Description and Analysis of Systems

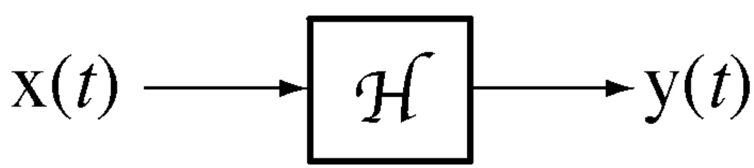
Systems

- Broadly speaking, a system is anything that responds when stimulated or excited
- The systems most commonly analyzed by engineers are artificial systems designed and built by humans
- Engineering system analysis is the application of mathematical methods to the design and analysis of systems

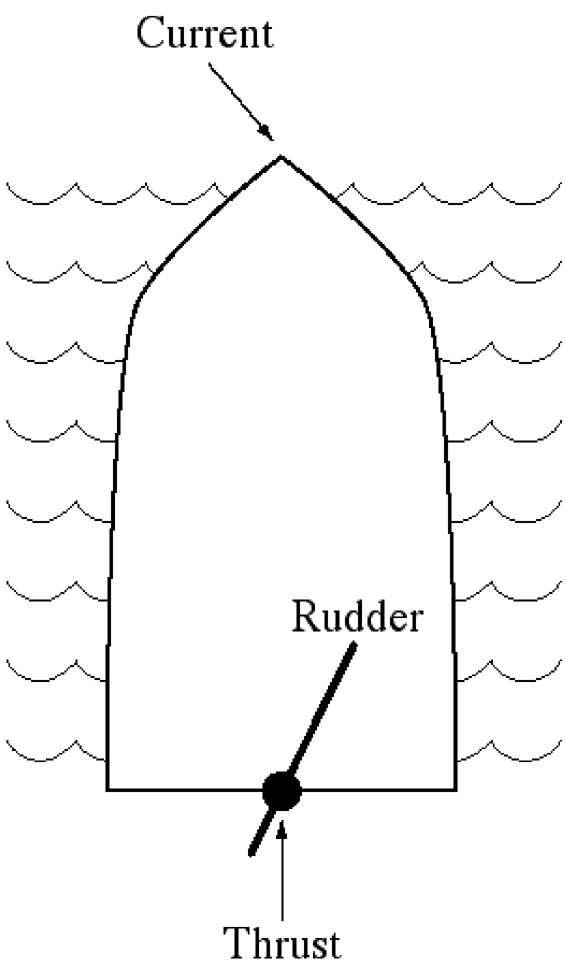
Systems

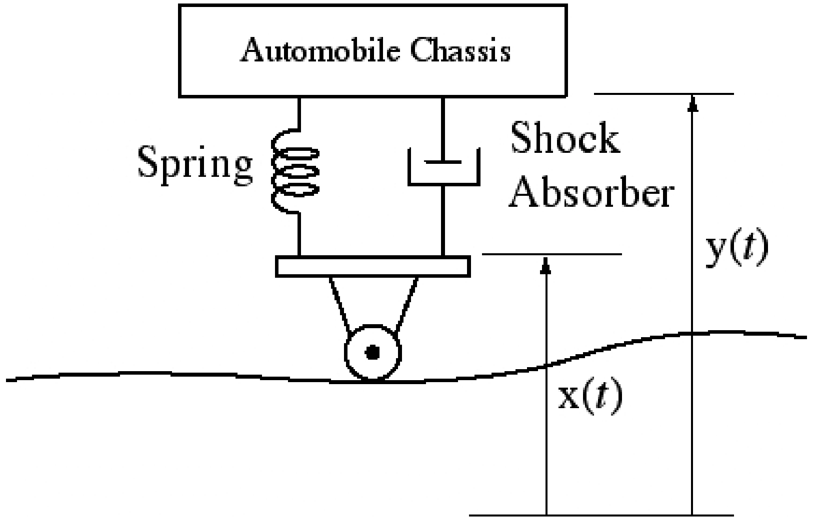
- Systems have inputs and outputs
- Systems accept excitation signals at their inputs and produce response signals at their outputs
- Systems are often usefully represented by *block diagrams*

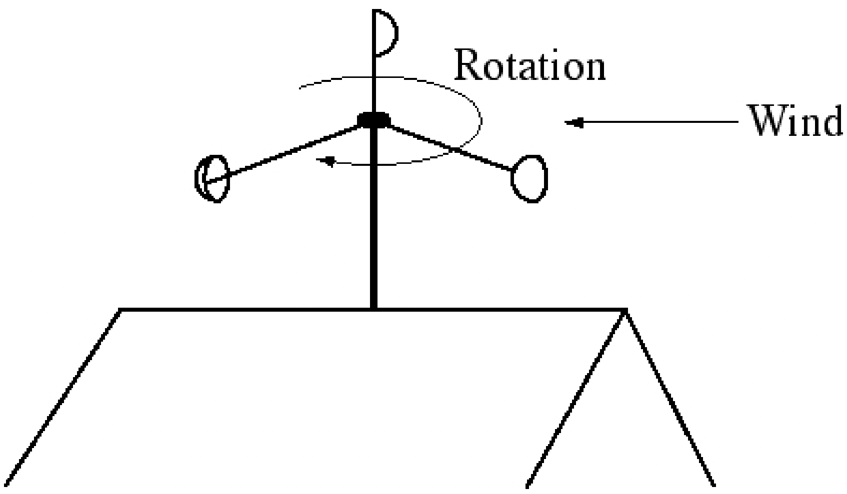
A single-input, single-output system block diagram



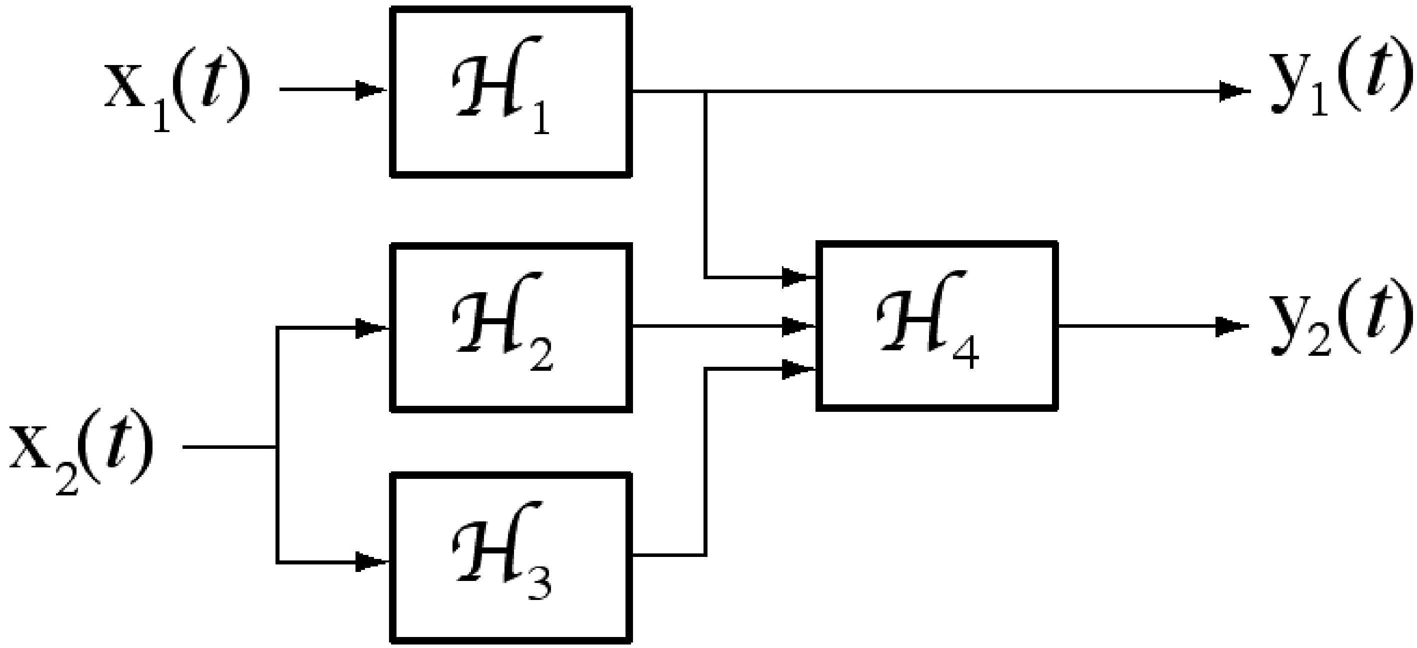
System Examples





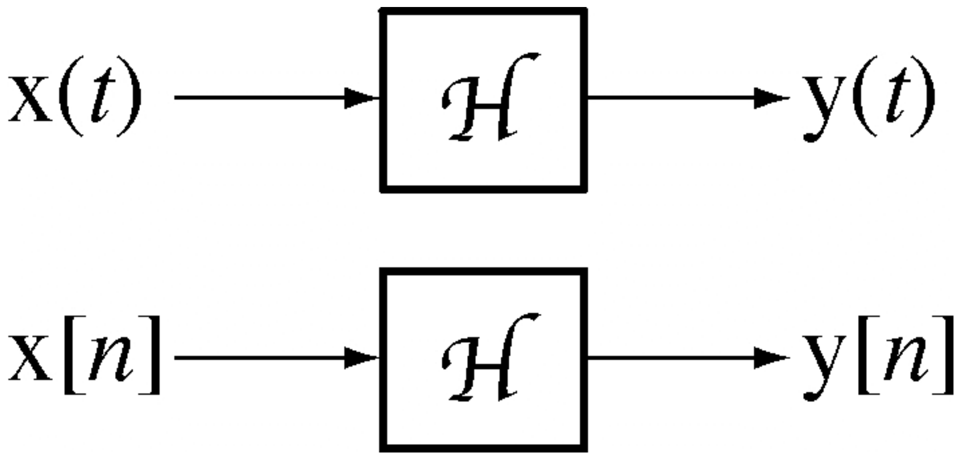


A Multiple-Input, Multiple-Output System Block Diagram



CT and DT Systems

CT systems respond to and produce CT signals



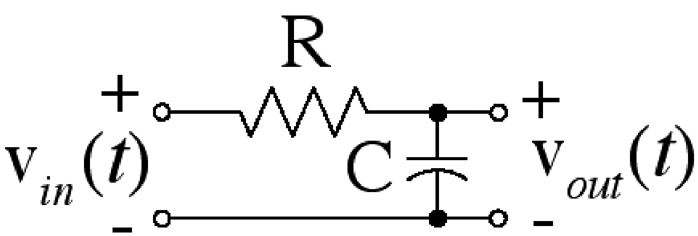
DT systems respond to and produce DT signals

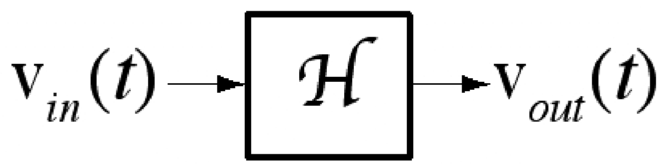
An Electrical Circuit Viewed as a System

An RC lowpass filter is a simple electrical system

It is excited by a voltage, $\mathrm{v}_{in}(t)$, and responds with a voltage, $\mathrm{v}_{out}(t)$

It can be viewed or modeled as a single-input, single-output system





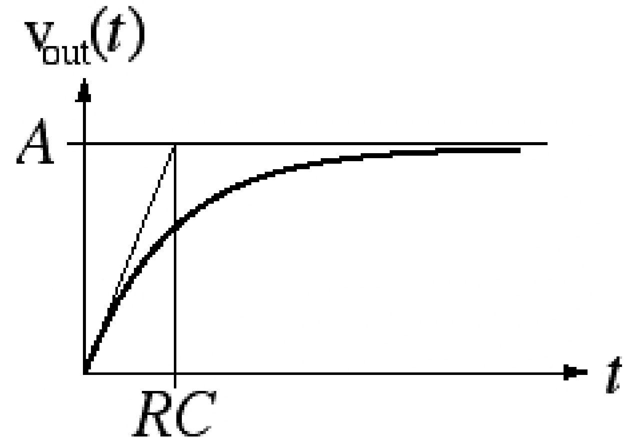
Response of an RC Lowpass Filter to a Step Excitation

If an RC lowpass filter is excited by a step of voltage,

$$\mathrm{v}_{in}(t) = A\,\mathrm{u}(t)$$

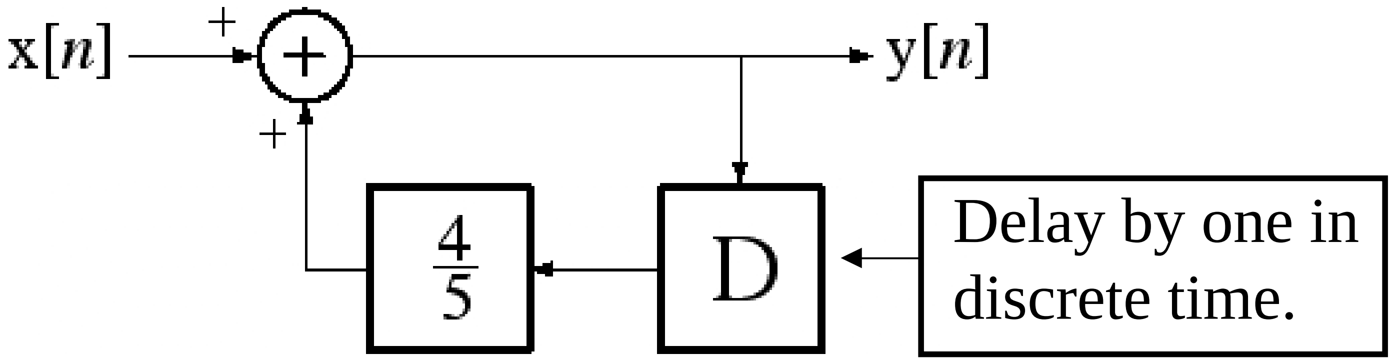
its response is

$$\mathrm{v}_{out}(t) = A\left(1 - e^{-\frac{t}{RC}}\right)\mathrm{u}(t)$$



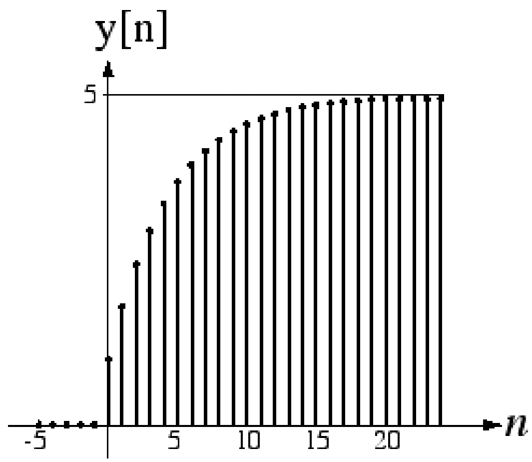
If the excitation is doubled, the response doubles.

