Chapter 2 - Mathematical Description of Signals

Selected Solutions

1. If $g(t) = 7e^{-2t-3}$ write out and simplify

   (a) $g(3) = 7e^{-2(3)-3} = 7e^{-9} \cong 8.639 \times 10^{-4}$

   (b) $g(2-t)$

   (c) $g\left(\frac{t}{10}+4\right) = 7e^{-\frac{t}{5}-11}$

   (d) $g(jt)$ (e) $\frac{g(jt)+g(-jt)}{2}$

   (f) $\frac{g\left(\frac{jt-3}{2}\right)+g\left(\frac{-jt-3}{2}\right)}{2} = 7\frac{e^{-jt}+e^{jt}}{2} = 7\cos(t)$

2. If $g(x) = x^2 - 4x + 4$ write out and simplify

   (a) $g(z)$

   (b) $g(u+v) = (u+v)^2 - 4(u+v) + 4 = u^2 + v^2 + 2uv - 4u - 4v + 4$

   (c) $g(e^{jt})$

   (d) $g(g(t)) = g(t^2 - 4t + 4) = (t^2 - 4t + 4)^2 - 4(t^2 - 4t + 4) + 4$

   $g(g(t)) = t^4 - 8t^3 + 20t^2 - 16t + 4$

   (e) $g(2)$

3. What would be the numerical value of "`g`" in each of the following MATLAB instructions?

   (a) `t = 3 ; g = sin(t) ;`

   (b) `x = 1:5 ; g = cos(pi*x) ;` `[-1,1,-1,1,-1]`

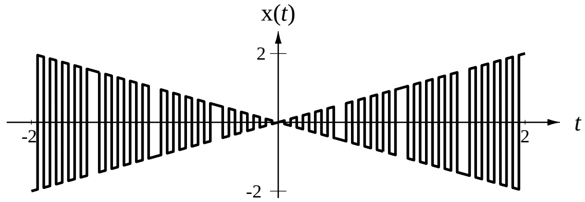
   (c) `f = -1:0.5:1 ; w = 2*pi*f ; g = 1./(1+j*w') ;`

4. Let two functions be defined by

$$\mathrm{x}_1(t) = \begin{cases} 1 & , \ \sin(20\pi t) \geq 0 \\ -1 & , \ \sin(20\pi t) < 0 \end{cases} \quad \text{and} \quad \mathrm{x}_2(t) = \begin{cases} t & , \ \sin(2\pi t) \geq 0 \\ -t & , \ \sin(2\pi t) < 0 \end{cases} .$$

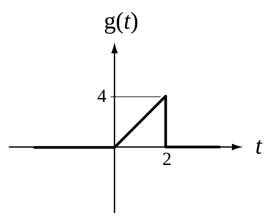
Graph the product of these two functions versus time over the time range, $-2 < t < 2$.

To graph these functions set up an array of times, *t*, compute the arrays of sin function values, check each sin function value and apply the constraints of being +/- 1 or being +/- *t*, to form the function arrays, x_1 and x_2, array- multiply the two function arrays and plot the product versus *t*.

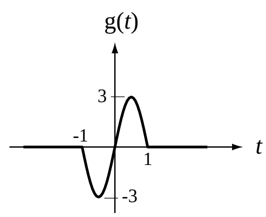


5. For each function, $\mathrm{g}(t)$, sketch $\mathrm{g}(-t)$, $-\mathrm{g}(t)$, $\mathrm{g}(t-1)$, and $\mathrm{g}(2t)$.

(a)



(b)



6. A function, $\mathrm{G}(f)$, is defined by

$$\mathrm{G}(f) = e^{-j2\pi f}\,\mathrm{rect}\left(\frac{f}{2}\right) .$$

Graph the magnitude and phase of $\mathrm{G}(f-10) + \mathrm{G}(f+10)$ over the range, $-20 < f < 20$.

First imagine what $\mathrm{G}(f)$ looks like. It consists of a rectangle centered at $f = 0$ of width, 2, multiplied by a complex exponential. Therefore for frequencies greater than one in magnitude it is zero. Its magnitude is simply the magnitude of the rectangle function because the magnitude of the complex exponential is one for any *f*.

$$\left|e^{-j2\pi f}\right| = \left|\cos(-2\pi f) + j\sin(-2\pi f)\right| = \left|\cos(2\pi f) - j\sin(2\pi f)\right|$$

$$\left|e^{-j2\pi f}\right| = \sqrt{\cos^2(2\pi f) + \sin^2(2\pi f)} = 1$$

The phase (angle) of $\mathrm{G}(f)$ is simply the phase of the complex exponential between $f = -1$ and $f = 1$ and undefined outside that range because the phase of the rectangle function is zero between $f = -1$ and $f = 1$ and undefined outside that range and the phase of a product is the sum of the phases. The phase of the complex exponential is

$$\angle e^{-j2\pi f} = \angle\left(\cos(2\pi f) - j\sin(2\pi f)\right) = \tan^{-1}\left(-\frac{\sin(2\pi f)}{\cos(2\pi f)}\right) = -\tan^{-1}\left(\frac{\sin(2\pi f)}{\cos(2\pi f)}\right)$$

$$\angle e^{-j2\pi f} = -\tan^{-1}\left(\tan(2\pi f)\right)$$

The inverse tangent function is multiple-valued. Therefore there are multiple correct answers for this phase. The simplest of them is found by choosing

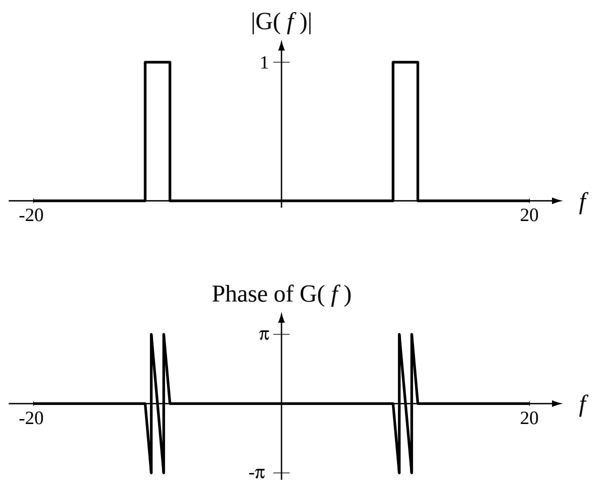
$$\angle e^{-j2\pi f} = -2\pi f$$

which is simply the coefficient of *j* in the original complex exponential expression.

A more general solution would be $\angle e^{-j2\pi f} = -2\pi f + 2n\pi$, n an integer .

The solution of the original problem is simply this solution except shifted up and down by 10 in *f* and added.

$$G(f-10) + G(f+10) = e^{-j2\pi(f-10)}\,\text{rect}\left(\frac{f-10}{2}\right) + e^{-j2\pi(f+10)}\,\text{rect}\left(\frac{f+10}{2}\right)$$



7. Sketch the derivatives of these functions.

(All sketches at end.)

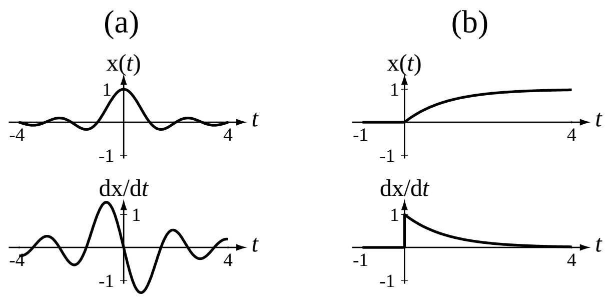
(a) Use the definition of the sinc function and the rule for differentiating a fraction.

(b) $g(t) = (1 - e^{-t})u(t)$ This function is constant zero for all time before time, $t = 0$, therefore its derivative during that time is zero. This function is a constant minus a decaying exponential after time, $t = 0$, and its derivative in that time is therefore also a positive decaying exponential.

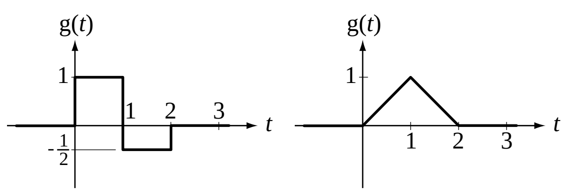
$$g'(t) = \begin{cases} e^{-t} & , \ t > 0 \\ 0 & , \ t < 0 \end{cases}$$

Strictly speaking, its derivative is not defined at exactly $t = 0$. Since the value of a physical signal at a single point has no impact on any physical system (as long as it is finite) we can choose any finite value at time, $t = 0$, without changing the effect of this signal on any physical system. If we choose 1/2, then we can write the derivative as

$$g'(t) = e^{-t}\,u(t) \quad .$$



8. Sketch the integral from negative infinity to time, *t*, of these functions which are zero for all time before time, $t = 0$.



This is the integral, $\int_{-\infty}^{t} g(\lambda)\,d\lambda$, which, in geometrical terms, is the accumulated area under the function, $g(t)$, from time, $-\infty$ to time, *t*. For the case of the two back-to-back rectangular pulses, there is no accumulated area until after time, $t = 0$, and then in the time interval, $0 < t < 1$, the area accumulates linearly with time up to a maximum area of one at time, $t = 1$. In the second time interval, $1 < t < 2$, the area is linearly declining at half the rate at which it increased in the first time interval, $0 < t < 1$, down to a value of 1/2 where it stays because there is no accumulation of area for $t > 2$.

In the second case of the triangular-shaped function, the area does not accumulate linearly, but rather non-linearly because the integral of a linear function is a second-degree polynomial. The rate of accumulation of area is increasing up to time, $t = 1$, and then decreasing (but still positive) until time, $t = 2$, at which time it stops completely. The final value of the accumulated area must be the total area of the triangle, which, in this case, is one.

9. Find the even and odd parts of these functions.

(a) $\mathrm{g}(t) = 2t^2 - 3t + 6$

$$\mathrm{g}_e(t) = \frac{2t^2 - 3t + 6 + 2(-t)^2 - 3(-t) + 6}{2} = \frac{4t^2 + 12}{2} = 2t^2 + 6$$

$$\mathrm{g}_o(t) = \frac{2t^2 - 3t + 6 - 2(-t)^2 + 3(-t) - 6}{2} = \frac{-6t}{2} = -3t$$

(b) $\mathrm{g}(t) = 20\cos\left(40\pi t - \frac{\pi}{4}\right)$

$$\mathrm{g}_e(t) = \frac{20\cos\left(40\pi t - \frac{\pi}{4}\right) + 20\cos\left(-40\pi t - \frac{\pi}{4}\right)}{2}$$

Use $\cos(z_1 + z_2) = \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)$ to separate even and odd parts.

$$\mathrm{g}_e(t) = \frac{\left\{\begin{array}{l} 20\left[\cos(40\pi t)\cos\left(-\frac{\pi}{4}\right) - \sin(40\pi t)\sin\left(-\frac{\pi}{4}\right)\right] \\ +20\left[\cos(-40\pi t)\cos\left(-\frac{\pi}{4}\right) - \sin(-40\pi t)\sin\left(-\frac{\pi}{4}\right)\right]\end{array}\right\}}{2}$$

which simplifies to

$$\mathrm{g}_e(t) = \frac{20}{\sqrt{2}}\cos(40\pi t)$$

Use the same trigonometric identity.

(c) $\mathrm{g}(t) = \frac{2t^2 - 3t + 6}{1 + t}$

(d) $\mathrm{g}(t) = \mathrm{sinc}(t)$ Use the definition of the sinc function.

