more than previously. This causes the lowpass filter output to move to a more negative voltage. That more negative voltage drives the VCO to lower its output signal frequency. That tends to make the phase difference return to quadrature (or near quadrature) at a lower frequency of the VCO output.

Eventually the loop is locked again with the VCO output frequency matching the input signal's frequency and with the lowpass filter's output signal at a lower voltage.

A stable phase locked-loop is operating in a locked condition and the output of the lowpass filter is a small positive voltage. The amplitude of the input signal suddenly doubles. The phase-locked loop is momentarily perturbed but soon quickly settles into a new locked state. What happens to the phase relationship between the input signal and the VCO output signal and what happens to the lowpass filter output signal?

The fact that the lowpass filter output is initially not zero means that the input signal and the VCO output signal are not exactly 90° apart. The VCO output leads the input signal by less than 90° . When the input signal amplitude doubles the output of the lowpass filter would quickly double if there were no feedback action. But the increase in the lowpass filter output momentarily increases the frequency of the VCO causing the phase of the VCO output to increase relative to the phase of the input signal, moving the phase difference closer to 90° . So the increase in input signal amplitude makes the two signals move closer to exact quadrature. The lowpass filter output returns to its original value.