A DT System



If the excitation, x[n], is the unit sequence, the response is

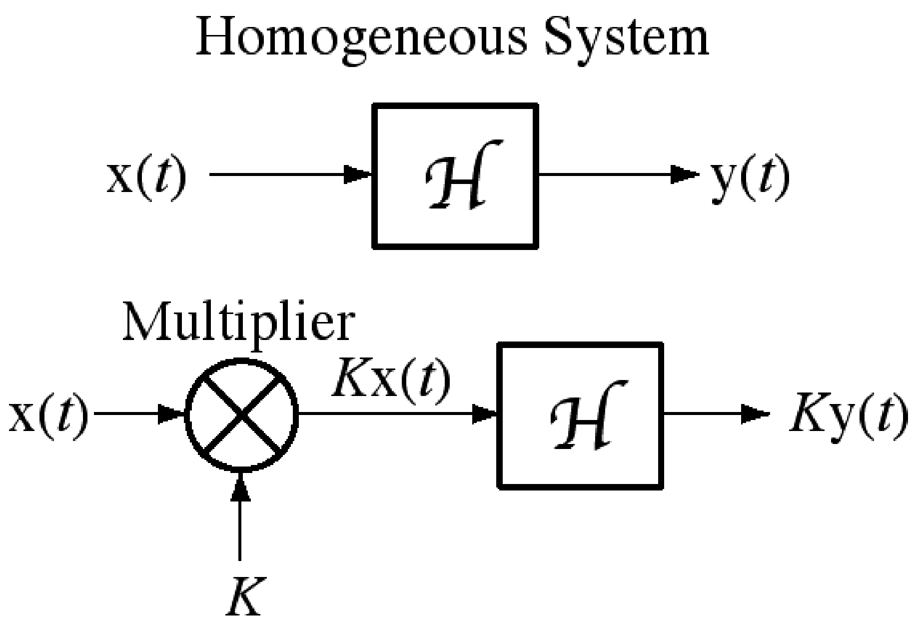
$$y[n] = \left[5 - 4\left(\frac{4}{5}\right)^n\right] u[n]$$



If the excitation is doubled, the response doubles.

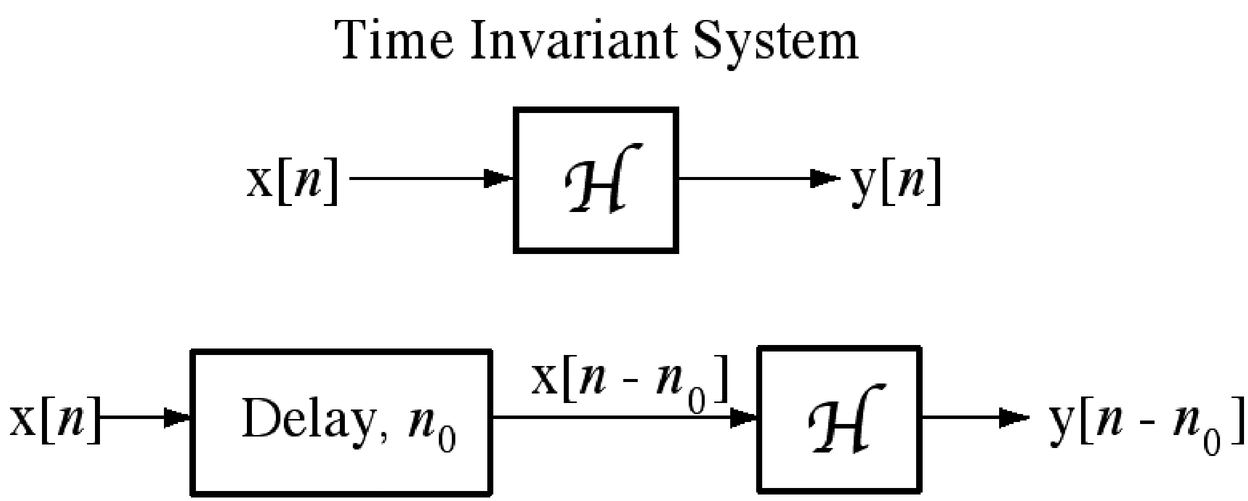
Homogeneity

- In a *homogeneous* system, multiplying the excitation by any constant (including *complex* constants), multiplies the response by the same constant.



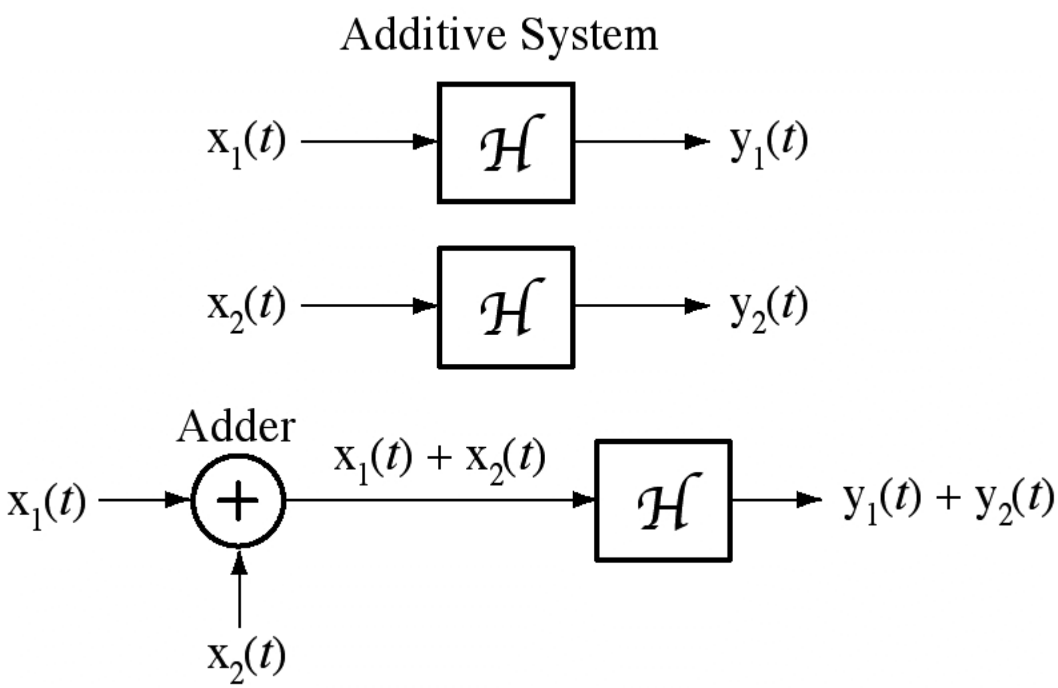
Time Invariance

- If an excitation causes a response and delaying the excitation simply delays the response by the same amount of time, regardless of the amount of delay, then the system is *time invariant*



Additivity

- If one excitation causes a response and another excitation causes another response and if, for any arbitrary excitations, the sum of the two excitations causes a response which is the sum of the two responses, the system is said to be *additive*



Linearity and LTI Systems

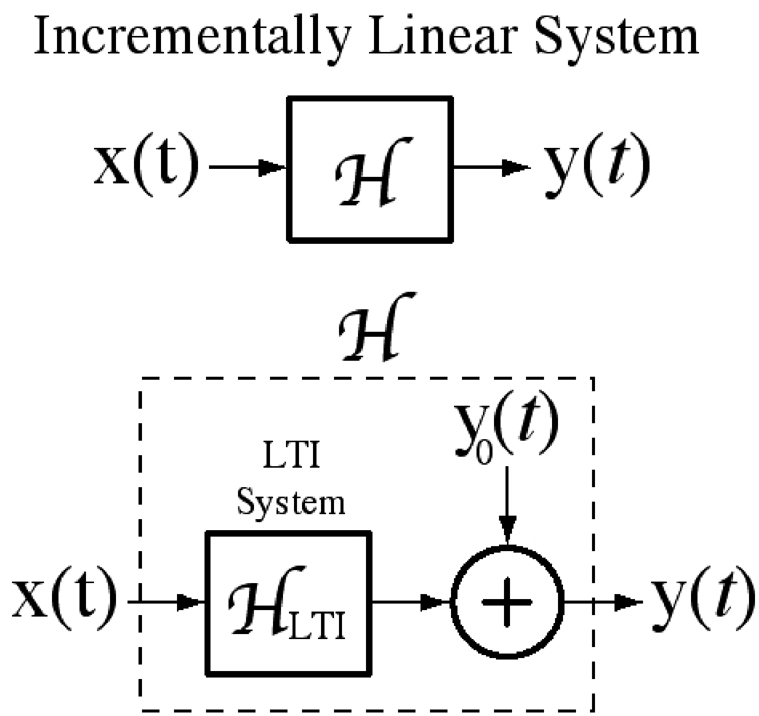
- If a system is both homogeneous and additive it is *linear*.
- If a system is both linear and time-invariant it is called an *LTI* system
- Some systems which are non-linear can be accurately approximated for analytical purposes by a linear system for small excitations

Stability

- Any system for which the response is bounded for any arbitrary bounded excitation, is called a *bounded-input-bounded-output* (BIBO) stable system

Incremental Linearity

- If a system can be modeled as a linear system with an offset added to its response it is called an incrementally linear system



Causality

- Any system for which the response occurs only during or after the time in which the excitation is applied is called a *causal* system.
- Strictly speaking, all real physical systems are causal

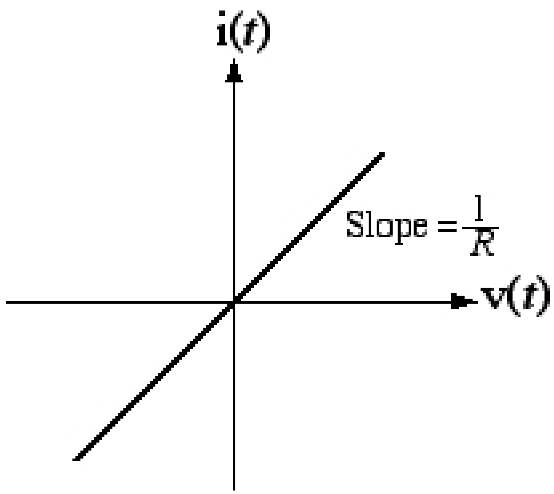
Memory

- If a system's response at any arbitrary time depends only on the excitation at that same time and not on the excitation or response at any other time is called a *static* system and is said to have no *memory*
- A system whose response at some arbitrary time does depend on the excitation or response at another time is called a *dynamic* system and is said to have memory

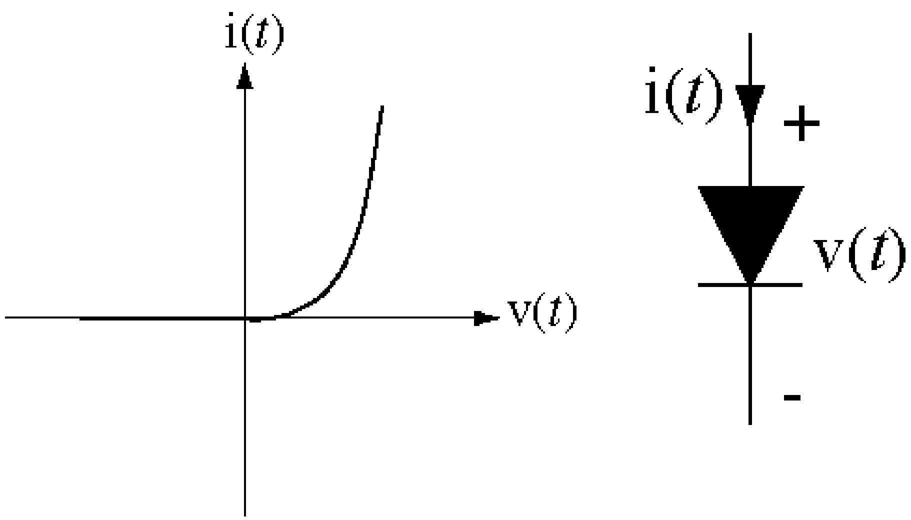
Static Non-Linearity

- Many real systems are non-linear because the relationship between excitation amplitude and response amplitude is non-linear