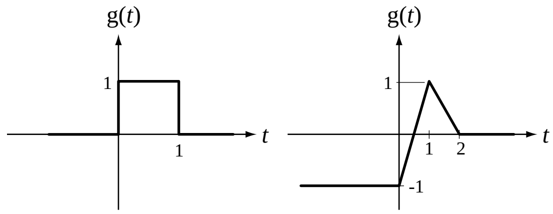
(e) $\mathrm{g}(t) = t(2 - t^2)(1 + 4t^2)$

$$\mathrm{g}(t) = \underbrace{\underbrace{\underset{\text{odd}}{t}\left(\underset{\text{even}}{2} - \underset{\text{even}}{t^2}\right)}_{\text{even}}\underbrace{\left(\underset{\text{even}}{1} + \underset{\text{even}}{4t^2}\right)}_{\text{even}}}_{\text{odd}}$$

Therefore $g(t)$ is odd, $g_e(t) = 0$ and $g_o(t) = t(2 - t^2)(1 + 4t^2)$

(f) $$g(t) = t(2-t)(1+4t) = t(-4t^2 + 7t + 2) = \underbrace{-4t^3}_{\text{odd}} + \underbrace{7t^2}_{\text{even}} + \underbrace{2t}_{\text{odd}}$$

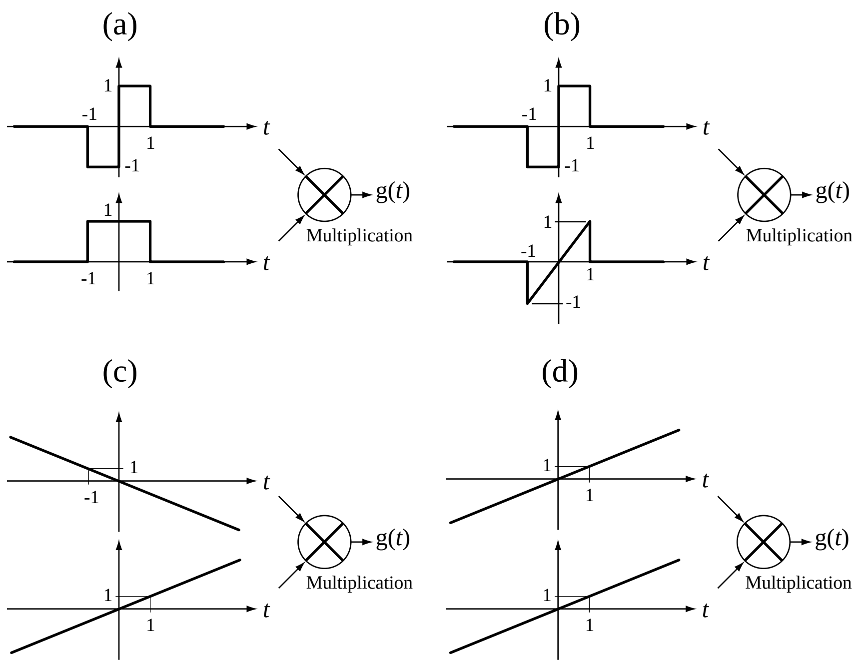
10. Sketch the even and odd parts of these functions.

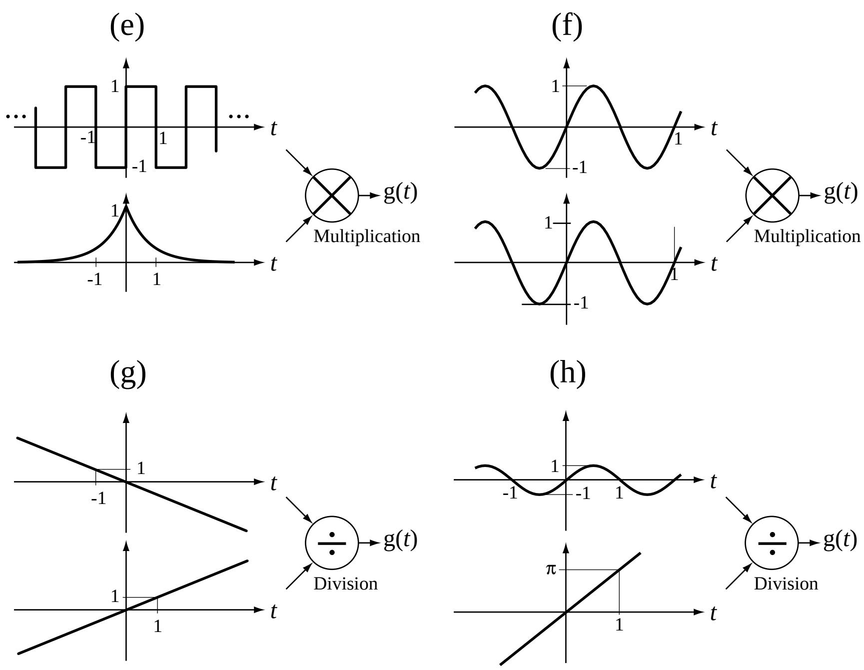


(a) (b)

To sketch the even part of graphically-defined functions like these, first sketch $g(-t)$. Then add it (graphically, point by point) to $g(t)$ and (graphically) divide the sum by two. Then, to sketch the odd part, subtract $g(-t)$ from $g(t)$ (graphically) and divide the difference by two.

11. Sketch the indicated product or quotient, $g(t)$, of these functions.





The product of two functions is simply a line drawn through many points, each of which is found by multiplying the values of the two individual functions at that same point in time. A quotient is found the same way except that the two point values are divided instead of multiplied.

12. Use the properties of integrals of even and odd functions to evaluate these integrals in the quickest way.

(a) $$\int_{-1}^{1}(2+t)\,dt = \int_{-1}^{1}\underbrace{2}_{\text{even}}\,dt + \underbrace{\int_{-1}^{1}\underbrace{t}_{\text{odd}}\,dt}_{=0} = 2\int_{0}^{1}2\,dt = 4$$

(b) $$\int_{-\frac{1}{20}}^{\frac{1}{20}}\left[4\cos(10\pi t)+8\sin(5\pi t)\right]dt$$

(c) $$\int_{-\frac{1}{20}}^{\frac{1}{20}}4t\cos(10\pi t)\,dt$$

(d) $$\int_{-\frac{1}{10}}^{\frac{1}{10}}\underbrace{\underbrace{t}_{\text{odd}}\;\underbrace{\sin(10\pi t)}_{\text{odd}}}_{\text{even}}\,dt = 2\int_{0}^{\frac{1}{10}}t\sin(10\pi t)\,dt$$

Integrate by parts or look up this form.

$$\int_{-\frac{1}{10}}^{\frac{1}{10}}t\sin(10\pi t)\,dt = \frac{1}{50\pi}$$

(e) $\int_{-1}^{1} e^{-|t|}dt$ (f) $\int_{-1}^{1} te^{-|t|}dt$

13. Find the fundamental period and fundamental frequency of each of these functions.

(a) $\mathrm{g}(t) = 10\cos(50\pi t)$ $f_0 = 25 \text{ Hz} \quad , \quad T_0 = \dfrac{1}{25} \text{ s}$

For a simple sinusoid of the form, $A\cos(2\pi f_0 t + \theta)$ or $A\sin(2\pi f_0 t + \theta)$ the frequency is simply f_0.

(b) $\mathrm{g}(t) = 10\cos\left(50\pi t + \dfrac{\pi}{4}\right)$

(c) $\mathrm{g}(t) = \cos(50\pi t) + \sin(15\pi t)$

The fundamental period of the sum of two periodic signals is the least common multiple (LCM) of their two individual fundamental periods. The fundamental frequency of the sum of two periodic signals is the greatest common divisor (GCD) of their two individual fundamental frequencies.

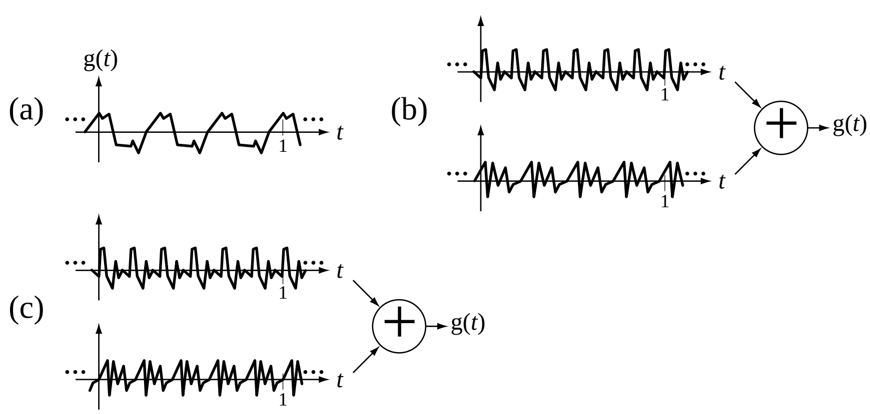
$$f_0 = \mathrm{GCD}\left(25, \frac{15}{2}\right) = 2.5 \text{ Hz} \quad , \quad T_0 = \frac{1}{2.5} = 0.4 \text{ s}$$

OR

$$T_0 = \mathrm{LCM}\left(\frac{1}{25}, \frac{2}{15}\right) = \mathrm{LCM}\left(\frac{6}{150}, \frac{20}{150}\right) = \frac{60}{150} = 0.4 \text{ s} \quad , \quad f_0 = \frac{1}{0.4} = 2.5 \text{ Hz} \quad ,$$

(d) $\mathrm{g}(t) = \cos(2\pi t) + \sin(3\pi t) + \cos\left(5\pi t - \dfrac{3\pi}{4}\right)$

14. Find the fundamental period and fundamental frequency of $\mathrm{g}(t)$.



(a) Counting periods we see that there are three periods in one second. Therefore $f_0 = 3\text{ Hz}$ and $T_0 = \frac{1}{3}\text{ s}$

(b) Count periods to find the fundamental frequency of each signal, then find the GCD of those two frequencies.

15. Plot these DT functions.

Use a calculator or MATLAB to plot these functions.

(a) $\mathrm{x}[n] = 4\cos\left(\frac{2\pi n}{12}\right) - 3\sin\left(\frac{2\pi(n-2)}{8}\right) \quad , \quad -24 \le n < 24$

```
n = -24:23 ;
x = 4*cos(2*pi*n/12) - 3*sin(2*pi*(n-2)/8) ;
stem(n,x,'k','filled') ;
```

(b) $\mathrm{x}[n] = 3ne^{-\left|\frac{n}{5}\right|} \quad , \quad -20 \le n < 20$

```
n = -20:19 ;
x = 3*n.*exp(-abs(n/5)) ;
stem(n,x,'k','filled') ;
```

(c) $\mathrm{x}[n] = 21\left(\frac{n}{2}\right)^2 + 14n^3 \quad , -5 \le n < 5$

16. Let $\mathrm{x}_1[n] = 5\cos\left(\frac{2\pi n}{8}\right)$ and $\mathrm{x}_2[n] = -8e^{-\left(\frac{n}{6}\right)^2}$. Plot the following combinations of those two signals over the DT range, $-20 \le n < 20$. If a signal has some defined and some undefined values, just plot the defined values.