V-I Diagram for a Linear Resistor

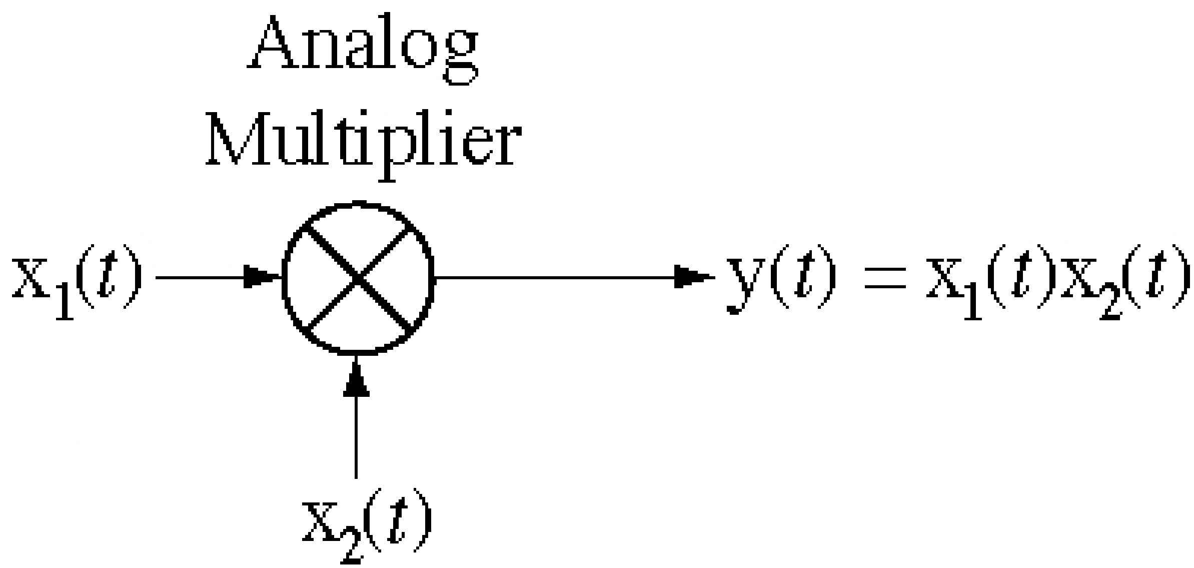


V-I Diagram for a Diode



Static Non-Linearity

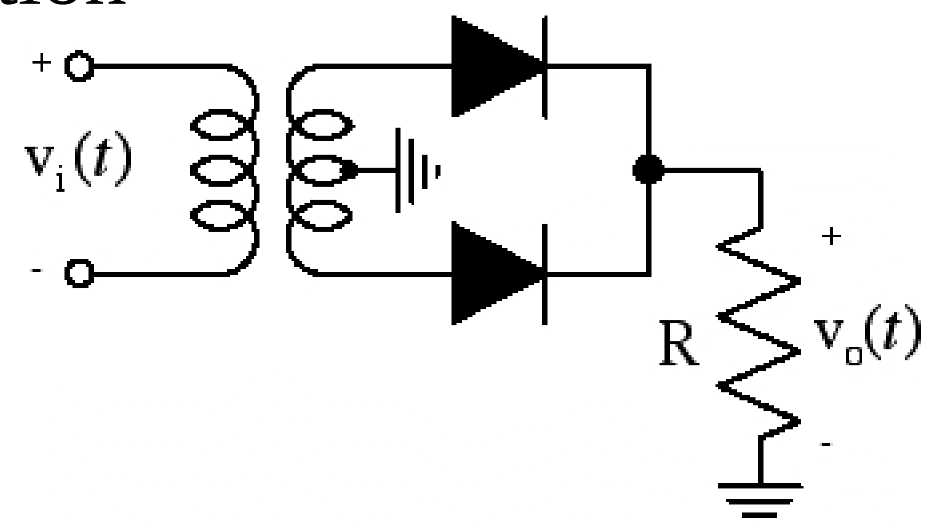
- In the analog multiplier below, if the two excitations are the same single excitation signal, the response signal is the square of that single excitation signal
- In that case, doubling the excitation would cause the response to increase by a factor of 4
- Such a system is not homogeneous and therefore not linear



Invertibility

- A system is said to be invertible if unique excitations produce unique responses.
- In other words, if a system is invertible, knowledge of the response is sufficient to determine the excitation

This full-wave rectifier is a non-invertible system



Eigenfunctions of LTI Systems

- The eigenfunction of an LTI system is the complex exponential
- The eigenvalues are either real or, if complex, occur in complex conjugate pairs
- Any LTI system excited by a complex sinusoid responds with another complex sinusoid of the same frequency, but generally a different amplitude and phase
- All these statements are true of both CT and DT systems

Impulse Response of DT Systems

Discrete-time LTI systems are described mathematically by difference equations of the form,

$$a_n \mathrm{y}[n] + a_{n-1} \mathrm{y}[n-1] + \cdots + a_{n-D} \mathrm{y}[n-D]$$
$$= b_n \mathrm{x}[n] + b_{n-1} \mathrm{x}[n-1] + \cdots + b_{n-N} \mathrm{x}[n-N]$$

For any excitation, x[n], the response, y[n], can be found by finding the response to x[n] as the only forcing function on the right-hand side and then adding scaled and time-shifted versions of that response to form y[n].

If x[n] is a unit impulse, the response to it as the only forcing function is simply the homogeneous solution of the difference equation with initial conditions applied. The impulse response is conventionally designated by the symbol, h[n].

Impulse Response of DT Systems

Since the impulse is applied to the system at time, $n = 0$, and that is the *only* excitation of the system, the impulse response is zero before time, $n = 0$.

$$\mathrm{h}[n] = 0 \quad , \quad n < 0$$

After time $n = 0$, the impulse has come and gone and the excitation is again zero. Therefore for $n > 0$, the solution of the difference equation describing the system is the homogeneous solution.

$$\mathrm{h}[n] = \mathrm{y}_h[n] \quad , \quad n > 0$$

Therefore, the impulse response is of the form,

$$\mathrm{h}[n] = \mathrm{y}_h[n]\mathrm{u}[n]$$

Impulse Response Example

Let a DT system be described by

$$3\,\mathrm{y}[n] + 2\,\mathrm{y}[n-1] + \mathrm{y}[n-2] = \mathrm{x}[n]$$

Impulse $\delta[n]$

The eigenfunction is the DT complex exponential, α^n

Substituting into the homogeneous difference equation,

$$3\alpha^n + 2\alpha^{n-1} + \alpha^{n-2} = 0$$

Dividing through by α^{n-2}

$$3\alpha^2 + 2\alpha + 1 = 0$$

Solving, $\alpha = -0.333 \pm j0.4714$

Impulse Response Example

The homogeneous solution is then of the form,

$$\mathrm{h}[n] = K_1(-0.333 + j0.4714)^n + K_2(-0.333 - j0.4714)^n$$

The constants can be found be applying initial conditions.
For the case of unit impulse excitation at time, $n = 0$,

$$3\mathrm{h}[0] + 2\underbrace{\mathrm{h}[0-1]}_{=0} + \underbrace{\mathrm{h}[0-2]}_{=0} = \mathrm{x}[0] = 1 \Rightarrow \mathrm{h}[0] = \frac{1}{3}$$

$$3\mathrm{h}[1] + 2\underbrace{\mathrm{h}[1-1]}_{=\frac{1}{3}} + \underbrace{\mathrm{h}[1-2]}_{=0} = \mathrm{x}[1] = 0 \Rightarrow \mathrm{h}[1] = -\frac{2}{9}$$

$$\mathrm{h}[0] = K_1(-0.333 + j0.4714)^0 + K_2(-0.333 - j0.4714)^0 = K_1 + K_2 = \frac{1}{3}$$

$$\mathrm{h}[1] = K_1(-0.333 + j0.4714) + K_2(-0.333 - j0.4714) = -\frac{2}{9}$$

$$K_1 = 0.1665 + j0.1181 \quad , \quad K_2 = 0.1665 - j0.1181$$

Impulse Response Example

The impulse response is then

$$\mathrm{h}[n]=\begin{bmatrix}(0.1665+j0.1181)(-0.333+j0.4714)^n\\+(0.1665-j0.1181)(-0.333-j0.4714)^n\end{bmatrix}\mathrm{u}[n]$$

which can also be written in the forms,

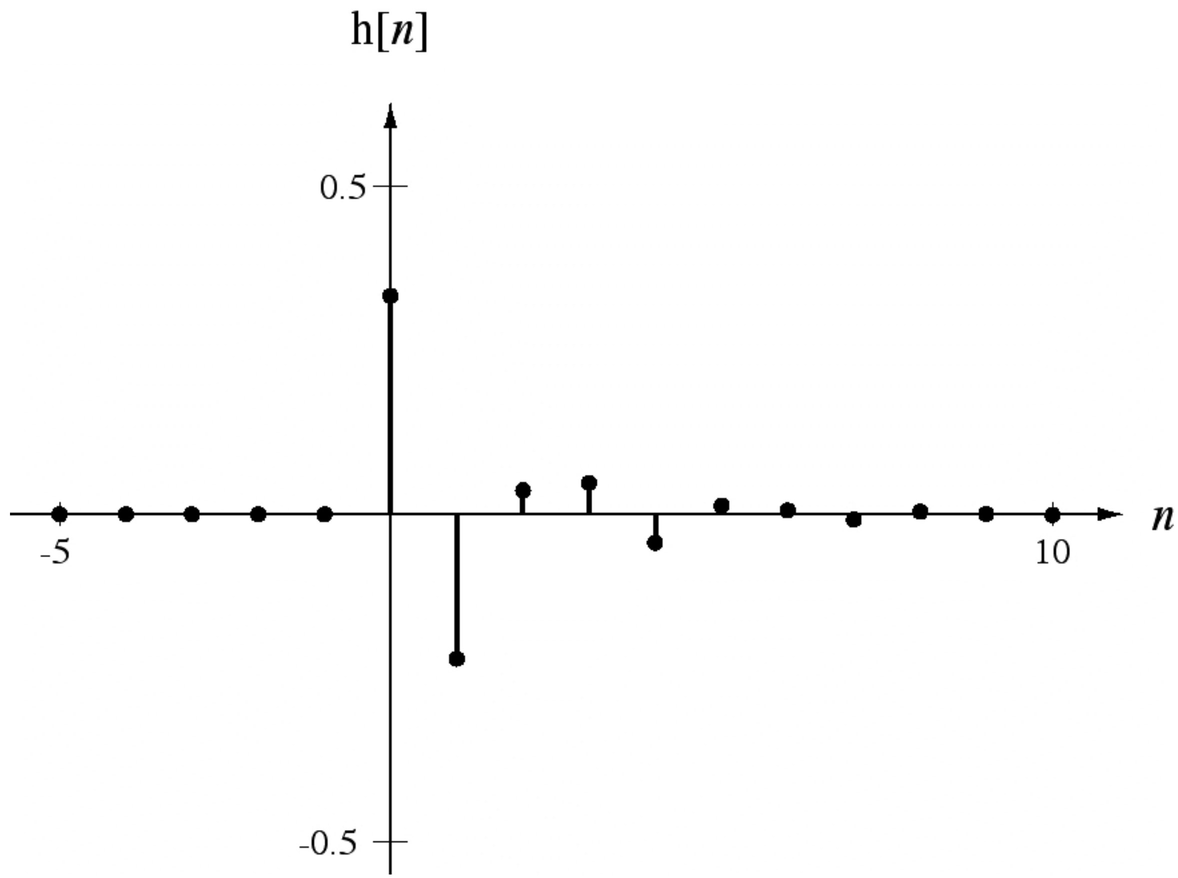
$$\mathrm{h}[n]=(0.5722)^n\begin{bmatrix}(0.1665+j0.1181)e^{j2.1858n}\\+(0.1665-j0.1181)e^{-j2.1858n}\end{bmatrix}\mathrm{u}[n]$$

$$\mathrm{h}[n]=(0.5722)^n\begin{bmatrix}0.1665\left(e^{j2.1858n}+e^{-j2.1858n}\right)\\+j0.1181\left(e^{j2.1858n}-e^{-j2.1858n}\right)\end{bmatrix}\mathrm{u}[n]$$

$$\mathrm{h}[n]=(0.5722)^n[0.333\cos(2.1858n)-0.2362\sin(2.1858n)]\mathrm{u}[n]$$