(a) $\mathrm{x}[n] = \mathrm{x}_1[n]\mathrm{x}_2[n]$

Make MATLAB functions for x_1 and x_2 and save them in files named `x1DT.m` and `x2DT.m`. (DT for discrete time.)

```
function y = x1DT(n),
     y = 5*cos(2*pi*n/8) ;
     I = find(round(n) ~= n) ; y(I) = NaN ;

function y = x2DT(n)
     y = -8*exp(-(n/6).^2) ;
     I = find(round(n) ~= n) ; y(I) = NaN ;
```

Then use the functions in this way in part (a).

```
n = -20:19 ; x = x1DT(n).*x2DT(n) ;
```

(b) $\mathrm{x}[n] = 4\,\mathrm{x}_1[n] + 2\,\mathrm{x}_2[n]$

```
n = -20:19 ; x = 4*x1DT(n) + 2*x2DT(n) ;
```

(c) $\mathrm{x}[n] = \mathrm{x}_1[2n]\,\mathrm{x}_2[3n]$

```
n = -20:19 ; x = x1DT(2*n).*x2DT(3*n) ;
```

(d) $\mathrm{x}[n] = \dfrac{\mathrm{x}_1[2n]}{\mathrm{x}_2[-n]}$

(e) $\mathrm{x}[n] = 2\,\mathrm{x}_1\left[\dfrac{n}{2}\right] + 4\,\mathrm{x}_2\left[\dfrac{n}{3}\right]$

```
n = -20:19 ; x = 2*x1DT(n/2) + 4*x2DT(n/3) ;
```

17. A function, $\mathrm{g}[n]$ is defined by

$$\mathrm{g}[n] = \begin{cases} -2 & , \; n < -4 \\ n & , \; -4 \le n < 1 \\ \dfrac{4}{n} & , \; 1 \le n \end{cases} .$$

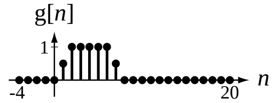
Sketch $\mathrm{g}[-n]$, $\mathrm{g}[2-n]$, $\mathrm{g}[2n]$ and $\mathrm{g}\left[\dfrac{n}{2}\right]$.

```
function y = gDT(n)
     I = find(round(n) ~= n) ;      %  Find all non-
                                    %  integer "n's"
     n(I) = NaN ;                   %  Set them all to
                                    %  "NaN"
     y1 = -2 ;
     y2 = n ;
     num3 = 4*ones(length(n),1) ; den3 = n ;
     I = find(den3 == 0) ; num3(I) = 1 ; den3(I) = 1 ;
     y3 = num3./den3 ;
     y = y1.*(n<-4) + y2.*(n>=-4 & n<1) + y3.*(n>=1) ;
```

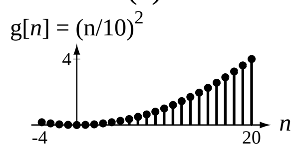
18. Sketch the backward differences of these DT functions.

(a)

(b)



(c)



(a) and (b) can be done easily by hand

(c) is also easy to sketch once you realize what is happening

$$\mathrm{g}[n]=\left(\frac{n}{10}\right)^2 \Rightarrow \mathrm{g}[n]-\mathrm{g}[n-1]=\left(\frac{n}{10}\right)^2-\left(\frac{n-1}{10}\right)^2=\frac{n^2}{100}-\frac{n^2}{100}+\frac{n}{50}-\frac{1}{100}=\frac{2n-1}{100}$$

19. Sketch the accumulation, $\mathrm{g}[n]$, from negative infinity to n of each of these DT functions.

(a) $\mathrm{h}[n]=\delta[n]$

$$\mathrm{g}[n]=\sum_{m=-\infty}^{n}\delta[m]=\begin{Bmatrix}1 \;,\; n\geq 0\\ 0 \;,\; n<0\end{Bmatrix}=\mathrm{u}[n]$$

(b) $\mathrm{h}[n]=\mathrm{u}[n]$

The answer is <u>not</u> ramp[n].

(c) $\mathrm{h}[n]=\cos\left(\frac{2\pi n}{16}\right)\mathrm{u}[n]$

The answer looks a lot like an offset DT sine wave, but not exactly.

(d) $\mathrm{h}[n]=\cos\left(\frac{2\pi n}{8}\right)\mathrm{u}[n]$ (e) $\mathrm{h}[n]=\cos\left(\frac{2\pi n}{16}\right)\mathrm{u}[n+8]$

20. Find and sketch the even and odd parts of these functions.

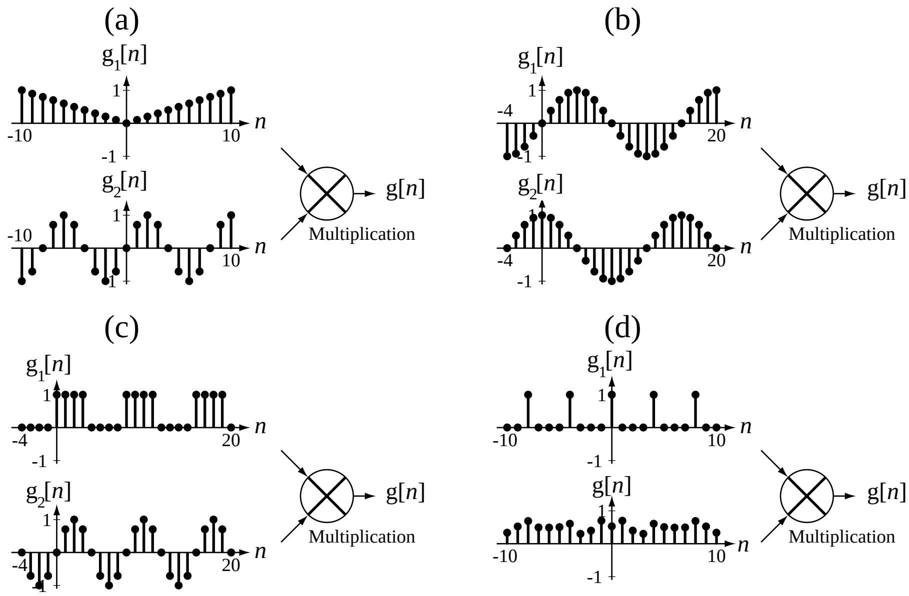
(a) $\mathrm{g}[n]=\mathrm{u}[n]-\mathrm{u}[n-4]$

$$\mathrm{g}_e[n]=\frac{\mathrm{u}[n]-\mathrm{u}[n-4]+\mathrm{u}[-n]-\mathrm{u}[-n-4]}{2}==\frac{1}{2}\left(\underbrace{\mathrm{u}[n]+\mathrm{u}[-n]}_{1+\delta[n]}-\mathrm{u}[n-4]-\mathrm{u}[n+4]\right)$$

(b) $g[n] = e^{-\frac{n}{4}} u[n]$

(c) $g[n] = \cos\left(\frac{2\pi n}{4}\right)$ (d) $g[n] = \sin\left(\frac{2\pi n}{4}\right) u[n]$

21. Sketch $g[n]$.



In each part multiply point-by-point to form the product.

22. Find the fundamental DT period and fundamental DT frequency of these functions.

(a) $g[n] = \cos\left(\frac{2\pi n}{10}\right)$

(b) $g[n] = \cos\left(\frac{\pi n}{10}\right)$

(c) $g[n] = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$

(d) $g[n] = e^{j\frac{2\pi n}{20}} + e^{-j\frac{2\pi n}{20}}$

(e) $g[n] = e^{-j\frac{2\pi n}{3}} + e^{-j\frac{2\pi n}{4}}$

These are almost identical to the previous exercises on finding the fundamental period of CT functions.

23. Graph the following functions and determine from the graphs the fundamental period of each one (if it is periodic).

(a) $\mathrm{g}[n] = 5\sin\left(\frac{2\pi n}{4}\right) + 8\cos\left(\frac{2\pi n}{6}\right)$ (b) $\mathrm{g}[n] = 5\sin\left(\frac{7\pi n}{12}\right) + 8\cos\left(\frac{14\pi n}{8}\right)$

(c) $\mathrm{g}[n] = \mathrm{Re}\left(e^{j\pi n} + e^{-j\frac{\pi n}{3}}\right)$

(d) $\mathrm{g}[n] = \mathrm{Re}\left(e^{jn} + e^{-j\frac{n}{3}}\right)$

These two exponentials do not have a finite LCM period.

24. Find the signal energy of these signals.

(a) $\mathrm{x}(t) = 2\,\mathrm{rect}(t)$ $E_x = \int_{-\infty}^{\infty} |2\,\mathrm{rect}(t)|^2\,dt = 4\int_{-\frac{1}{2}}^{\frac{1}{2}} dt = 4$

(b) $\mathrm{x}(t) = A(\mathrm{u}(t) - \mathrm{u}(t-10))$

(c) $\mathrm{x}(t) = \mathrm{u}(t) - \mathrm{u}(10-t)$

This function has an infinite duration with a non-zero value. Therefore its energy must be infinite.

(d) $\mathrm{x}(t) = \mathrm{rect}(t)\cos(2\pi t)$

(e) $\mathrm{x}(t) = \mathrm{rect}(t)\cos(4\pi t)$ (f) $\mathrm{x}(t) = \mathrm{rect}(t)\sin(2\pi t)$

(g) $\mathrm{x}[n] = A\,\mathrm{rect}_{N_0}[n]$

Just looking at this simple function, it obviously has $2N_0 + 1$ DT impulses, each of strength, A. Squaring each one and then adding the squares the energy must be $(2N_0 + 1)A^2$.

(h) $\mathrm{x}[n] = A\delta[n]$

(i) $\mathrm{x}[n] = \mathrm{comb}_{N_0}[n]$

(j) $\mathrm{x}[n] = \mathrm{ramp}[n]$

This function takes off at $n = 0$ and rises linearly for all positive time. Therefore its energy must be infinite.

(k) $\mathrm{x}[n] = \mathrm{ramp}[n] - 2\,\mathrm{ramp}[n-4] + \mathrm{ramp}[n-8]$

Even though each of these three individual functions has infinite energy, the sum of the three functions does not. This can be seen by drawing a graph of the function. This is a dramatic demonstration that the energy of a sum of functions is not necessarily the sum of the energies of the functions. For energy signals which are "uncorrelated" the energy of the sum is the sum of the energies. Correlation will be introduced and mathematically defined in Chapter 8.

25. Find the signal power of these signals.

(a) $\mathrm{x}(t) = A$ $\qquad P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |A|^2 dt = \lim_{T\to\infty} \frac{A^2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} dt = \lim_{T\to\infty} \frac{A^2}{T} T = A^2$

(b) $\mathrm{x}(t) = \mathrm{u}(t)$

(c) $\mathrm{x}(t) = A\cos(2\pi f_0 t + \theta)$

The average signal power of a periodic power signal is unaffected if it is shifted in time. Therefore we could find the average signal power of $A\cos(2\pi f_0 t)$ instead, which is somewhat easier algebraically. In doing the integral, use the trigonometric identity for the product of two sinusoids.

(d) $\mathrm{x}(t) = A\sum_{n=-\infty}^{\infty} \mathrm{rect}(t - 2n)$

It will help to visualize this signal before beginning the analysis. It is a "square" wave with fundamental period, $T_0 = 2$, alternating between 0 and *A*, spending half its time at each level. So the square of the signal alternates between 0 and A^2, spending half its time at each level. Therefore, without any math, its average signal power is obviously $\frac{A^2}{2}$.

(e) $\mathrm{x}(t) = 2A\left[-\frac{1}{2} + \sum_{n=-\infty}^{\infty} \mathrm{rect}(t - 2n)\right]$

(f) $\mathrm{x}[n] = A$ $\qquad$ (g) $\mathrm{x}[n] = \mathrm{u}[n]$

(h) $\mathrm{x}[n] = A\sum_{m=-\infty}^{\infty} \mathrm{rect}_2[n - 8m]$

Sketch the function and the square of the function first. Then you can find the average signal power without much work.