$$\mathrm{h}[n]=0.4083(0.5722)^n\cos(2.1858n+0.6169)$$

Impulse Response Example

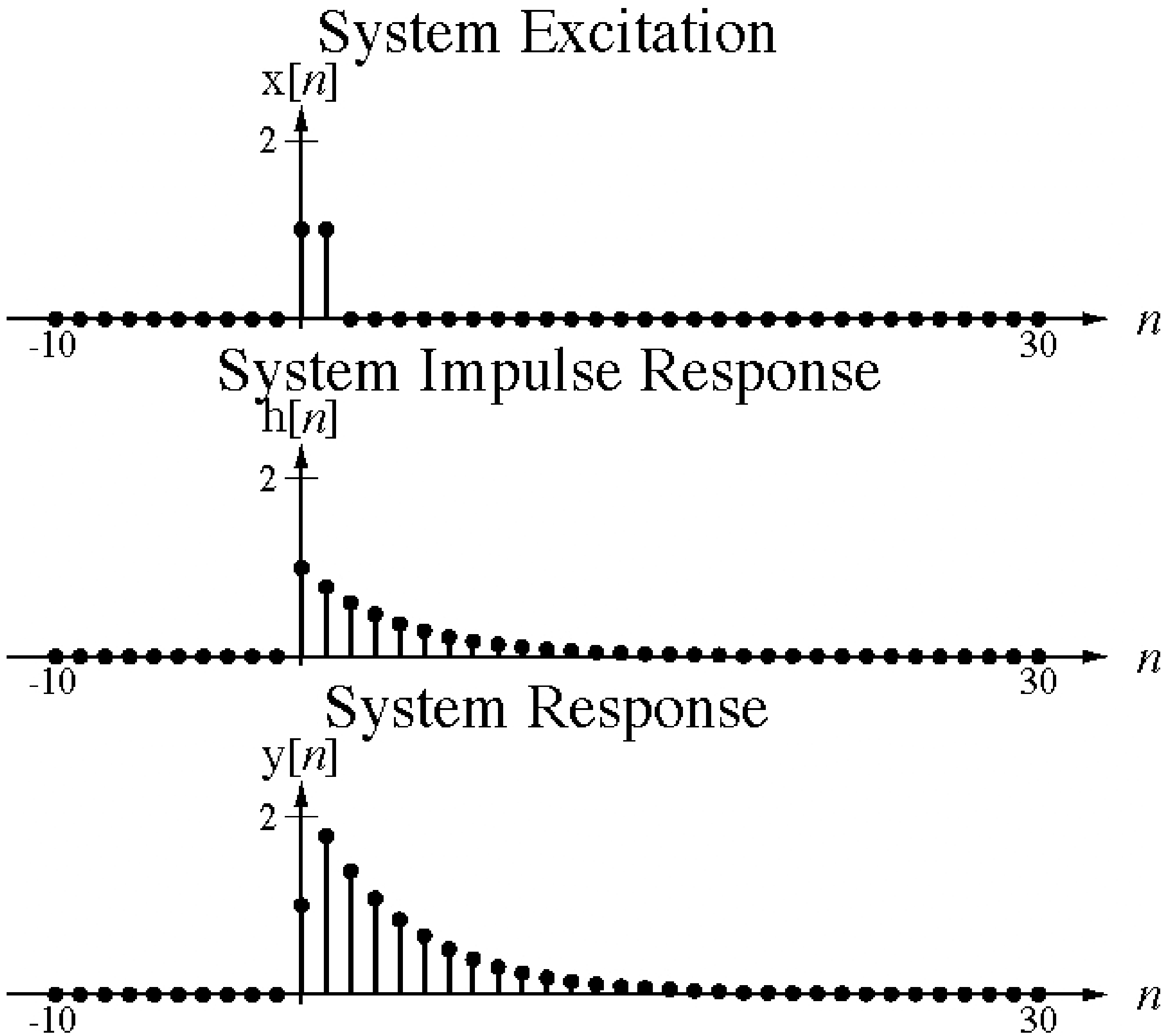


$$h[n] = 0.4083(0.5722)^n \cos(2.1858n + 0.6169)$$

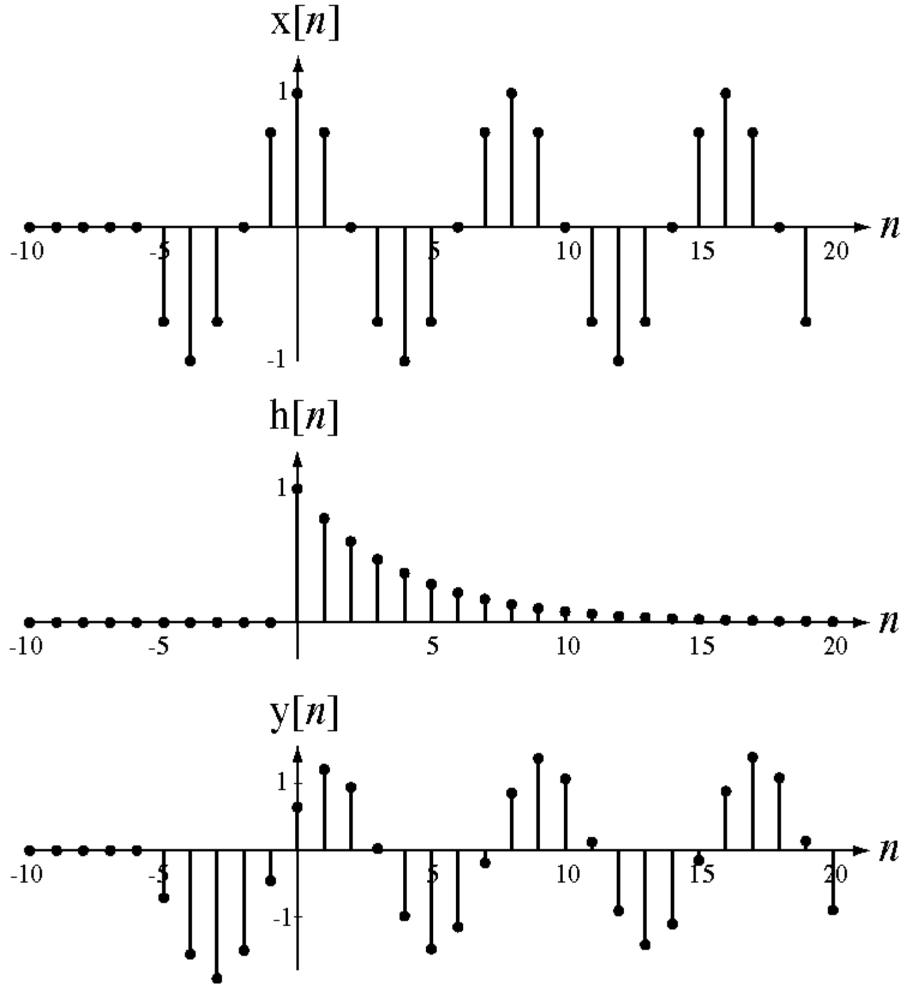
System Response

- Once the response to a unit impulse is known, the response of any discrete-time LTI system to any arbitrary excitation can be found
- Any arbitrary excitation is simply a sequence of amplitude-scaled and time-shifted DT impulses
- Therefore the response is simply a sequence of amplitude-scaled and time-shifted DT impulse *responses*

Simple System Response Example



More Complicated System Response Example



System Excitation

System Impulse Response

System Response

The Convolution Sum

The response, y[n], to an arbitrary excitation, x[n], is of the form,

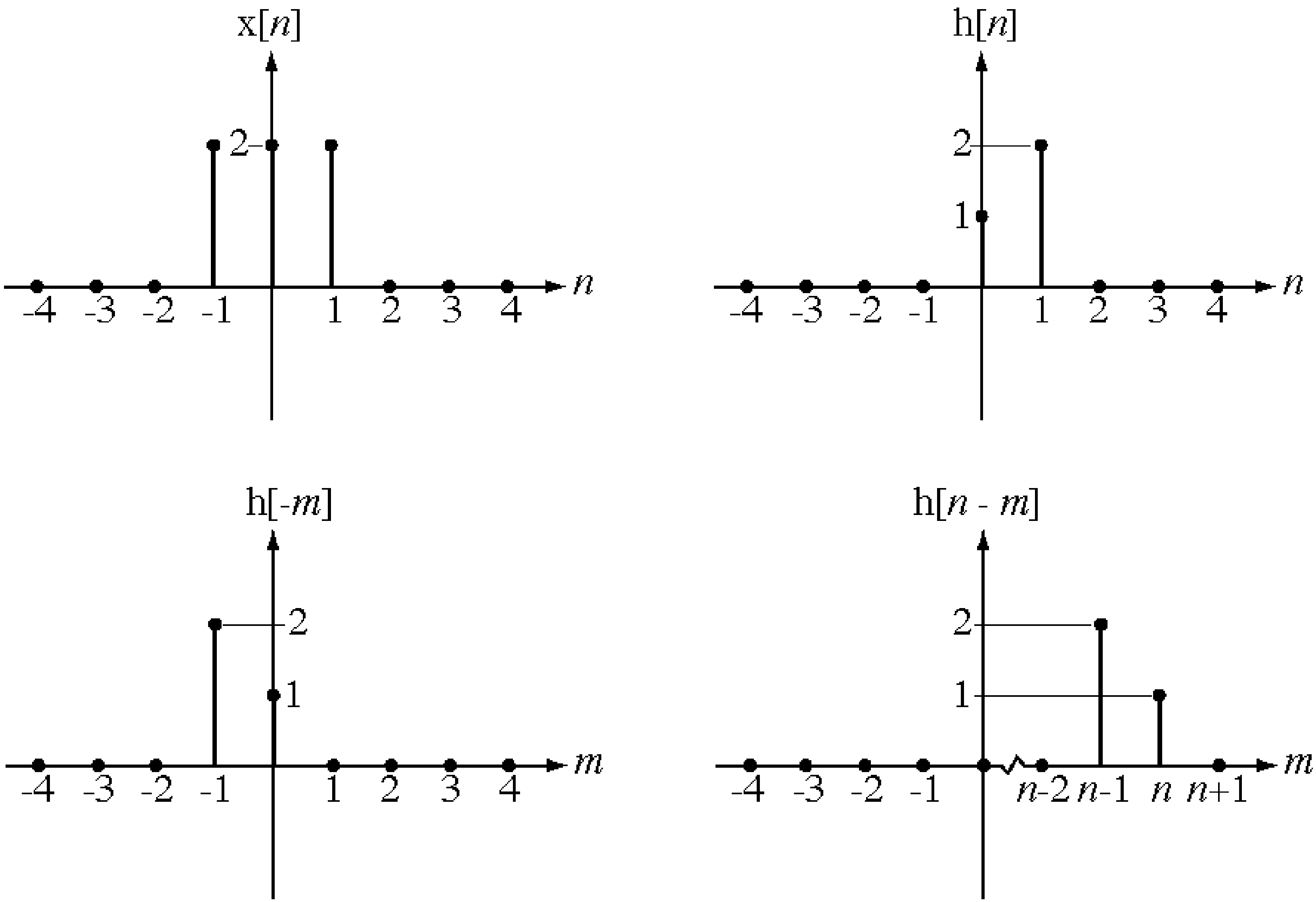
$$\mathrm{y}[n] = \cdots \mathrm{x}[-1]\mathrm{h}[n+1] + \mathrm{x}[0]\mathrm{h}[n] + \mathrm{x}[1]\mathrm{h}[n-1] + \cdots$$

where h[n] is the impulse response. This can be written in a more compact form,

$$\mathrm{y}[n] = \sum_{m=-\infty}^{\infty} \mathrm{x}[m]\mathrm{h}[n-m]$$

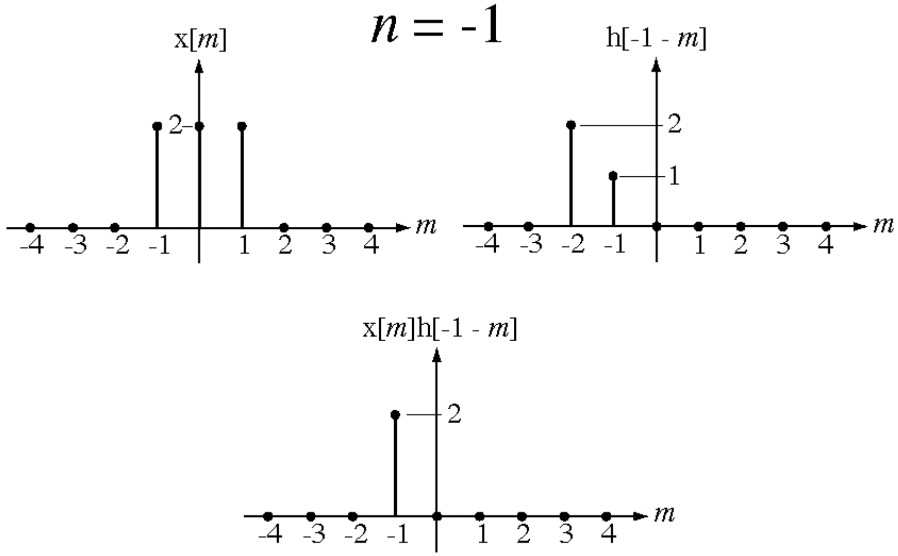
called the *convolution sum*.

A Convolution Sum Example



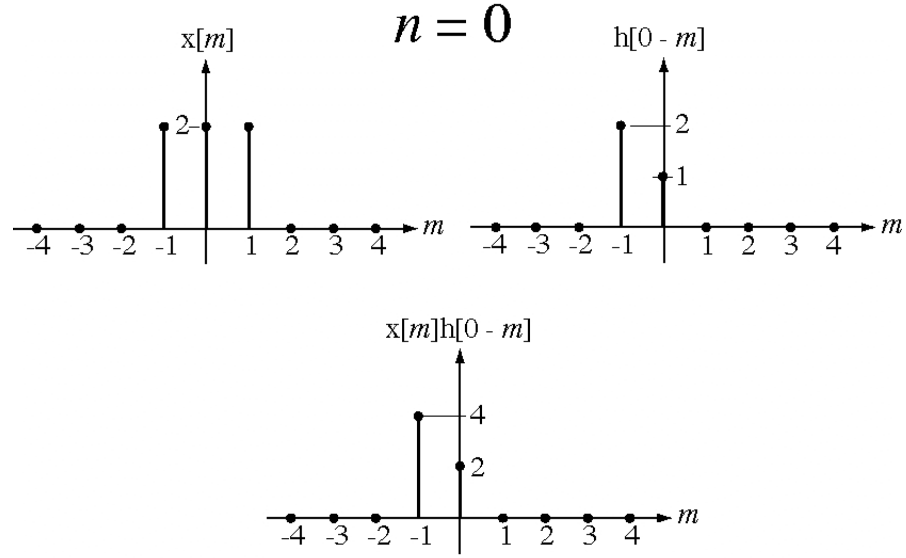
A Convolution Sum Example

$n = -1$



$y[-1] = 2$

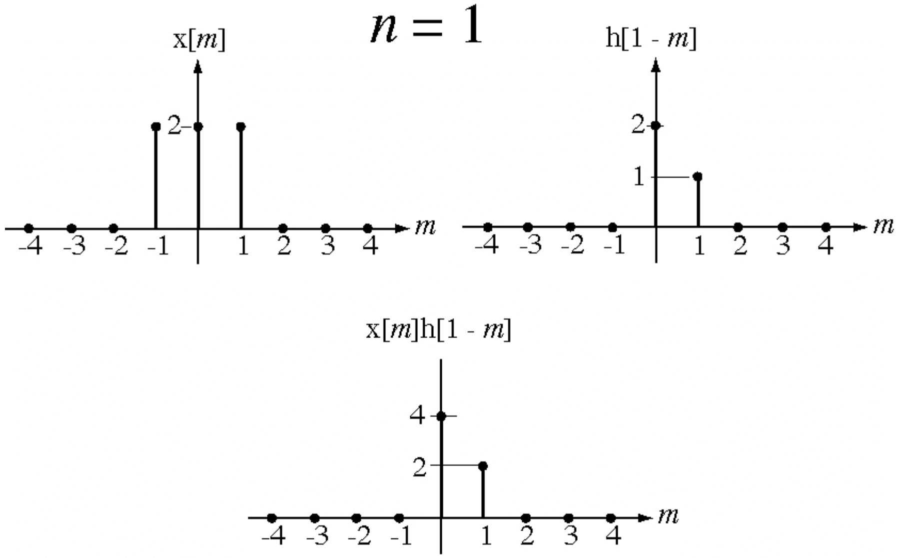
$n = 0$



$y[0] = 6$

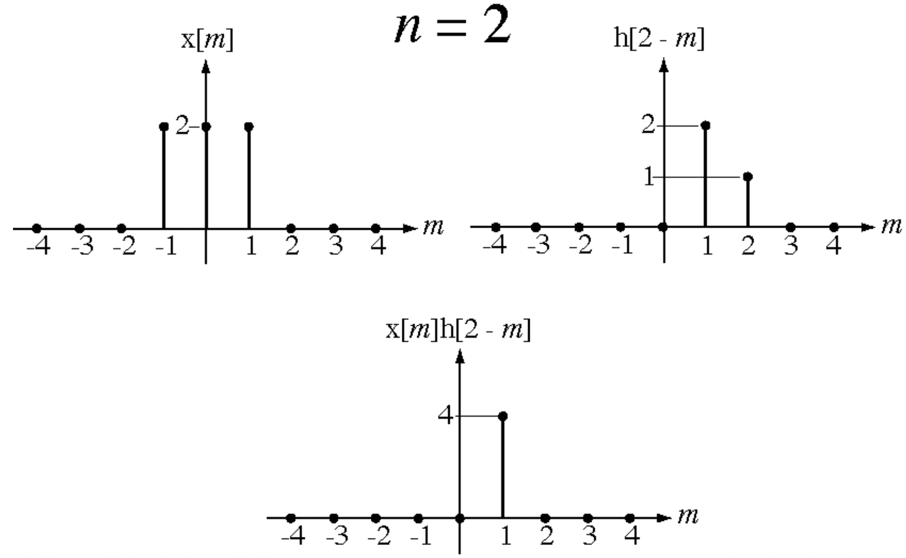
A Convolution Sum Example

$n = 1$



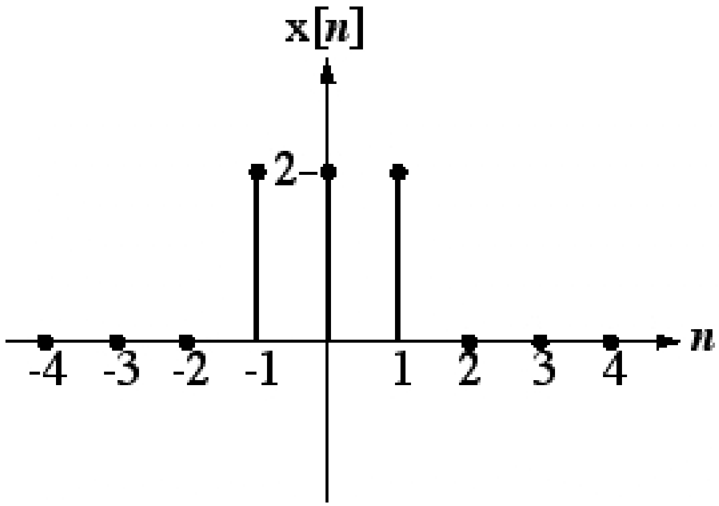
$y[1] = 6$

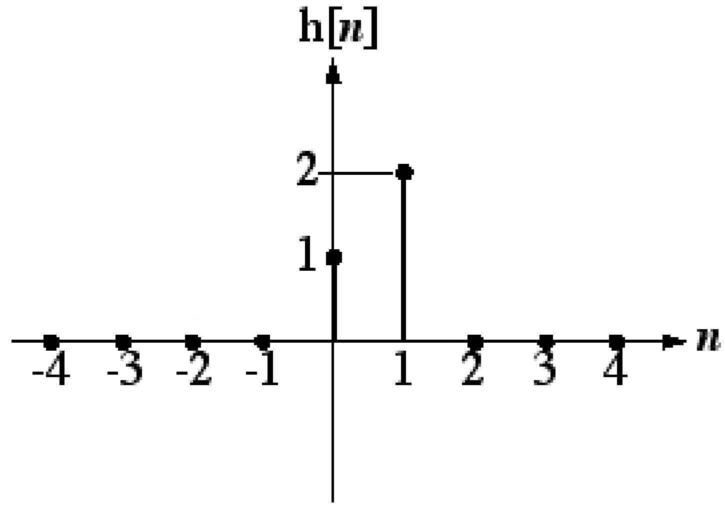
$n = 2$

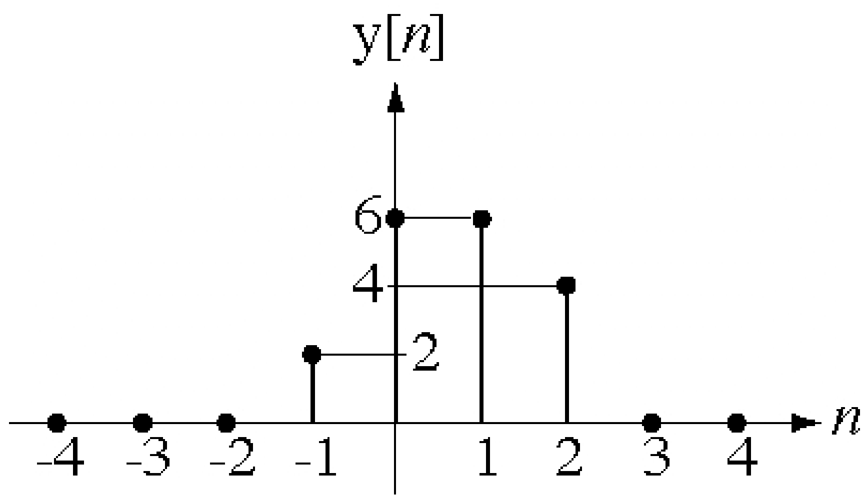


$y[2] = 4$

A Convolution Sum Example







Convolution Sum Properties

Discrete-time convolution is defined mathematically by

$$\mathrm{y}[n] = \mathrm{x}[n] * \mathrm{h}[n] = \sum_{m=-\infty}^{\infty} \mathrm{x}[m]\mathrm{h}[n-m]$$

The following properties can be proven from the definition.

$$\mathrm{x}[n] * A\delta[n-n_0] = A\,\mathrm{x}[n-n_0]$$

Let

$$\mathrm{y}[n] = \mathrm{x}[n] * \mathrm{h}[n]$$

then

$$\mathrm{y}[n-n_0] = \mathrm{x}[n] * \mathrm{h}[n-n_0] = \mathrm{x}[n-n_0] * \mathrm{h}[n]$$

$$\mathrm{y}[n] - \mathrm{y}[n-1] = \mathrm{x}[n] * \left(\mathrm{h}[n] - \mathrm{h}[n-1]\right) = \left(\mathrm{x}[n] - \mathrm{x}[n-1]\right) * \mathrm{h}[n]$$

and the sum of the impulse strengths in y is the product of the sum of the impulse strengths in x and the sum of the impulse strengths in h.

Convolution Sum Properties (continued)

Commutativity

$$\mathrm{x}[n] * \mathrm{y}[n] = \mathrm{y}[n] * \mathrm{x}[n]$$

Associativity

$$\left(\mathrm{x}[n] * \mathrm{y}[n]\right) * \mathrm{z}[n] = \mathrm{x}[n] * \left(\mathrm{y}[n] * \mathrm{z}[n]\right)$$

Distributivity

$$\left(\mathrm{x}[n] + \mathrm{y}[n]\right) * \mathrm{z}[n] = \mathrm{x}[n] * \mathrm{z}[n] + \mathrm{y}[n] * \mathrm{z}[n]$$

System Interconnections

If the response of one system is the excitation of another system the two systems are said to be *cascade* connected

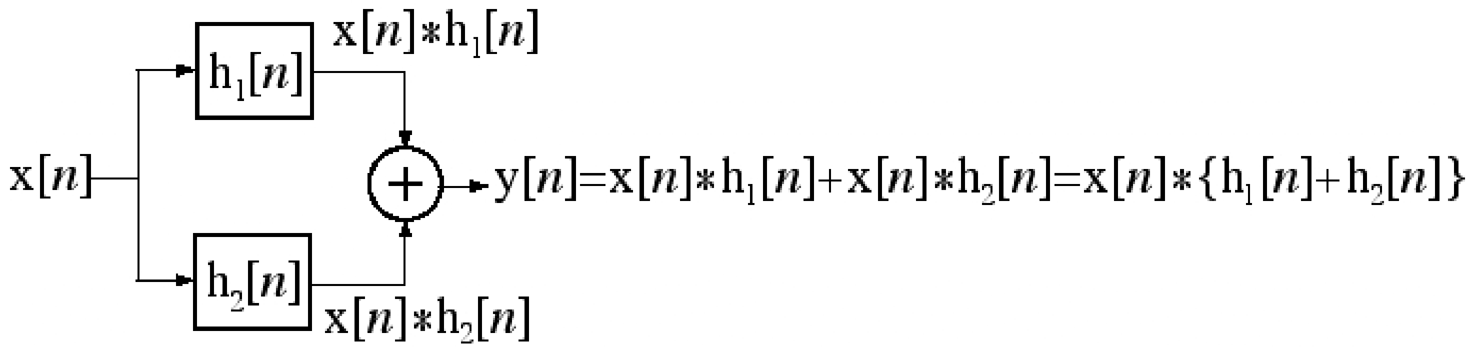
$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow x[n]*h_1[n] \longrightarrow \boxed{h_2[n]} \longrightarrow y[n]=\{x[n]*h_1[n]\}*h_2[n]$$

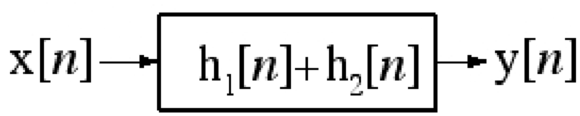
$$x[n] \longrightarrow \boxed{h_1[n]*h_2[n]} \longrightarrow y[n]$$

The cascade connection of two systems can be viewed as a single system whose impulse response is the convolution of the two individual system impulse responses. This is a direct consequence of the associativity property of convolution.

System Interconnections

If two systems are excited by the same signal and their responses are added they are said to be *parallel* connected.





The parallel connection of two systems can be viewed as a single system whose impulse response is the sum of the two individual system impulse responses. This is a direct consequence of the distributivity property of convolution.

Stability and Impulse Response

It can be shown that a BIBO-stable DT system has an impulse response that is absolutely summable. That is,

$$\sum_{n=-\infty}^{\infty} \left|\mathrm{h}[n]\right| \quad \text{is finite.}$$

Unit Impulse Response and Unit Sequence Response

In any discrete-time LTI system let an excitation, x[n], produce the response, y[n]. Then the excitation x[n] - x[n - 1] will produce the response y[n] - y[n - 1] .

It follows then that the unit impulse response is the first backward difference of the unit sequence response and, conversely that the unit sequence response is the accumulation of the unit impulse response.

Complex Exponential Response

Let a discrete-time LTI system be excited by a complex exponential of the form,

$$\mathrm{x}[n] = z^n$$

The response is the convolution of the excitation with the impulse response or

$$\mathrm{y}[n] = \sum_{m=-\infty}^{\infty} z^m \, \mathrm{h}[n-m] = \sum_{m=-\infty}^{\infty} z^{n-m} \, \mathrm{h}[m]$$

which can be written as

$$\mathrm{y}[n] = z^n \underbrace{\sum_{m=-\infty}^{\infty} \mathrm{h}[m] z^{-m}}_{\text{complex constant}}$$

Complex Exponential Response

The response of a discrete-time LTI system to a complex exponential excitation is another complex exponential of the same functional form but multiplied by a complex constant. That complex constant is

$$\sum_{n=-\infty}^{\infty} \mathrm{h}[n] z^{-n}$$

Later this will be called the *z transform* of the impulse response and will be one of the important transform methods.

Impulse Response of CT Systems

Example

Let a CT system be described by

$$a_2\,\mathrm{y}''(t) + a_1\,\mathrm{y}'(t) + a_0\,\mathrm{y}(t) = \mathrm{x}(t)$$

and let the excitation be a unit impulse at time, $t = 0$. Then the response, y, is the impulse response, h.

$$a_2\,\mathrm{h}''(t) + a_1\,\mathrm{h}'(t) + a_0\,\mathrm{h}(t) = \delta(t)$$

Since the impulse occurs at time, $t = 0$, and nothing has excited the system before that time, the impulse response before time, $t = 0$, is zero. After time, $t = 0$, the impulse has occurred and gone away. Therefore there is no excitation and the impulse response is the homogeneous solution of the differential equation.