(i) $\mathrm{x}[n] = \mathrm{comb}_{N_0}[n]$ $\qquad$ (j) $\mathrm{x}[n] = \mathrm{ramp}[n]$

26. Using MATLAB, plot the CT signal, $\mathrm{x}(t) = \sin(2\pi t)$, over the time range, $0 < t < 10$, with

the following choices of the time resolution, Δt, of the plot. Explain why the plots look the way they do.

(a) $\Delta t = \frac{1}{24}$ (b) $\Delta t = \frac{1}{12}$ (c) $\Delta t = \frac{1}{4}$

(d) $\Delta t = \frac{1}{2}$ This graph is all zero. Why?

(e) $\Delta t = \frac{2}{3}$ Why do the graphs in (e) and (f) both have different fundamental periods than the original function?

(f) $\Delta t = \frac{5}{6}$

(g) $\Delta t = 1$ This graph is all zero. Why?

27. Given the function definitions on the left, find the function values on the right.

(a) $g(t) = 100\sin\left(200\pi t + \frac{\pi}{4}\right)$

$$g(0.001) = 100\sin\left(200\pi \times 0.001 + \frac{\pi}{4}\right) = 100\sin\left(\frac{\pi}{5} + \frac{\pi}{4}\right) = 98.77$$

(b) $g(t) = 13 - 4t + 6t^2$ $\quad g(2)$

(c) $g(t) = -5e^{-2t}e^{-j2\pi t}$ $\quad g\left(\frac{1}{4}\right)$

28. Sketch these CT exponential and trigonometric functions.

(a) $g(t) = 10\cos(100\pi t)$ (b) $g(t) = 40\cos(60\pi t) + 20\sin(60\pi t)$

(c) $g(t) = 5e^{-\frac{t}{10}}$ (d) $g(t) = 5e^{-\frac{t}{2}}\cos(2\pi t)$

29. Sketch these CT singularity and related functions.

(a) $g(t) = 2\,u(4 - t)$

(b) $g(t) = u(2t)$ Why do the graphs of $u(2t)$ and $u(t)$ look alike?

(c) $g(t) = 5\,\mathrm{sgn}(t - 4)$ (d) $g(t) = 1 + \mathrm{sgn}(4 - t)$

(e) $g(t) = 5\,\text{ramp}(t+1)$ (f) $g(t) = -3\,\text{ramp}(2t)$

(g) $g(t) = 2\delta(t+3)$

(h) $g(t) = 6\delta(3t+9)$ Remember the scaling property of the impulse. This graph is exactly the same as the graph in part (g).

(i) $g(t) = -4\delta(2(t-1))$ (j) $g(t) = 2\,\text{comb}\left(t - \frac{1}{2}\right)$

(k) $g(t) = 8\,\text{comb}(4t)$ Remember the scaling property of the impulse.

(l) $g(t) = -3\,\text{comb}\left(\frac{t+1}{2}\right)$

(m) $g(t) = 2\,\text{rect}\left(\frac{t}{3}\right)$ (n) $g(t) = 4\,\text{rect}\left(\frac{t+1}{2}\right)$

(o) $g(t) = \text{tri}(4t)$ (p) $g(t) = -6\,\text{tri}\left(\frac{t-1}{2}\right)$

(q) $g(t) = 5\,\text{sinc}\left(\frac{t}{2}\right)$ (r) $g(t) = -\text{sinc}(2(t+1))$

(s) $g(t) = -10\,\text{drcl}(t,4)$ (t) $g(t) = 5\,\text{drcl}\left(\frac{t}{4},7\right)$

(u) $g(t) = -3\,\text{rect}(t-2)$ (v) $g(t) = 0.1\,\text{rect}\left(\frac{t-3}{4}\right)$

(w) $g(t) = -4\,\text{tri}\left(\frac{3+t}{2}\right)$

(x) $g(t) = 4\,\text{sinc}(5(t-3))$ (y) $g(t) = 4\,\text{sinc}(5t-3)$

The answers in parts (x) and (y) are different.

30. Sketch these CT functions.

(a) $g(t) = u(t) - u(t-1)$

(b) $g(t) = \text{rect}\left(t - \frac{1}{2}\right)$ The graphs in (a) and (b) are identical.

(c) $g(t) = -4\,\text{ramp}(t)\,u(t-2)$ (d) $g(t) = \text{sgn}(t)\sin(2\pi t)$

(e) $g(t) = 5e^{-\frac{t}{4}} u(t)$ (f) $g(t) = \text{rect}(t)\cos(2\pi t)$

(g) $g(t) = -6\,\text{rect}(t)\cos(3\pi t)$ (h) $g(t) = \text{rect}(t)\,\text{tri}(t)$

(i) $g(t) = \text{rect}(t)\,\text{tri}\left(t + \frac{1}{2}\right)$

(j) $g(t) = u\left(t + \frac{1}{2}\right)\text{ramp}\left(\frac{1}{2} - t\right)$ The graphs in (i) and (j) are identical.

(k) $g(t) = \text{tri}^2(t)$ This graph does not look like a triangle.

(l) $g(t) = \text{sinc}^2(t)$ (m) $g(t) = |\text{sinc}(t)|$

(n) $g(t) = \frac{d}{dt}(\text{tri}(t))$ (o) $g(t) = \text{rect}\left(t + \frac{1}{2}\right) - \text{rect}\left(t - \frac{1}{2}\right)$

(p) $g(t) = \left[\int_{-\infty}^{t} \delta(\lambda + 1) - 2\delta(\lambda) + \delta(\lambda - 1)\right] d\lambda$

The graphs in (n), (o) and(p) are identical.

(q) $3\,\text{tri}\left(\frac{2t}{3}\right) + 3\,\text{rect}\left(\frac{t}{3}\right)$

(r) $6\,\text{tri}\left(\frac{t}{3}\right)\text{rect}\left(\frac{t}{3}\right)$ The graphs in (q) and (r) are identical.

(s) $4\,\text{sinc}(2t)\,\text{sgn}(-t)$ (t) $2\,\text{ramp}(t)\,\text{rect}\left(\frac{t-1}{2}\right)$

(u) $4\,\text{tri}\left(\frac{t-2}{2}\right)u(2-t)$ The graphs in (t) and (u) are identical.

(v) $3\,\text{rect}\left(\frac{t}{4}\right) - 6\,\text{rect}\left(\frac{t}{2}\right)$ (w) $g(t) = 10\,\text{drcl}\left(\frac{t}{4}, 5\right)\text{rect}\left(\frac{t}{8}\right)$

31. Using MATLAB, for each function below plot the original function and the transformed function.

(a)

$g(t) = 10\cos(20\pi t)\,\text{tri}(t)$ $5\,g(2t)$ vs. t

(b)

$$g(t) = \begin{cases} -2 \ , \ t < -1 \\ 2t \ , \ -1 < t < 1 \\ 3 - t^2 \ , \ 1 < t < 3 \\ -6 \ , \ t > 3 \end{cases} \qquad -3g(4-t) \quad \text{vs. } t$$

(c)

$$g(t) = \mathrm{Re}\left(e^{j\pi t} + e^{j1.1\pi t}\right) \qquad g\left(\frac{t}{4}\right) \ \text{vs. } t$$

(d)

$$G(f) = \left|\frac{5}{f^2 - j2 + 3}\right| \qquad \left|G(10(f-10)) + G(10(f+10))\right| \qquad \text{vs. } f$$

```
%     Plotting functions and transformations of those functions

close all ;

%     (a) part
tmin = -2 ; tmax = 2 ; N = 400 ;
dt = (tmax - tmin)/N ; t = tmin + dt*[0:N]' ;
g0 = ga(t) ; g1 = 5*ga(2*t) ;
subplot(2,1,1) ; p = plot(t,g0,'k') ; set(p,'LineWidth',2) ; grid ;
ylabel('g(t)') ;
subplot(2,1,2) ; p = plot(t,g1,'k') ; set(p,'LineWidth',2) ; grid ;
xlabel('t') ; ylabel('5g(2t)') ;

function y = ga(t)
      g = (1-abs(t)).*(-1 < t & t < 1) ;
      y = 10*cos(20*pi*t).*g ;
```

32. Let two signals be defined by

$$x_1(t) = \begin{cases} 1 \ , \ \cos(2\pi t) \geq 1 \\ 0 \ , \ \cos(2\pi t) < 1 \end{cases} \quad \text{and} \quad x_2(t) = \sin\left(\frac{2\pi t}{10}\right) .$$

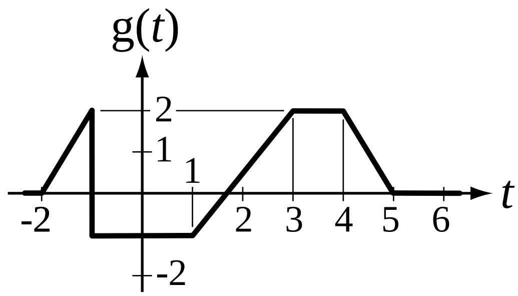
Plot these products over the time range, $-5 < t < 5$.

(a) $x_1(2t)\,x_2(-t)$ (b) $x_1\left(\frac{t}{5}\right)x_2(20t)$

(c) $x_1\left(\frac{t}{5}\right)x_2(20(t+1))$ (d) $x_1\left(\frac{t-2}{5}\right)x_2(20t)$

33. Given the graphical definition of a function, graph the indicated transformation(s).

(a)

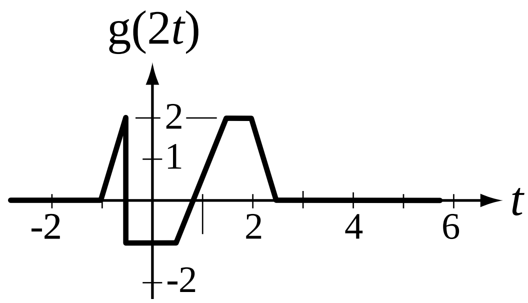


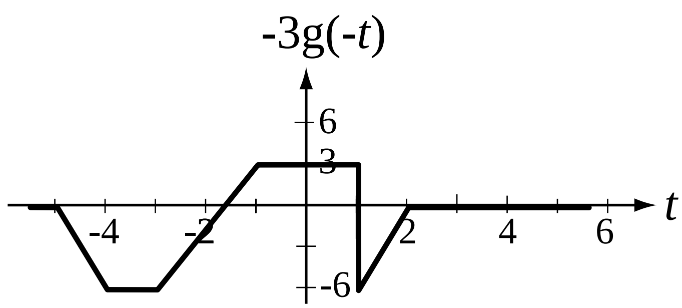
$g(t) \to g(2t)$

$g(t) \to -3g(-t)$

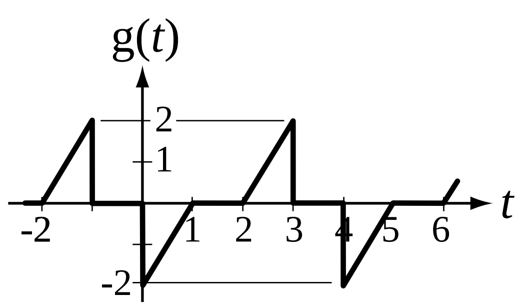
$g(t) = 0$, $t > 6$ or $t < -2$

The transformation, $g(t) \to g(2t)$, simply compresses the time scale by a factor of 2. The transformation $g(t) \to -3g(-t)$ time inverts the signal, amplitude inverts the signal and then multiplies the amplitude by 3.