Impulse Response of CT Systems

Example

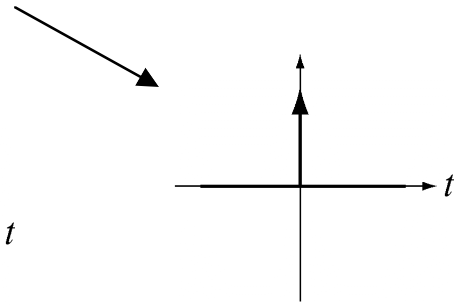
$$a_2 \, \mathrm{h}''(t) + a_1 \, \mathrm{h}'(t) + a_0 \, \mathrm{h}(t) = \delta(t)$$

What happens *at* time, $t = 0$? The equation must be satisfied at *all* times. So the left side of the equation must be a unit impulse. We already know that the left side is zero before time, $t = 0$ because the system has never been excited. We know that the left side is zero after time, $t = 0$, because it is the solution of the homogeneous equation whose right side is zero. These two facts are both consistent with an impulse. The impulse response *might* have in it an impulse or derivatives of an impulse since all of these occur only *at* time, $t = 0$. What the impulse response *does* have in it depends on the form of the differential equation.

Impulse Response of CT Systems

Example

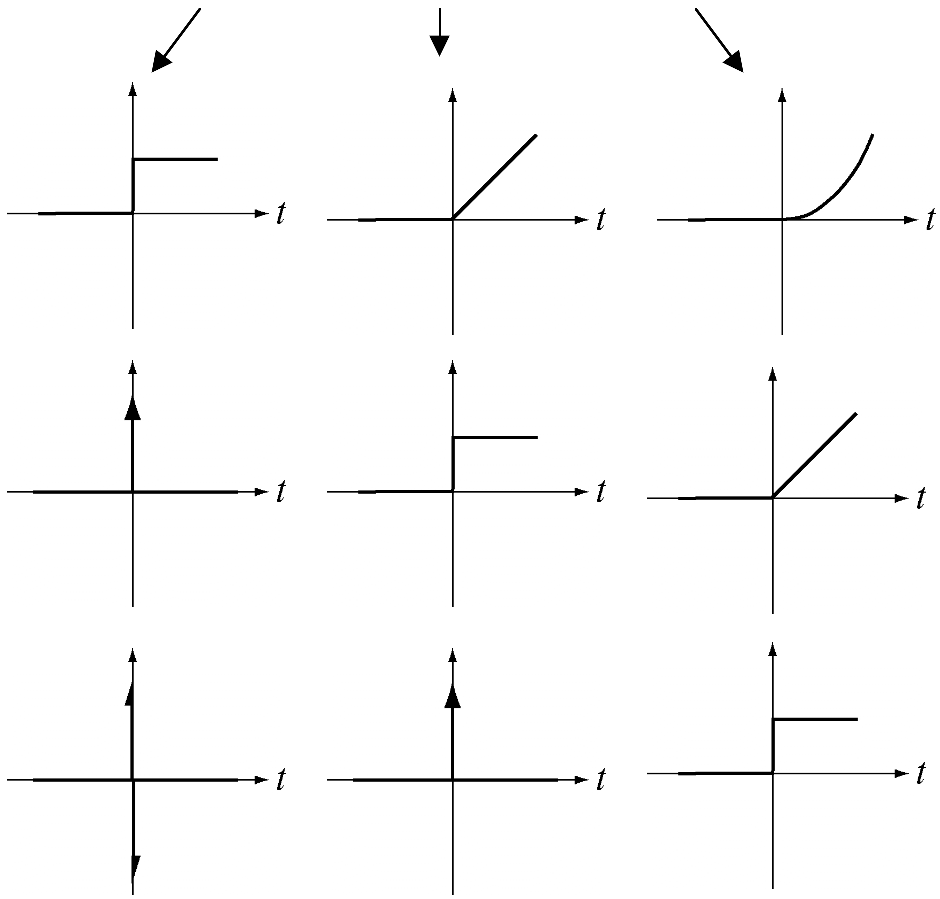
$$a_2\,\mathrm{h}''(t) + a_1\,\mathrm{h}'(t) + a_0\,\mathrm{h}(t) = \delta(t)$$



Won’t work

Will work

Won’t work



(These graphs are intended to depict the behavior of the impulse response and its derivatives in an infinitesimal interval centered at t = 0.)

Impulse Response of CT Systems

Continuous-time LTI systems are described by differential equations of the general form,

$$a_n \, \mathrm{y}^{(n)}(t) + a_{n-1} \, \mathrm{y}^{(n-1)}(t) + \cdots + a_1 \, \mathrm{y}'(t) + a_0 \, \mathrm{y}(t)$$
$$= b_m \, \mathrm{x}^{(m)}(t) + b_{m-1} \, \mathrm{x}^{(m-1)}(t) + \cdots + b_1 \, \mathrm{x}'(t) + b_0 \, \mathrm{x}(t)$$

For all times, $t < 0$:

If the excitation, x(t), is an impulse, then for all time, $t < 0$, it is zero. The response, y(t), is zero before time, $t = 0$, because there has never been an excitation before that time.

Impulse Response of CT Systems

For all time, $t > 0$:

The excitation is zero. The response is the homogeneous solution of the differential equation.

At time, $t = 0$:

The excitation is an impulse. In general it would be possible for the response to contain an impulse plus derivatives of an impulse because these all occur *at* time, $t = 0$ and are zero before and after that time. Whether or not the response contains an impulse or derivatives of an impulse at time, $t = 0$, depends on the form of the differential equation,

$$a_n \, \mathrm{y}^{(n)}(t) + a_{n-1} \, \mathrm{y}^{(n-1)}(t) + \cdots + a_1 \, \mathrm{y}'(t) + a_0 \, \mathrm{y}(t)$$
$$= b_m \, \mathrm{x}^{(m)}(t) + b_{m-1} \, \mathrm{x}^{(m-1)}(t) + \cdots + b_1 \, \mathrm{x}'(t) + b_0 \, \mathrm{x}(t)$$

Impulse Response of CT Systems

$$a_n\,\mathrm{y}^{(n)}(t)+a_{n-1}\,\mathrm{y}^{(n-1)}(t)+\cdots+a_1\,\mathrm{y}'(t)+a_0\,\mathrm{y}(t)$$
$$=b_m\,\mathrm{x}^{(m)}(t)+b_{m-1}\,\mathrm{x}^{(m-1)}(t)+\cdots+b_1\,\mathrm{x}'(t)+b_0\,\mathrm{x}(t)$$

Case 1: $m < n$

If the response contained an impulse at time, $t = 0$, then the nth derivative of the response would contain the nth derivative of an impulse. Since the excitation contains only the mth derivative of an impulse and $m < n$, the differential equation cannot be satisfied at time, $t = 0$. Therefore the response cannot contain an impulse or any derivatives of an impulse.

Impulse Response of CT Systems

$$a_n \mathrm{y}^{(n)}(t) + a_{n-1} \mathrm{y}^{(n-1)}(t) + \cdots + a_1 \mathrm{y}'(t) + a_0 \mathrm{y}(t)$$
$$= b_m \mathrm{x}^{(m)}(t) + b_{m-1} \mathrm{x}^{(m-1)}(t) + \cdots + b_1 \mathrm{x}'(t) + b_0 \mathrm{x}(t)$$

Case 2: $m = n$

In this case the highest derivative of the excitation and response are the same and the response could contain an impulse at time, $t = 0$, but no derivatives of an impulse.

Case 3: $m > n$

In this case, the response could contain an impulse at time, $t = 0$, plus derivatives of an impulse up to the (m - n)th derivative.

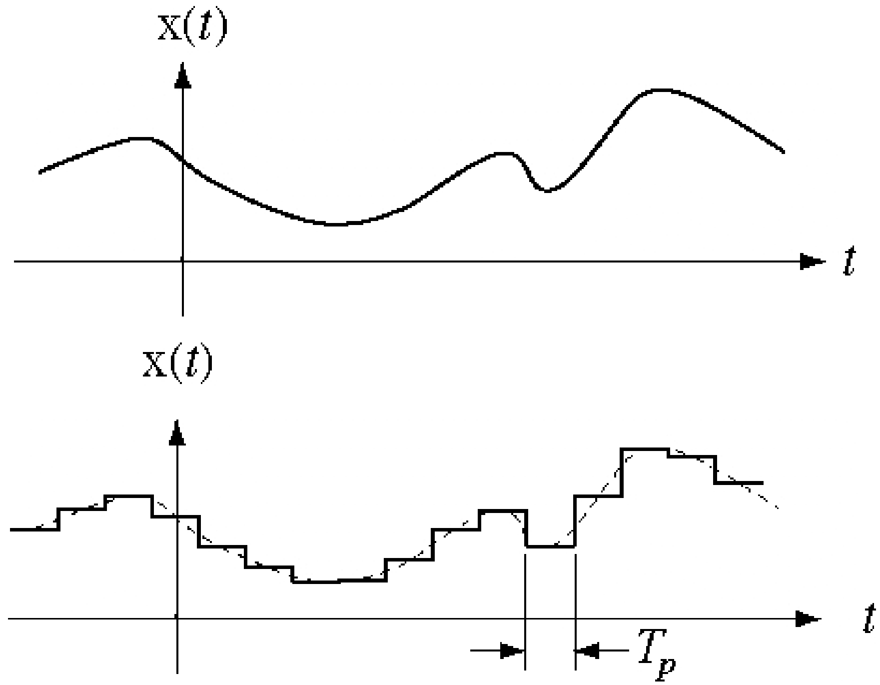
Cases 2 and 3 are rare in the analysis of real systems.

The Convolution Integral

If a continuous-time LTI system is excited by an arbitrary excitation, the response could be found approximately by approximating the excitation as a sequence of contiguous rectangular pulses of width, T_p .

Exact Excitation

Approximate Excitation



The Convolution Integral

Approximating the excitation as a pulse train can be expressed mathematically by

$$\mathrm{x}(t) \cong \cdots + \mathrm{x}(-T_p)\,\mathrm{rect}\left(\frac{t+T_p}{T_p}\right) + \mathrm{x}(0)\,\mathrm{rect}\left(\frac{t}{T_p}\right) + \mathrm{x}(T_p)\,\mathrm{rect}\left(\frac{t-T_p}{T_p}\right) + \cdots$$

or

$$\mathrm{x}(t) \cong \sum_{n=-\infty}^{\infty} \mathrm{x}(nT_p)\,\mathrm{rect}\left(\frac{t-nT_p}{T_p}\right)$$

The excitation can be written in terms of pulses of width, T_p, and unit area as

$$\mathrm{x}(t) \cong \sum_{n=-\infty}^{\infty} T_p\,\mathrm{x}(nT_p)\underbrace{\frac{1}{T_p}\,\mathrm{rect}\left(\frac{t-nT_p}{T_p}\right)}_{\text{shifted unit}-\text{area pulse}}$$

The Convolution Integral

Let the response to an unshifted pulse of unit area and width, T_p be the "unit pulse response",

$$\mathrm{h}_p(t)$$

Then, invoking linearity, the response to the overall excitation is (approximately) a sum of shifted and scaled unit pulse responses of the form,

$$\mathrm{y}(t) \cong \sum_{n=-\infty}^{\infty} T_p \,\mathrm{x}(nT_p)\,\mathrm{h}_p(t - nT_p)$$

As T_p approaches zero, the unit pulses become unit *impulses*, the unit pulse response becomes the unit *impulse* response, h(t), and the excitation and response become exact.

The Convolution Integral

As T_p approaches zero, the expressions for the approximate excitation and response approach the limiting exact forms,

Superposition Integral

$$\mathrm{x}(t) = \int_{-\infty}^{\infty} \mathrm{x}(\tau)\delta(t-\tau)\,d\tau$$

Convolution Integral

$$\mathrm{y}(t) = \int_{-\infty}^{\infty} \mathrm{x}(\tau)\mathrm{h}(t-\tau)\,d\tau$$

Notice the similarity of the forms of the convolution integral for CT systems and the convolution sum for DT systems,

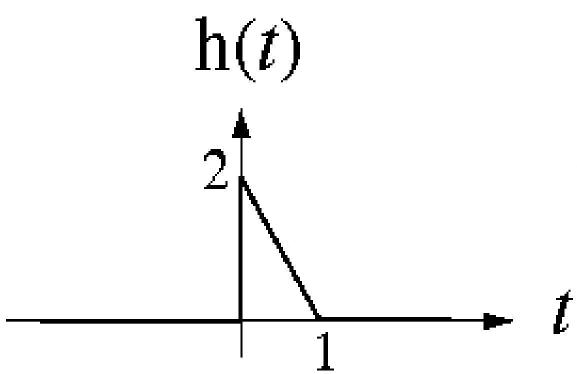
$$\mathrm{y}(t) = \int_{-\infty}^{\infty} \mathrm{x}(\tau)\mathrm{h}(t-\tau)\,d\tau \qquad \mathrm{y}[n] = \sum_{m=-\infty}^{\infty} \mathrm{x}[m]\mathrm{h}[n-m]$$

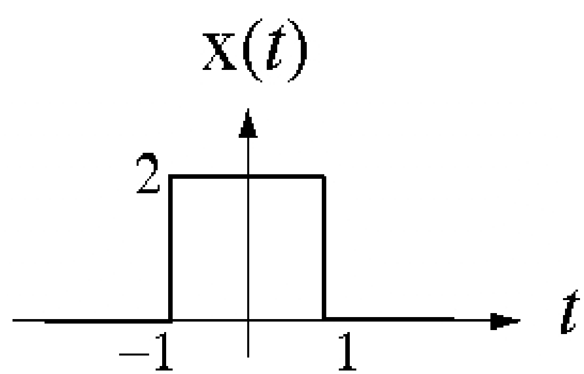
A Graphical Illustration of the Convolution Integral

The convolution integral is defined by

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

For illustration purposes let the excitation, x(t), and the impulse response, h(t), be the two functions below.

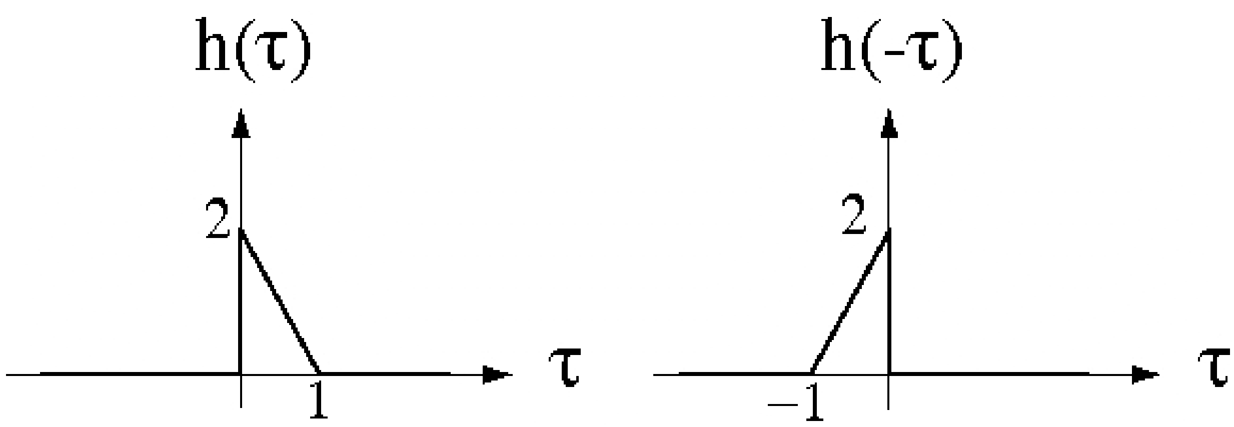




A Graphical Illustration of the Convolution Integral

In the convolution integral there is a factor, $\mathrm{h}(t-\tau)$

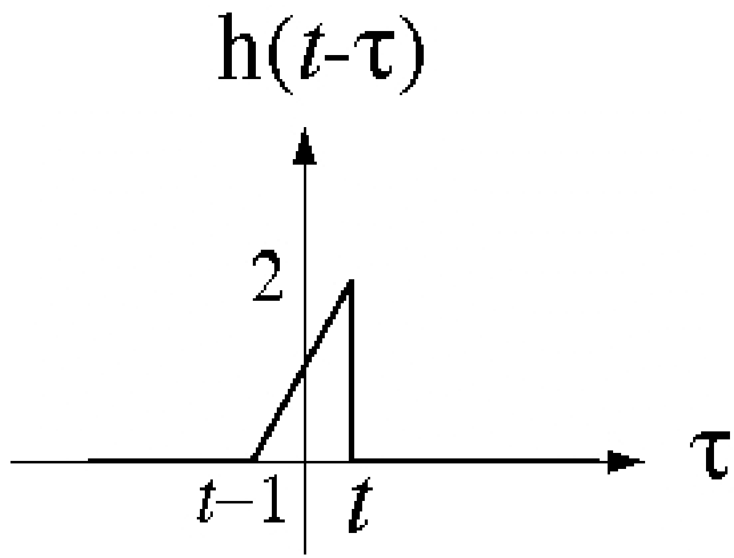
We can begin to visualize this quantity in the graphs below.



A Graphical Illustration of the Convolution Integral

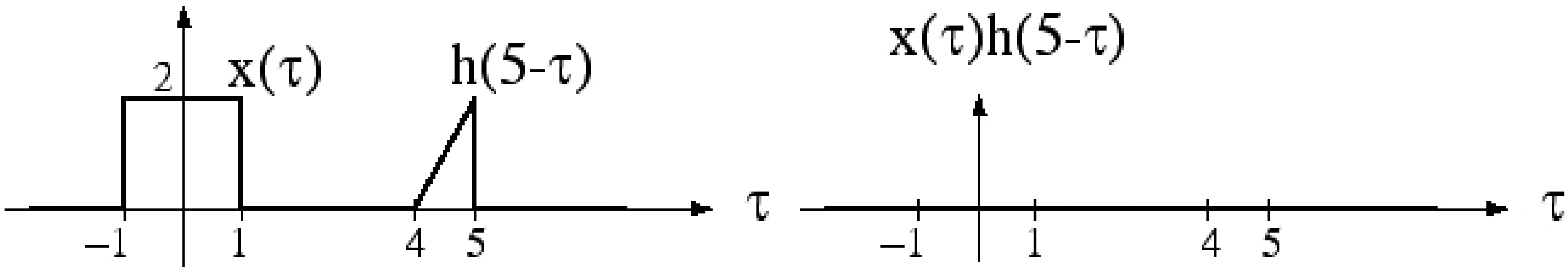
The functional transformation in going from h(τ) to h(*t* - τ) is

$$\mathrm{h}(\tau) \xrightarrow{\tau \to -\tau} \mathrm{h}(-\tau) \xrightarrow{\tau \to \tau - t} \mathrm{h}(-(\tau - t)) = \mathrm{h}(t - \tau)$$



A Graphical Illustration of the Convolution Integral

The convolution value is the area under the product of x(t) and h(t - τ). This area depends on what t is. First, as an example, let t = 5.



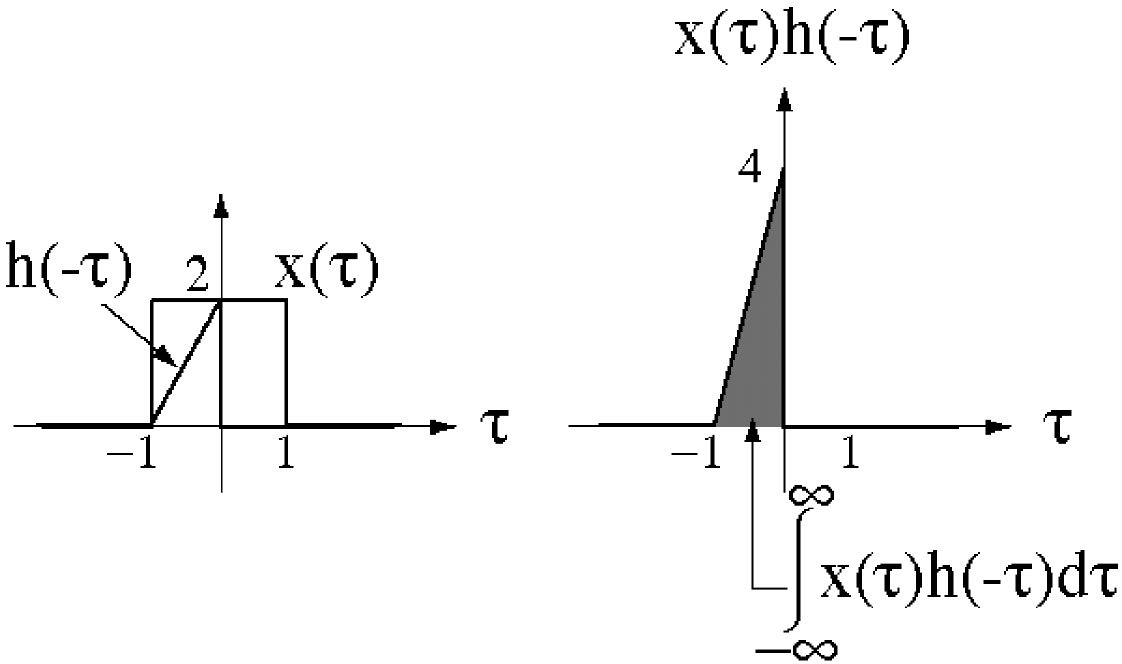
For this choice of t the area under the product is zero. Therefore if

$$y(t) = x(t) * h(t)$$

then y(5) = 0.

A Graphical Illustration of the Convolution Integral

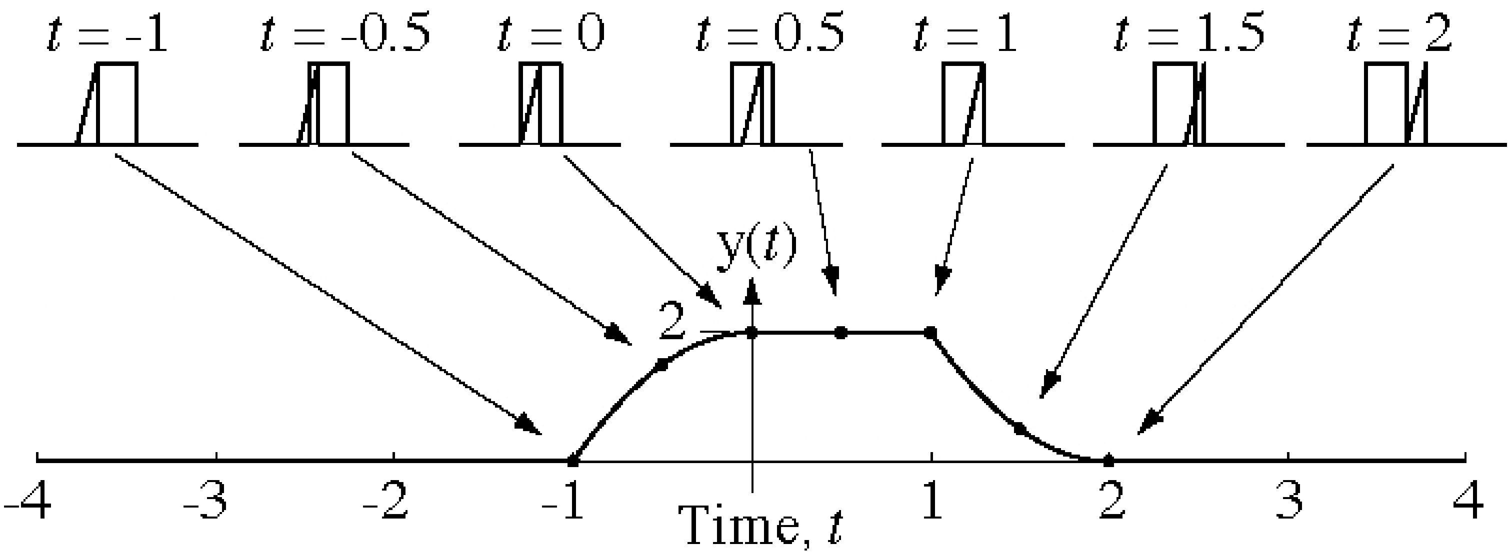
Now let t = 0.



Therefore y(0) = 2, the area under the product.

A Graphical Illustration of the Convolution Integral

The process of convolving to find y(t) is illustrated below.



Convolution Integral Properties

$$\mathrm{x}(t) * A\delta(t - t_0) = A\,\mathrm{x}(t - t_0)$$

$$\text{If } \mathrm{g}(t) = \mathrm{g}_0(t) * \delta(t) \text{ then } \mathrm{g}(t - t_0) = \mathrm{g}_0(t - t_0) * \delta(t) = \mathrm{g}_0(t) * \delta(t - t_0)$$

$$\text{If } \mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) \text{ then } \mathrm{y}'(t) = \mathrm{x}'(t) * \mathrm{h}(t) = \mathrm{x}(t) * \mathrm{h}'(t)$$

$$\text{and } \mathrm{y}(at) = |a|\,\mathrm{x}(at) * \mathrm{h}(at)$$

Commutativity

$$\mathrm{x}(t) * \mathrm{y}(t) = \mathrm{y}(t) * \mathrm{x}(t)$$

Associativity

$$[\mathrm{x}(t) * \mathrm{y}(t)] * \mathrm{z}(t) = \mathrm{x}(t) * [\mathrm{y}(t) * \mathrm{z}(t)]$$

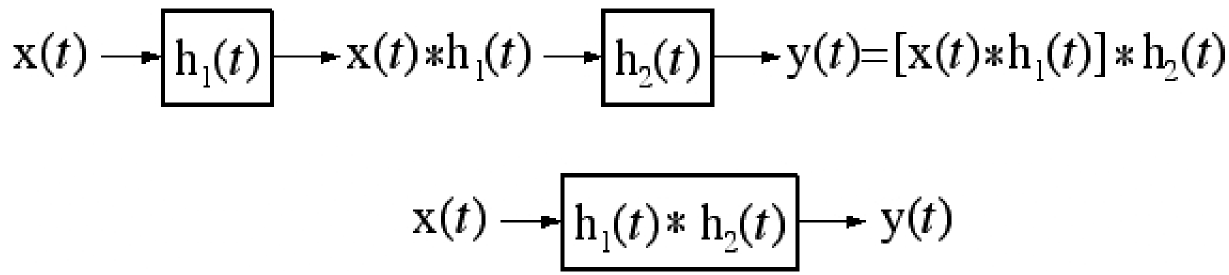
Distributivity

$$[\mathrm{x}(t) + \mathrm{y}(t)] * \mathrm{z}(t) = \mathrm{x}(t) * \mathrm{z}(t) + \mathrm{y}(t) * \mathrm{z}(t)$$