(b)

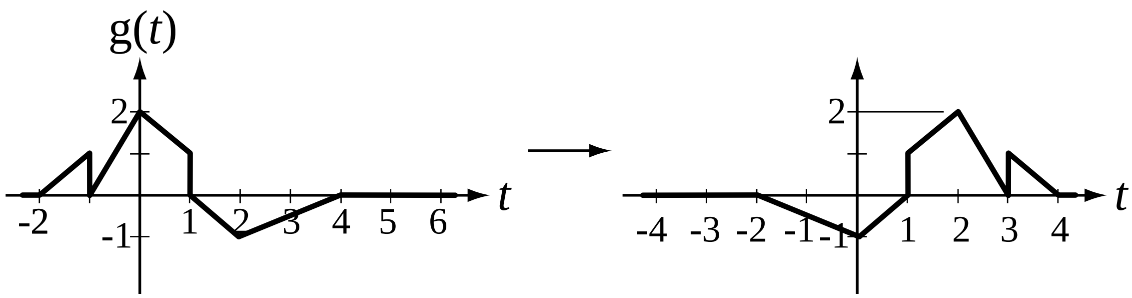


$g(t) \to g(t+4)$

$g(t) \to -2g\left(\frac{t-1}{2}\right)$

34. For each pair of functions graphed below determine what transformation has been done and write a correct functional expression for the transformed function.

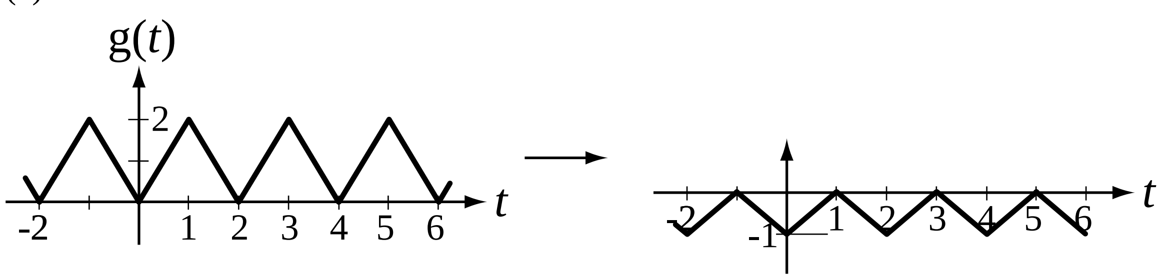
(a)



It should be visually obvious that the transformed signal has been time inverted and time shifted. By identifying a few corresponding points on both curves we see that after the time inversion the shift is to the right by 2. This corresponds to two successive transformations, $t \to -t$, followed by $t \to t - 2$. The overall effect of the two successive transformations is then $t \to -(t-2) = 2 - t$. Therefore the transformation is

$$\mathrm{g}(t) \to \mathrm{g}(2-t) \quad .$$

(b)



35. Sketch the magnitude and phase of each function versus f.

(a) $\mathrm{G}(f) = \mathrm{sinc}(f) e^{-j\frac{\pi f}{8}}$

(b) $\mathrm{G}(f) = \dfrac{jf}{1 + j\dfrac{f}{10}}$

This function is a ratio of two functions, jf and $1 + jf/10$. The magnitude of the ratio is the ratio of the magnitudes. At very low values of f, the ratio approaches 0 because the numerator approaches 0 and the denominator approaches 1. At very high values of f the denominator is approximately $jf/10$ and the magnitude of the ratio approaches 10. All these statements are equally true for positive and negative f. Therefore the magnitude is an even function of f.

The phase of the ratio is the phase of the numerator minus the phase of the denominator. For any positive f, the phase of the numerator is the phase of j times a positive constant. That is some number on the positive imaginary axis in the complex plane. So the phase is $\dfrac{\pi}{2}$ radians or 90°. For very small positive f, the denominator is approximately just the real number, 1, whose phase is 0. Therefore for very small positive *f* approaching 0 the phase approaches $\frac{\pi}{2}$. For very large positive *f*, the phase of the denominator approaches $\frac{\pi}{2}$ also and the difference between the numerator and denominator phases approaches 0. The behavior for negative *f* is similar except that the phase of the numerator is now $-\frac{\pi}{2}$. So the phase for negative *f* is exactly the negative of the phase for the corresponding positive *f*. That is, the phase is an odd function of *f*.

(c) $\mathrm{G}(f) = \left[\mathrm{rect}\left(\frac{f-1000}{100}\right) + \mathrm{rect}\left(\frac{f+1000}{100}\right)\right] e^{-j\frac{\pi f}{500}}$

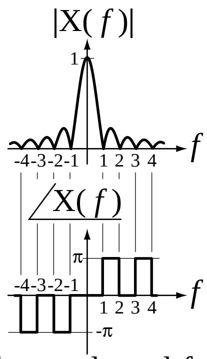
(d) $\mathrm{G}(f) = \frac{1}{250 - f^2 + j3f}$

(e) $\mathrm{G}(f) = \mathrm{comb}(100f)\,\mathrm{sinc}(25f)\,e^{j\frac{\pi f}{50}}$

This function has non-zero values only at the impulse locations in the comb function. Therefore the phase is only defined at those same points.

36. Graph versus f, in the range, $-4 < f < 4$, the magnitude and phase of

(a) $\mathrm{X}(f) = \mathrm{sinc}(f)$



The phase in this plot is the phase of a purely real function. If we only plotted purely real functions we would not need to graph magnitude and phase separately. A simple real plot of the function would be sufficient and clearer. But most transforms that we will later graph are complex functions and magnitude and phase plots are good ways of representing them. Since this function is purely real its value always lies on the real axis of the complex plane. When it is positive the simplest phase answer is 0. When it is negative the simplest phase answer is either positive or negative π radians. Later, in the study of transform methods applied to systems, we will find that we always have an even magnitude and an odd phase. For that reason, it is consistent and logical to choose phase values so as to make the plot an odd function. Here that is done by making the phase for negative values of $\mathrm{sinc}(f)$ be π for positive *f* and $-\pi$ for negative *f*.

(b) $\mathrm{X}(f) = 2\,\mathrm{sinc}(f)\,e^{-j4\pi f}$

(c) $\mathrm{X}(f) = 5\,\mathrm{rect}(2f)\,e^{+j2\pi f}$

(d) $\mathrm{X}(f) = 10\sin c^2\left(\frac{f}{4}\right)$

(e) $\mathrm{X}(f) = j5\delta(f+2) - j5\delta(f-2)$ The phase is undefined except at two points.

(f) $\mathrm{X}(f) = 2\,\mathrm{comb}(4f)e^{-j\pi f}$ The phase is undefined except where the impulses occur.

37. Sketch the even and odd parts of these CT signals.

(a) $\mathrm{x}(t) = \mathrm{rect}(t-1)$

(b) $\mathrm{x}(t) = \mathrm{tri}\left(t - \frac{3}{4}\right) + \mathrm{tri}\left(t + \frac{3}{4}\right)$ This is an even function.

(c) $\mathrm{x}(t) = 4\,\mathrm{sinc}\left(\frac{t-1}{2}\right)$ Use the definition of the sinc function to reduce this to some sine functions in fractions, then use the trigonometric identity,

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y),$$

to simplify that expression. This takes about 5 lines of complicated algebra to finish for the even part. The complexity is about the same for the odd part.

(d) $\mathrm{x}(t) = 2\sin\left(4\pi t - \frac{\pi}{4}\right)\mathrm{rect}(t)$

Use the trigonometric identity,

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

to simplify the expression and separate even and odd parts.

38. Let the CT unit impulse function be represented by the limit,

$$\delta(x) = \lim_{a\to 0}\frac{1}{a}\mathrm{tri}\left(\frac{x}{a}\right) \quad , \quad a > 0 \quad .$$

The function, $\frac{1}{a}\mathrm{tri}\left(\frac{x}{a}\right)$ has an area of one regardless of the value of *a*.

(a) What is the area of the function, $\delta(4x) = \lim_{a\to 0}\frac{1}{a}\mathrm{tri}\left(\frac{4x}{a}\right)$?

This is a triangle with the same height as $\frac{1}{a}\mathrm{tri}\left(\frac{x}{a}\right)$ but 1/4 times the base width.

(b) What is the area of the function, $\delta(-6x) = \lim_{a\to 0}\frac{1}{a}\mathrm{tri}\left(\frac{-6x}{a}\right)$?

This is a triangle with the same height as $\frac{1}{a}\text{tri}\left(\frac{x}{a}\right)$ but reversed in time with 1/6 the base width. What is the effect of the reversal in time?

(c) What is the area of the function, $\delta(bx) = \lim_{a \to 0} \frac{1}{a}\text{tri}\left(\frac{bx}{a}\right)$ for b positive and for b negative ?

39. Using a change of variable and the definition of the unit impulse, prove that

$$\delta\left(a\left(t - t_0\right)\right) = \frac{1}{|a|}\delta\left(t - t_0\right) \ .$$

Start with this definition of the impulse.

$$\delta(x) = 0 \ \ , \ \ x \neq 0 \ \ , \ \ \int_{-\infty}^{\infty} \delta(x)dx = 1$$

to establish the time of occurrence of the impulse, $\delta\left(a\left(t - t_0\right)\right)$. Then, use a change of variable to make $\delta\left(a\left(t - t_0\right)\right)$ have the same form as the impulse in the integral definition above and consider the cases of positive a and negative a separately.

40. Using the results of Exercise 39, show that

(a)

$$\text{comb}(ax) = \frac{1}{|a|}\sum_{n=-\infty}^{\infty} \delta\left(x - \frac{n}{a}\right)$$

(b) the average value of $\text{comb}(ax)$ is one, independent of the value of a

The average value of any periodic function is the area under the function over one period, divided by the period.

(c) a comb function of the form, $\frac{1}{a}\text{comb}\left(\frac{t}{a}\right)$ is a sequence of *unit* impulses spaced a units apart.

and (d) even though $\delta(at) = \frac{1}{|a|}\delta(t)$, $\text{comb}(ax) \neq \frac{1}{|a|}\text{comb}(x)$

41. Sketch the generalized derivative of $\text{g}(t) = 3\sin\left(\frac{\pi t}{2}\right)\text{rect}(t)$.

The generalized derivative is exactly the same as the derivative at all points of continuity. At points of discontinuity, the generalized derivative is an impulse whose strength is the size of the discontinuity.

42. Sketch the following CT functions.

(a) $g(t) = 3\delta(3t) + 6\delta(4(t-2))$

(b) $g(t) = 2\,\text{comb}\left(-\frac{t}{5}\right)$

The impulses in this function all have strength, 10.

(c) $g(t) = \text{comb}(t)\,\text{rect}\left(\frac{t}{11}\right)$ (d) $g(t) = 5\,\text{sinc}\left(\frac{t}{4}\right)\left[\frac{1}{2}\text{comb}\left(\frac{t}{2}\right)\right]$

(e) $g(t) = \frac{1}{2}\int_{-\infty}^{t}\left[\text{comb}\left(\frac{\lambda}{2}\right) - \text{comb}\left(\frac{\lambda-1}{2}\right)\right]d\lambda$

This is a rectangular wave.

43. What is the numerical value of each of the following integrals?

Use the sampling property of the impulse in each case.

(a) $\int_{-\infty}^{\infty}\delta(t)\cos(48\pi t)\,dt$ (b) $\int_{-\infty}^{\infty}\delta(t-5)\cos(\pi t)\,dt$

(c) $\int_{0}^{20}\delta(t-8)\,\text{tri}\left(\frac{t}{32}\right)dt$

The sampling property of the impulse holds as long as the range of integration includes the impulse.

(d) $\int_{0}^{20}\delta(t-8)\,\text{rect}\left(\frac{t}{16}\right)dt$

(e) $\int_{-2}^{2}\delta(t-1.5)\,\text{sinc}(t)\,dt$ (f) $\int_{-2}^{2}\delta(t-1.5)\,\text{sinc}(4t)\,dt$

44. What is the numerical value of each of the following integrals?

(a) $\int_{-\infty}^{\infty}\text{comb}(t)\cos(48\pi t)\,dt$ (b) $\int_{-\infty}^{\infty}\text{comb}(t)\sin(2\pi t)\,dt$

(c) $\int_{0}^{20}\text{comb}\left(\frac{t-2}{4}\right)\text{rect}(t)\,dt$

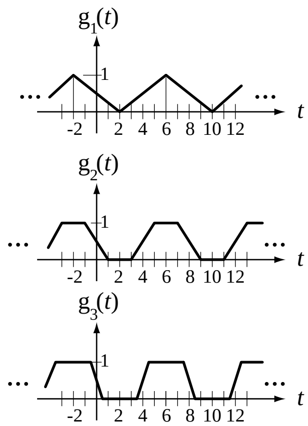
(d) $\int_{-2}^{2}\text{comb}(t)\,\text{sinc}(t)\,dt$ Only one of the comb impulses hits the sinc function at a value other than zero.

45. Sketch the derivatives of these functions.

(a) $g(t) = \sin(2\pi t)\operatorname{sgn}(t)$ (b) $g(t) = 2\operatorname{tri}\left(\frac{t}{2}\right) - 1$ (c) $g(t) = |\cos(2\pi t)|$

Sketch the functions first and then determine the derivatives with a combination of graphical and analytical analysis.

46. Sketch the derivatives of these functions. Compare the average values of the magnitudes of the derivatives.



Average derivative is zero in each case.

47. A function, $g(t)$, has this description:

It is zero for $t < -5$. It has a slope of –2 in the range, $-5 < t < -2$. It has the shape of a sine wave of unit amplitude and with a frequency of $\frac{1}{4}$ Hz plus a constant in the range, $-2 < t < 2$. For $t > 2$ it decays exponentially toward zero with a time constant of 2 seconds. It is continuous everywhere.

Write an exact mathematical description of this function.

Specify the functions behavior in each of the 4 regions described above.

(a) Graph $g(t)$ in the range, $-10 < t < 10$.

(b) Graph $g(2t)$ in the range, $-10 < t < 10$.

Compressed in time by a factor of two.

(c) Graph $2g(3-t)$ in the range, $-10 < t < 10$.

Time inverted , shifted and amplitude scaled.

(d) Graph $-2g\left(\frac{t+1}{2}\right)$ in the range, $-10 < t < 10$.

Amplitude scale, time scale and then time shift.

48. Find the even and odd parts of each of these CT functions.

(a) $g(t) = 10\sin(20\pi t)$

This function is obviously odd.

$$g_e(t) = \frac{10\sin(20\pi t) + 10\sin(-20\pi t)}{2} = 0 \quad , \quad x_o(t) = \frac{10\sin(20\pi t) - 10\sin(-20\pi t)}{2} = 10\sin(20\pi t)$$

(g) $g(t) = \dfrac{\cos(\pi t)}{\pi t}$

An even function divided by an odd function is an odd function. So the even part should be zero.

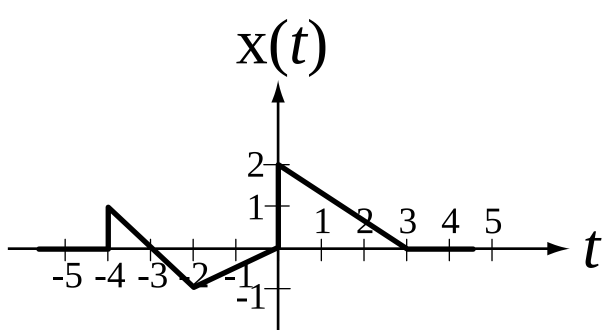
$$g_e(t) = \frac{\frac{\cos(\pi t)}{\pi t} + \frac{\cos(-\pi t)}{-\pi t}}{2} = \frac{\frac{\cos(\pi t)}{\pi t} + \frac{\cos(\pi t)}{-\pi t}}{2} = 0$$

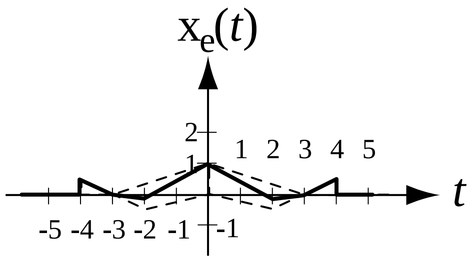
$$g_o(t) = \frac{\frac{\cos(\pi t)}{\pi t} - \frac{\cos(-\pi t)}{-\pi t}}{2} = \frac{\frac{\cos(\pi t)}{\pi t} + \frac{\cos(\pi t)}{\pi t}}{2} = \frac{\cos(\pi t)}{\pi t}$$

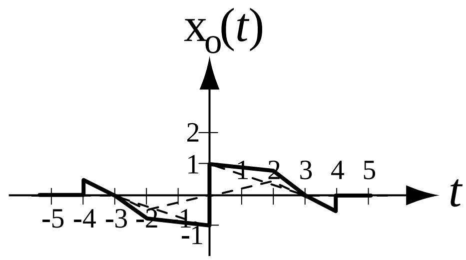
49. Is there a function that is both even and odd simultaneously? Discuss.

Think trivial.

50. Find and sketch the even and odd parts of the CT function, x(t).



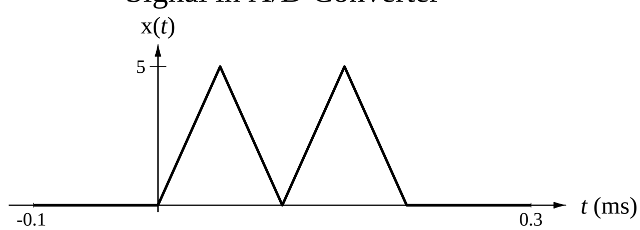




51. For each of the following signals decide whether it is periodic and, if it is, find the period.

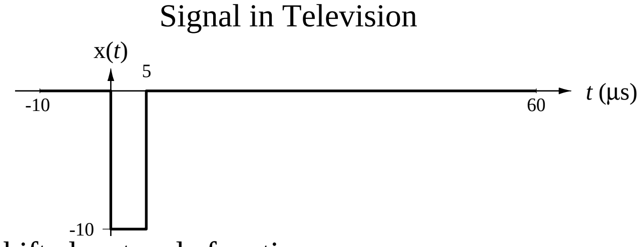
(a) $g(t) = 28\sin(400\pi t)$ Periodic. Fundamental frequency = 200 Hz, Period = 5 ms.
(b) $g(t) = 14 + 40\cos(60\pi t)$
(c) $g(t) = 5t - 2\cos(5000\pi t)$ Not periodic because 5t is not periodic.
(d) $g(t) = 28\sin(400\pi t) + 12\cos(500\pi t)$
(e) $g(t) = 10\sin(5t) - 4\cos(7t)$
(f) $g(t) = 4\sin(3t) + 3\sin(\sqrt{3}t)$ Not periodic because least common multiple is infinite.

52. The voltage illustrated in Figure E52 occurs in an analog-to-digital converter. Write a mathematical description of it.



Sum of two triangle functions.

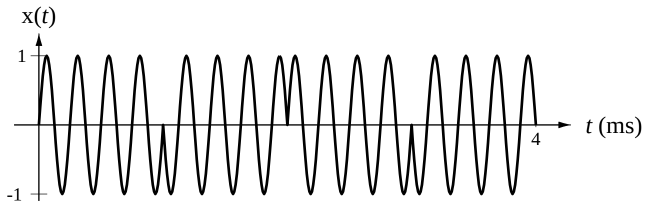
53. A signal occurring in a television set is illustrated in Figure E52. Write a mathematical description of it.



One scaled and shifted rectangle function.

54. The signal illustrated in Figure E54 is part of a binary-phase-shift-keyed (BPSK) binary data transmission. Write a mathematical description of it.

BPSK Signal



Sum of four products of a sine wave with a shifted rectangle function.

55. This signal illustrated in Figure E55 is the response of an RC lowpass filter to a sudden change in excitation. Write a mathematical description of it.

On a decaying exponential, a tangent line at any point intersects the final value one time constant later. Theconstant value before the decaying exponential is -4 V and the slope of the tangent line at 4 ns is -2.67V/4 ns or -2/3 V/ns.

RC Filter Signal

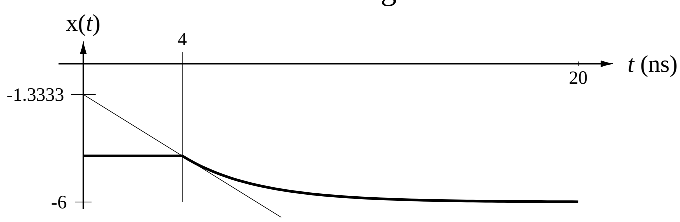


Figure E55 Transient response of an RC filter

$$00\pi t)\text{rect}\left(t-0.5\times10^{-3}\right)-\sin(8000\pi t)\text{rect}\left(t-1.5\times10^{-3}\right)+\sin(8000\pi t)\text{rect}\left(t-2.5\times10^{-3}\right)-\sin(8000\pi t)\text{rect}\left(t-3.5\right.$$

$$\text{x}(t)=-4-2\left(1-e^{-\frac{t-4}{3}}\right)\text{u}(t-4)$$

56. Find the signal energy of each of these signals:

(a) $2\,\text{rect}(-t)$, $E=\int_{-\infty}^{\infty}\left[2\,\text{rect}(-t)\right]^2 dt=4\int_{-\frac{1}{2}}^{\frac{1}{2}} dt=4$

(b) $\text{x}(t)=\text{rect}(8t)$ (c) $\text{x}(t)=3\text{rect}\left(\frac{t}{4}\right)$

(d) $\text{tri}(2t)$

Keep in mind that the square of a rectangle function is another rectangle function but the square of a triangle function is not another triangle function. Use the definition of the triangle function and the fact that a triangle function is even to reduce the work.

(e) $\text{x}(t)=3\text{tri}\left(\frac{t}{4}\right)$

(f) $2\sin(200\pi t)$

This is a power signal and, as such, has infinite signal energy.

(g) $\delta(t)$ (Hint: First find the signal energy of a signal which approaches an impulse some limit, then take the limit.)

(h) $x(t) = \frac{d}{dt}(\text{rect}(t))$

This requires a generalized derivative and therefore contains impulses. Use the result of part (g).

(i) $x(t) = \int_{-\infty}^{t} \text{rect}(\lambda)\, d\lambda$

The rectangle function has finite energy but its integral does not.

(j) $x(t) = e^{(-1-j8\pi)t}\,u(t)$

Even though this function is complex-valued, the formula still applies.