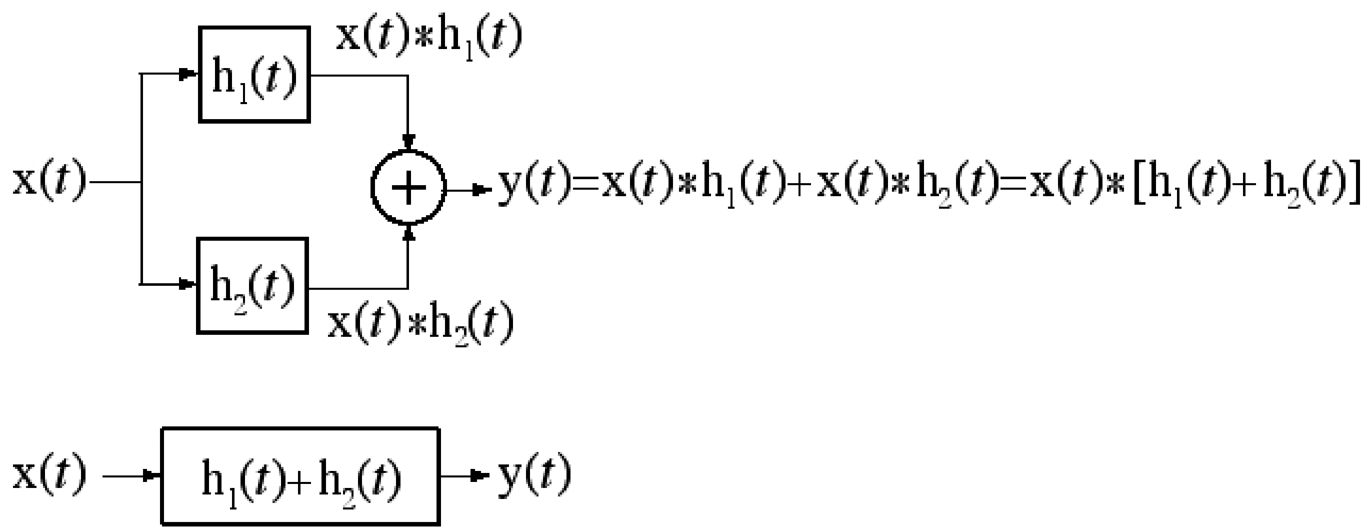
System Interconnections

The system-interconnection properties for CT systems are exactly the same as for DT systems.

Cascade Connection



Parallel Connection



Stability and Impulse Response

A CT system is BIBO stable if its impulse response is absolutely integrable. That is if

$$\int_{-\infty}^{\infty} \left|\mathrm{h}(t)\right| dt \quad \text{is finite.}$$

Unit Impulse Response and Unit Step Response

In any continuous-time LTI system let an excitation, $\mathrm{x}(t)$, produce the response, $\mathrm{y}(t)$. Then the excitation

$$\frac{d}{dt}\left(\mathrm{x}(t)\right)$$

will produce the response

$$\frac{d}{dt}\left(\mathrm{y}(t)\right)$$

It follows then that the unit impulse response is the first derivative of the unit step response and, conversely that the unit step response is the integral of the unit impulse response.

Complex Exponential Response

Let a continuous-time LTI system be excited by a complex exponential of the form,

$$\mathrm{x}(t) = e^{st}$$

The response is the convolution of the excitation with the impulse response or

$$\mathrm{y}(t) = \mathrm{h}(t) * e^{st} = \int_{-\infty}^{\infty} \mathrm{h}(\tau) e^{s(t-\tau)} d\tau = e^{st} \underbrace{\int_{-\infty}^{\infty} \mathrm{h}(\tau) e^{-s\tau} d\tau}_{\text{complex constant}}$$

The quantity, $\int_{-\infty}^{\infty} \mathrm{h}(\tau) e^{-s\tau} d\tau$, will later be designated the

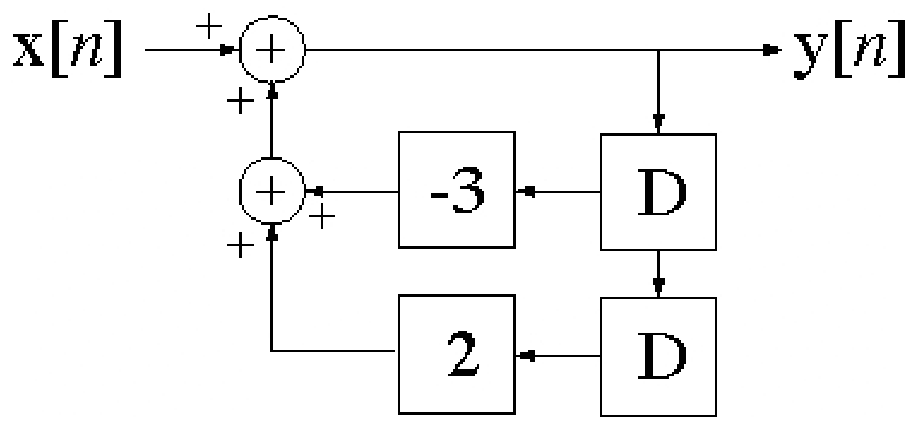
Laplace transform of the impulse response and will be an important transform method for CT system analysis.

Block Diagrams

A very useful method for describing and analyzing systems is the *block diagram*. A block diagram can be drawn directly from the difference or differential equation which describes the system. For example, if the system is described by

$$\mathrm{y}[n]+3\mathrm{y}[n-1]-2\mathrm{y}[n-2]=\mathrm{x}[n]$$

it can also be described by the block diagram below in which "D" represents a delay of one in discrete time.

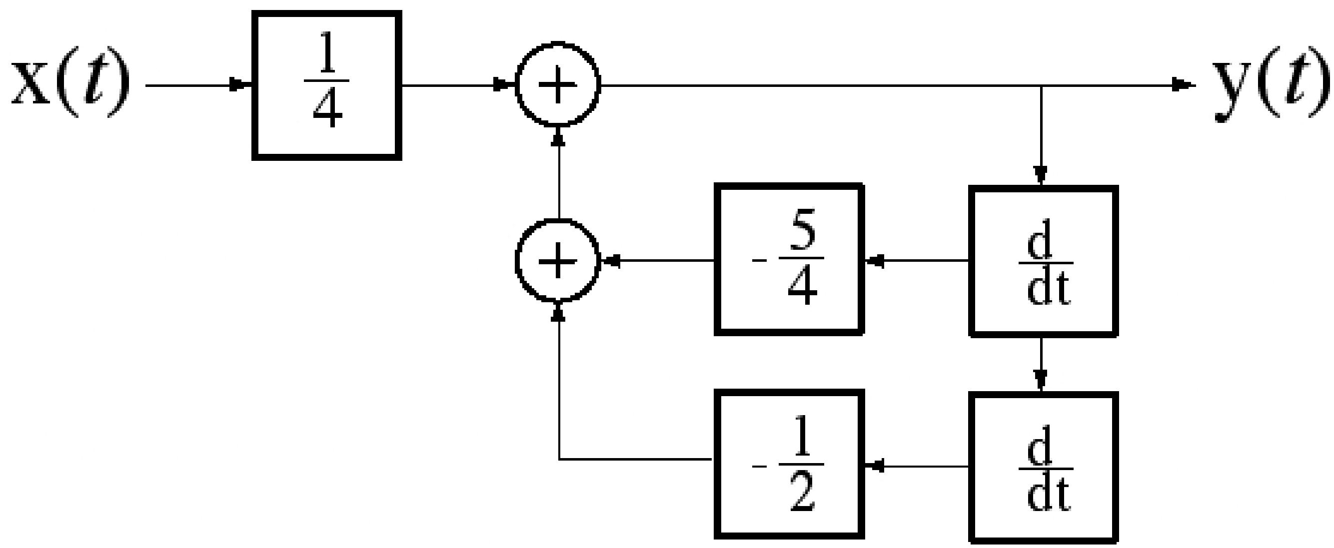


Block Diagrams

If a CT system is described by the differential equation,

$$2\,y''(t) + 5\,y'(t) + 4\,y(t) = x(t)$$

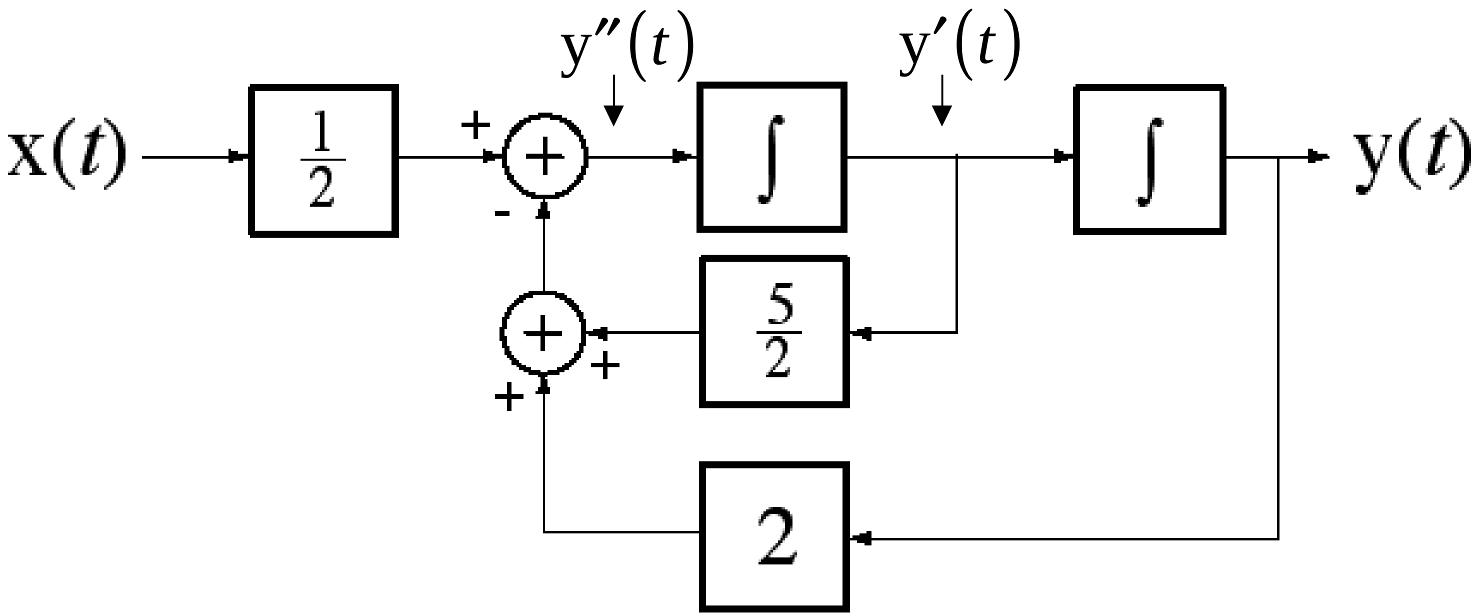
it can also be described by the block diagram below.



But this form of block diagram is not the preferred form.

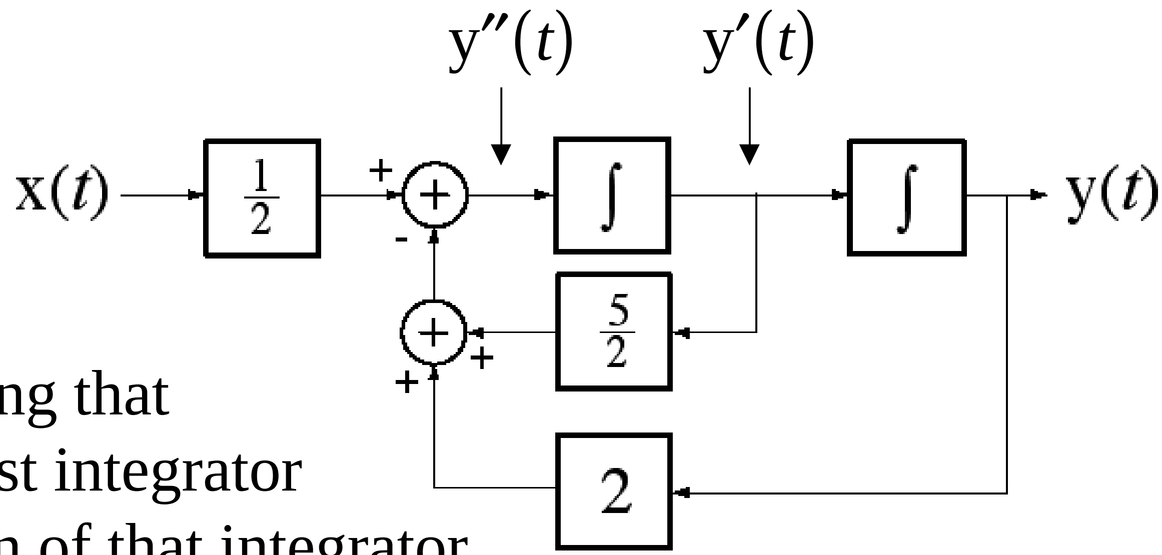
Block Diagrams

Although the previous block diagram is correct, it is not the preferred way of representing CT systems in block-diagram form. A preferred form is illustrated below.



This form is preferred because, as a practical matter, integrators are more desirable elements for an actual hardware simulation than differentiators.

Block Diagrams



This block diagram can be converted into the differentialequation of the system by realizing that if the response of the last integrator is y(t) that the excitation of that integrator must be $y'(t)$ and, similarly, the excitation of the previous integrator must be $y''(t)$. Then, describing the action of the summer, the differential equation is

$$y''(t) = \frac{1}{2}x(t) - \frac{5}{2}y'(t) - 2\,y(t) \Rightarrow 2\,y''(t) + 5\,y'(t) + 4\,y(t) = x(t)$$

and this is the original differential equation.