57. Find the average signal power of each of these signals.

(a) $x(t) = 2\sin(200\pi t)$ This is a periodic function. Therefore

$$P_x = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} [2\sin(200\pi t)]^2\, dt = \frac{4}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\frac{1}{2} - \frac{1}{2}\cos(400\pi t)\right] dt$$

$$P_x = \frac{2}{T}\left[t - \frac{\sin(400\pi t)}{400\pi}\right]_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{2}{T}\left[\frac{T}{2} - \frac{\sin(200\pi T)}{400\pi} + \frac{T}{2} + \frac{\sin(-200\pi T)}{400\pi}\right] = 2$$

For any sinusoid, the average signal power is half the square of the amplitude.

(b) $x(t) = \text{comb}(t)$

Use the result of 56 (g).

(c) $x(t) = e^{j100\pi t}$

58. Sketch these DT exponential and trigonometric functions.

(a) $g[n] = -4\cos\left(\frac{2\pi n}{10}\right)$

(b) $g[n] = -4\cos(2.2\pi n)$ (c) $g[n] = -4\cos(1.8\pi n)$

The graphs in (b) and (c) are identical. Why?

(d) $\mathrm{g}[n] = 2\cos\left(\frac{2\pi n}{6}\right) - 3\sin\left(\frac{2\pi n}{6}\right)$

(e) $\mathrm{g}[n] = \left(\frac{3}{4}\right)^n$

(f) $\mathrm{g}[n] = 2(0.9)^n \sin\left(\frac{2\pi n}{4}\right)$

59. Sketch these DT singularity functions.

(a) $\mathrm{g}[n] = 2\mathrm{u}[n+2]$

(b) $\mathrm{g}[n] = \mathrm{u}[5n]$

Why does $\mathrm{u}[5n] = \mathrm{u}[n]$?

(c) $\mathrm{g}[n] = -2\,\mathrm{ramp}[-n]$

(d) $\mathrm{g}[n] = 10\,\mathrm{ramp}\left[\frac{n}{2}\right]$

Don’t plot undefined values.

(e) $\mathrm{g}[n] = 7\delta[n-1]$

(f) $\mathrm{g}[n] = 7\delta[2(n-1)]$

The DT impulse does not have a scaling property.

(g) $\mathrm{g}[n] = -4\delta\left[\frac{2}{3}n\right]$

(h) $\mathrm{g}[n] = -4\delta\left[\frac{2}{3}n - 1\right]$

(i) $\mathrm{g}[n] = 8\,\mathrm{comb}_4[n]$

(j) $\mathrm{g}[n] = 8\,\mathrm{comb}_4[2n]$

(k) $\mathrm{g}[n] = \mathrm{rect}_4[n]$

(l) $\mathrm{g}[n] = 2\,\mathrm{rect}_5\left[\frac{n}{3}\right]$

(m) $\mathrm{g}[n] = \mathrm{tri}\left(\frac{n}{5}\right)$

(n) $\mathrm{g}[n] = -\mathrm{sinc}\left(\frac{n}{4}\right)$

(o) $\mathrm{g}[n] = \mathrm{sinc}\left(\frac{n+1}{4}\right)$

(p) $\mathrm{g}[n] = \mathrm{drcl}\left(\frac{n}{10}, 9\right)$

60. Sketch these combinations of DT functions.

(a) $g[n] = u[n] + u[-n]$

This function is not a constant for all *n*.

(b) $g[n] = u[n] - u[-n]$

(c) $g[n] = \cos\left(\frac{2\pi n}{12}\right)\mathrm{comb}_3[n]$

This is a "sampled" DT cosine.

(d) $g[n] = \cos\left(\frac{2\pi n}{12}\right)\mathrm{comb}_3\left[\frac{n}{2}\right]$

(e) $g[n] = 5e^{-\frac{n}{16}}\sin\left(\frac{2\pi n}{8}\right)u[n]$ (f) $g[n] = \sin\left(\frac{2\pi n}{4}\right)u[n]$

(g) $g[n] = \sum_{m=0}^{n}\left(\mathrm{comb}_4[m] - \mathrm{comb}_4[m-2]\right)$

This is a DT rectangular wave whose rectangles each have width, 2.

(h) $g[n] = \sum_{m=0}^{n}\cos\left(\frac{2\pi m}{12}\right)u[m]$

(i) $g[n] = \sum_{m=0}^{n}\left(\mathrm{comb}_4[m] - \mathrm{comb}_4[m-2]\right)$

(j) $g[n] = \sum_{m=0}^{n}\left(\mathrm{comb}_4[m] + \mathrm{comb}_3[m]\right)\mathrm{rect}_4[m]$

(k) $g[n] = \mathrm{comb}_2[n+1] - \mathrm{comb}_2[n]$

(l) $g[n] = \sum_{m=-\infty}^{n+1}\delta[m] - \sum_{m=-\infty}^{n}\delta[m]$

The accumulation of a unit DT impulse is a unit sequence and the first difference of a unit sequence is an impulse.

61. Sketch the magnitude and phase of each function versus *k*.

(a) $G[k] = 20\sin\left(\frac{2\pi k}{8}\right)e^{-j\frac{\pi k}{4}}$ (b) $G[k] = 20\cos\left(\frac{2\pi k}{8}\right)\mathrm{sinc}\left(\frac{k}{40}\right)$

(c) $\mathrm{G}[k] = (\delta[k+8] - 2\delta[k+4] + \delta[k] - 2\delta[k-4] + \delta[k-8])e^{j\frac{\pi k}{8}}$

62. Given the function definitions on the left, find the function values on the right.

(a)

$$\mathrm{g}[n] = \frac{3n+6}{10}e^{-2n} \qquad \mathrm{g}[3]$$

(b) $\mathrm{g}[n] = \mathrm{Re}\left(\left(\frac{1+j}{\sqrt{2}}\right)^n\right)$

A complex number of the form, $x + jy$, raised to the nth power can be expressed as $(re^{j\theta})^n = r^n e^{jn\theta}$ where r is the magnitude of the number, $\sqrt{x^2+y^2}$, and θ is the angle of the number, $\tan^{-1}\left(\frac{y}{x}\right)$.

(c)

$$\mathrm{g}[n] = (j2\pi n)^2 + j10\pi n - 4 \qquad \mathrm{g}[4]$$

63. Using MATLAB, for each function below plot the original function and the transformed function.

(a)

$$\mathrm{g}[n] = \begin{cases} 5 \ , & n \le 0 \\ 5 - 3n \ , & 0 < n \le 4 \\ -23 + n^2 \ , & 4 < n \le 8 \\ 41 \ , & n > 8 \end{cases} \qquad \mathrm{g}[3n] \ \text{ vs. } n$$

(b)

$$\mathrm{g}[n] = 10\cos\left(\frac{2\pi n}{20}\right)\cos\left(\frac{2\pi n}{4}\right) \qquad 4\,\mathrm{g}[2(n+1)] \ \text{ vs. } n$$

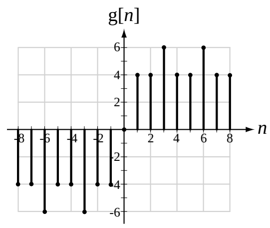
(c)

$$\mathrm{g}[n] = \left|8e^{j\frac{2\pi n}{16}}\mathrm{u}[n]\right| \qquad \mathrm{g}\left[\frac{n}{2}\right] \qquad \text{vs. } n$$

This function has some undefined values.

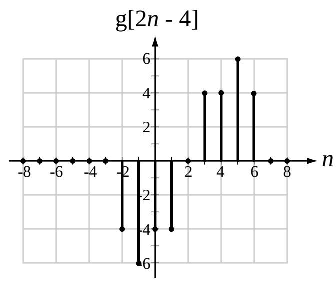
64. Given the graphical definition of a function, g[n], graph the indicated function(s), h[n].

(a)

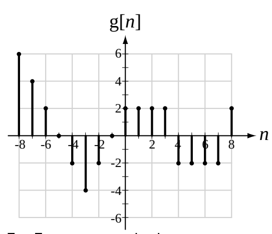


$$h[n] = g[2n - 4]$$

$$g[n] = 0 \quad , \quad |n| > 8$$



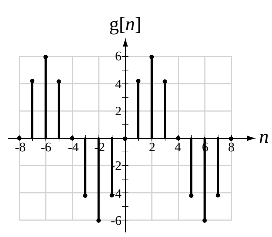
(b)



$$h[n] = g\left[\frac{n}{2}\right]$$

$$g[n] = 0 \quad , \quad |n| > 8$$

(c)



$$h[n] = g\left[\frac{n}{2}\right]$$

$g[n]$ is periodic with fundamental period, 8

65. Sketch the accumulation from negative infinity to n of each of these DT functions.

(a) $g[n] = \cos(2\pi n)u[n]$ (b) $g[n] = \cos(4\pi n)u[n]$

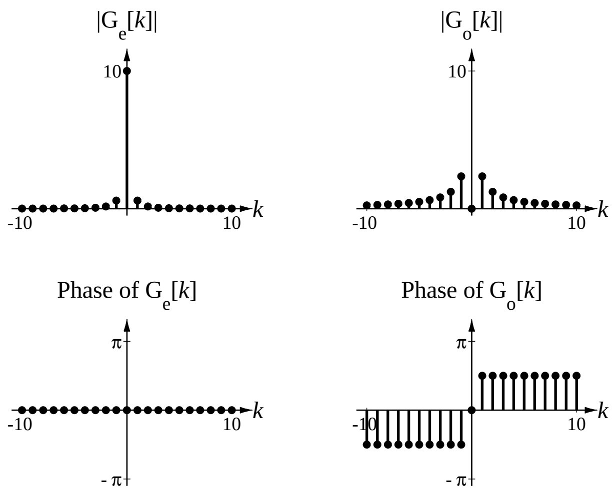
The answers in (a) and (b) are identical. Why?

66. Find and sketch the magnitude and phase of the even and odd parts of each of this "discrete-k" function.

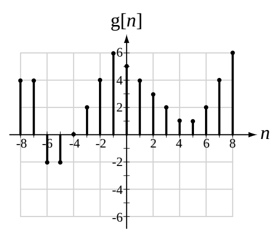
$$G[k] = \frac{10}{1 - j4k}$$

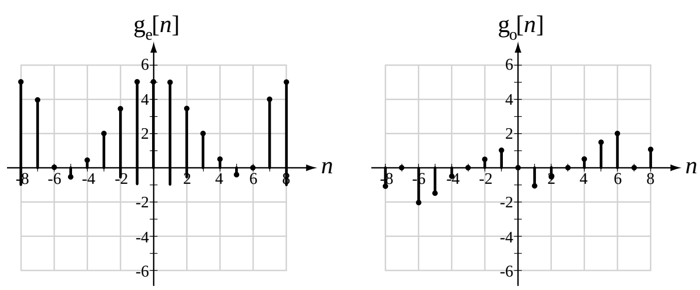
$$\mathrm{G}_e[k] = \frac{\dfrac{10}{1-j4k} + \dfrac{10}{1+j4k}}{2} = \frac{10}{(1-j4k)(1+j4k)} = \frac{10}{1+16k^2}$$

$$\mathrm{G}_o[k] = \frac{\dfrac{10}{1-j4k} - \dfrac{10}{1+j4k}}{2} = \frac{j40k}{(1-j4k)(1+j4k)} = \frac{j40k}{1+16k^2}$$



67. Find and sketch the even and odd parts of the DT function below.





68. Using MATLAB, plot each of these DT functions. If a function is periodic, find the period analytically and verify the period from the plot.

(a) $\mathrm{g}[n] = \sin\left(\dfrac{3\pi n}{2}\right)$

(b) $\mathrm{g}[n] = \sin\left(\frac{2\pi n}{3}\right) + \cos\left(\frac{10\pi n}{3}\right)$

$$\mathrm{g}[n] = \sin\left(\frac{2\pi n}{3}\right) + \cos\left(\frac{6\pi n}{3} + \frac{4\pi n}{3}\right) = \underbrace{\sin\left(\frac{2\pi n}{3}\right)}_{\text{Period}=3} + \underbrace{\cos\left(\frac{4\pi n}{3}\right)}_{\text{Period}=3}$$

Period is 3

(c) $\mathrm{g}[n] = 5\cos\left(\frac{2\pi n}{8}\right) + 3\sin\left(\frac{2\pi n}{5}\right)$

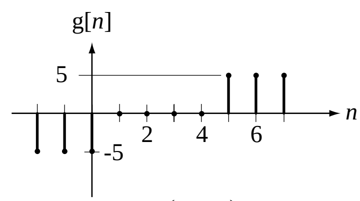
(d) $\mathrm{g}[n] = 10\cos\left(\frac{n}{4}\right)$ Not periodic. Why not?

(e) $\mathrm{g}[n] = -3\cos\left(\frac{2\pi n}{7}\right)\sin\left(\frac{2\pi n}{6}\right)$ (A trigonometric identity for the product of two sinusoids will be useful here.)

69. Sketch the following DT functions.

(a) $\mathrm{g}[n] = 5\delta[n-2] + 3\delta[n+1]$ (b) $\mathrm{g}[n] = 5\delta[2n] + 3\delta[4(n-2)]$

(c) $\mathrm{g}[n] = 5(\mathrm{u}[n-1] - \mathrm{u}[4-n])$



(d) $\mathrm{g}[n] = 8\,\mathrm{rect}_4[n+1]$ (e) $\mathrm{g}[n] = 8\cos\left(\frac{2\pi n}{7}\right)$

(f) $\mathrm{g}[n] = -10e^{\frac{n}{4}}\mathrm{u}[n]$

The graphs for (f) and (g) are identical. Why?

(g) $\mathrm{g}[n] = -10(1.284)^n\,\mathrm{u}[n]$

(h) $\mathrm{g}[n] = \left|\left(\frac{j}{4}\right)^n \mathrm{u}[n]\right|$

(i) $\mathrm{g}[n] = \mathrm{ramp}[n+2] - 2\,\mathrm{ramp}[n] + \mathrm{ramp}[n-2]$

(j) $\mathrm{g}[n] = \mathrm{rect}_2[n]\mathrm{comb}_2[n]$ (k) $\mathrm{g}[n] = \mathrm{rect}_2[n]\mathrm{comb}_2[n+1]$

(l) $\mathrm{g}[n] = 3\sin\left(\frac{2\pi n}{3}\right)\mathrm{rect}_4[n]$ (m) $\mathrm{g}[n] = 5\cos\left(\frac{2\pi n}{8}\right)\mathrm{u}\left[\frac{n}{2}\right]$

70. Graph versus k, in the range, $-10 < k < 10$, the magnitude and phase of

(a) $\mathrm{X}[k] = \mathrm{sinc}\left(\frac{k}{2}\right)$ (b) $\mathrm{X}[k] = \mathrm{sinc}\left(\frac{k}{2}\right)e^{-j\frac{2\pi k}{4}}$

(c) $\mathrm{X}[k] = \mathrm{rect}_3[k]e^{-j\frac{2\pi k}{3}}$

Phase is undefined for any k greater than 3.

(d) $\mathrm{X}[k] = \frac{1}{1 + j\frac{k}{2}}$ (e) $\mathrm{X}[k] = \frac{jk}{1 + j\frac{k}{2}}$

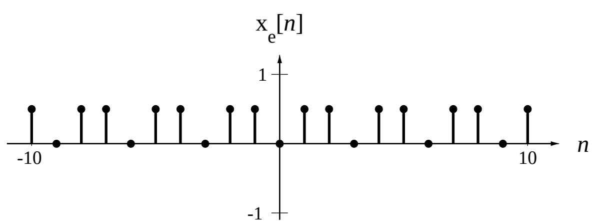
(f) $\mathrm{X}[k] = \mathrm{comb}_2[k]e^{-j\frac{2\pi k}{4}}$

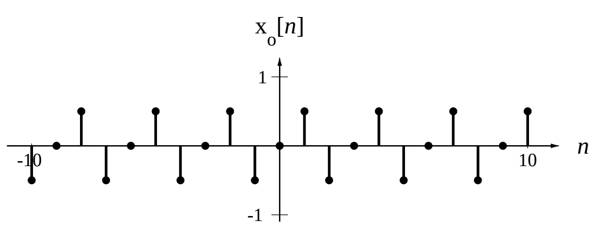
71. Sketch the even and odd parts of these signals.

(a) $\mathrm{x}[n] = \mathrm{rect}_5[n+2]$

(b) $\mathrm{x}[n] = \mathrm{comb}_3[n-1]$

$$\mathrm{x}_e[n] = \frac{\mathrm{comb}_3[n-1] + \mathrm{comb}_3[-n-1]}{2} = \frac{1}{2}\left(\mathrm{comb}_3[n-1] + \mathrm{comb}_3[n+1]\right)$$





(c) $\mathrm{x}[n] = 15\cos\left(\frac{2\pi n}{9} + \frac{\pi}{4}\right)$ (d) $\mathrm{x}[n] = \sin\left(\frac{2\pi n}{4}\right)\mathrm{rect}_5[n-1]$

72. What is the numerical value of each of the following accumulations?

(a) $\sum_{n=0}^{10} \mathrm{ramp}[n]$

(b) $\sum_{n=0}^{6} \frac{1}{2^n} = 1 + \frac{1}{2} + \cdots + \frac{1}{2^6}$.

Using $\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & , \ \alpha = 1 \\ \dfrac{1-\alpha^N}{1-\alpha} & , \ \text{otherwise} \end{cases}$

$$\sum_{n=0}^{6} \frac{1}{2^n} = \frac{1-\left(\frac{1}{2}\right)^7}{1-\frac{1}{2}} = \frac{1-\frac{1}{128}}{\frac{1}{2}} = \frac{127}{256}$$

(c) $\sum_{n=-\infty}^{\infty} \frac{\mathrm{u}[n]}{2^n}$ (d) $\sum_{n=-10}^{10} \mathrm{comb}_3[n]$

(e) $\sum_{n=-10}^{10} \mathrm{comb}_3[2n]$ (f) $\sum_{n=-\infty}^{\infty} \mathrm{sinc}(n)$

73. Find the signal energy of each of these signals:

(a) $\mathrm{x}[n] = 5\,\mathrm{rect}_4[n]$ $E_x = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = 25 \sum_{n=-\infty}^{\infty} |\mathrm{rect}_4[n]|^2 = 25 \sum_{n=-4}^{4} (1) = 225$

(b) $\mathrm{x}[n] = 2\delta[n] + 5\delta[n-3]$

(c) $\mathrm{x}[n] = \frac{\mathrm{u}[n]}{n}$ The energy is infinite.

(d) $\mathrm{x}[n] = \left(-\frac{1}{3}\right)^n \mathrm{u}[n]$ (e) $\mathrm{x}[n] = \cos\left(\frac{\pi n}{3}\right)(\mathrm{u}[n] - \mathrm{u}[n-6])$

74. Find the average signal power of each of these signals:

(a) $\mathrm{x}[n] = \mathrm{u}[n]$

$$P_x = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} |\mathrm{x}[n]|^2 = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} |\mathrm{u}[n]|^2 = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=0}^{N-1} (1) = \frac{1}{2}$$

(b) $\mathrm{x}[n] = (-1)^n$

(c) $\mathrm{x}[n] = A\cos(2\pi F_0 n + \theta)$

This is a rather long and involved exercise.

$$P_x = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} |A\cos(2\pi F_0 n + \theta)|^2 = \lim_{N\to\infty} \frac{A^2}{2N} \sum_{n=-N}^{N-1} \cos^2(2\pi F_0 n + \theta)$$

Using the trigonometric identity,

$$\cos(x)\cos(y) = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right],$$

$$P_x = \lim_{N\to\infty}\frac{A^2}{2N}\sum_{n=-N}^{N-1}\frac{1}{2}\left[1+\cos(4\pi F_0 n + 2\theta)\right] = \frac{A^2}{2} + \lim_{N\to\infty}\frac{A^2}{4N}\sum_{n=-N}^{N-1}\cos(4\pi F_0 n + 2\theta)$$

So the average power is a constant, $\frac{A^2}{2}$, plus another term,

$$\lim_{N\to\infty}\frac{A^2}{4N}\sum_{n=-N}^{N-1}\cos(4\pi F_0 n + 2\theta),$$

whose value depends on the parameters, F_0 and θ.

Case 1. $2F_0$, an integer.

$$P_x = \frac{A^2}{2} + \lim_{N\to\infty}\frac{A^2}{4N}\sum_{n=-N}^{N-1}\cos(2\theta) = \frac{A^2}{2}\left[1+\cos(2\theta)\right]$$

Case 2. $2F_0$, not an integer.

Subcase 1. The signal is periodic. If it is periodic with period, $N_0 = \frac{1}{F_0}$. The square of the function is periodic with period, $N_0 = \frac{1}{2F_0}$. For a periodic signal, $P_x = \frac{1}{N}\sum_{n=k}^{k+N-1}\left|\mathrm{x}[n]\right|^2$ where "N" is any integer number of periods of the signal and where "k" is any integer. Then the average power is

$$P_x = \frac{A^2}{2} + \frac{A^2}{4N}\sum_{n=k}^{k+N-1}\cos(2\theta)$$

The summation, $\frac{A^2}{4N}\sum_{n=k}^{k+N-1}\cos(2\theta)$ is zero because the samples are taken at equal angular intervals over exactly an integer number of periods. Therefore the average power is $P_x = \frac{A^2}{2}$.

Subcase 2. The signal is not periodic. This is the hardest case to examine. We cannot use the periodic formula so we must use

$$P_x = \frac{A^2}{2} + \lim_{N\to\infty}\frac{A^2}{4N}\sum_{n=-N}^{N-1}\cos(4\pi F_0 n + 2\theta)$$

The summation can be geometrically visualized as the real part of a sum of vectors in the complex plane, all with unit length and separated from their nearest neighbors by the angle, $4\pi F_0$. Since this angle is not any integer multiple of π radians (that case has already been considered), the summation is of an infinity of vectors at angles which uniformly fill up the full 2π radians of a full circle. We don't know what that sum is in the limit, but we do know it cannot grow to infinity because of the cancellation of the vectors arrayed in the circle and the sum is being divided by *N* which is going to infinity. Therefore, in the limit, the average power is $P_x = \frac{A^2}{2}$, just as in the periodic case.

$$P_x = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} \left| \mathrm{x}[n] \right|^2 = \frac{1}{4} \sum_{n=0}^{3} \left| \mathrm{x}[n] \right|^2 = \frac{1}{4} \sum_{n=0}^{3} \left| e^{-j\frac{\pi n}{2}} \right|^2 = \frac{1}{4} \sum_{n=0}^{3} 1 = 1$$

(d) $\mathrm{x}[n] = \begin{cases} A & , \quad n = \cdots,0,1,2,3,8,9,10,11,16,17,18,19,\cdots \\ 0 & , \quad n = \cdots,4,5,6,7,12,13,14,15,20,21,22,23,\cdots \end{cases}$

(e) $\mathrm{x}[n] = e^{-j\frac{\pi n}{2}}$