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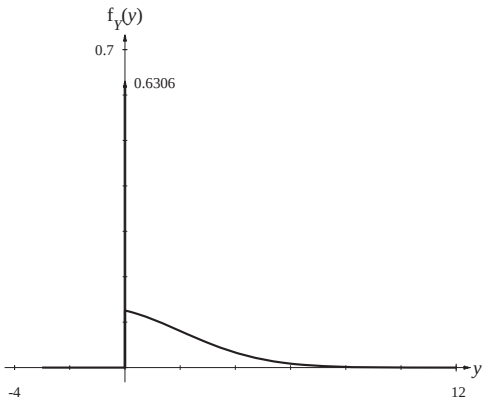
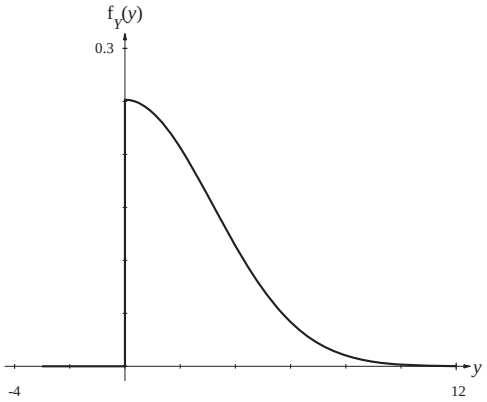
Answers to Exercises in Chapter 6 - Stochastic Processes

6-1. A Gaussian stochastic process $X(t)$ with a mean of -1 V and a standard deviation of 3 V is the excitation of the systems below. For each system graph the pdf of the response $Y(t)$.

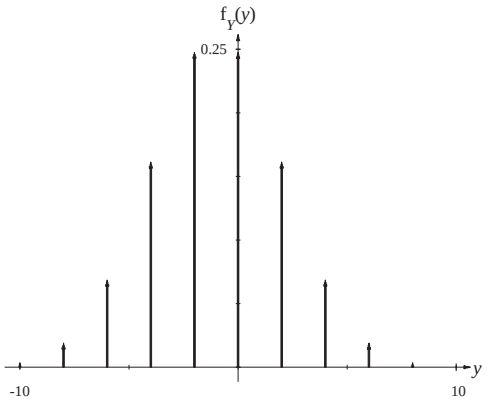
(a) A half-wave rectifier whose transfer function is $Y = \max(X, 0)$.



(b) A full-wave rectifier whose transfer function is $Y = |X|$.



(c) A quantizer whose transfer function is $Y = \lfloor (X+1)/2 \rfloor \times 2$, where the notation $\lfloor \lambda \rfloor$ means "the most positive integer less than or equal to λ".



6-2. Classify (to the extent possible) the following stochastic processes as

1. CT or DT,
2. CV, DV or mixed,
3. Deterministic or non-deterministic.

(a) The number of photons that have arrived at a photodetector since some starting time.
CTDV Non-deterministic

(b) The time required for a group of 100 consecutive photons to arrive at a photodetector since the end of the last group of 100 photons. CTCV Non-deterministic

(c) The number of photons arriving at a photodetector during each second.
DTDV Non-deterministic

(d) The stochastic process X and the three stochastic processes Y of Exercise 6-1
X - CV, Otherwise unknown

The pdf of X is Gaussian, indicating that it is CV.
There is not enough information to know whether or not X or any of the Y's are deterministic or whether they are CT or DT.

$Y = \max\left(X, 0\right)$ Mixed, Otherwise unknown

$Y = \left|X\right|$ is CV, Otherwise unknown

$Y = \left\lfloor \left(X + 1\right) / 2 \right\rfloor \times 2$ is DV, Otherwise unknown

(e) The number of automobile accidents during the preceding week, reported at the end of each week in Los Angeles. DTDV Non-deterministic

(f) The Johnson (thermal) noise of a resistor. CTCV Non-deterministic

(g) The binary number on the data bus of a computer CPU at the end of each clock cycle.
DTDV Otherwise unknown

(h) The wind speed at the top of Mt. Everest. CTCV Non-deterministic

(i) The nth number in the arithmetic progression $\left\{0,1,2,3,5,8,13,21,34,\cdots\right\}$.
DV Deterministic Otherwise unknown

(j) A stochastic process with sample functions

$$\mathrm{X}\left(t\right) = \sum_{n=-\infty}^{\infty} A\,\mathrm{rect}\left(t - t_0 - 10n\right)$$

where A is Rayleigh distributed random variable and t_0 is uniformly distributed random variable.
CTCV Deterministic

6-3. A deterministic stochastic process has sample functions

$$\mathrm{X}\left(t\right) = A + B\cos\left(\omega t + \theta\right)$$

where A, B and θ are all independent random variables. A is Gaussian with mean 3 and standard deviation 2, B is exponential with mean 5 and θ is uniform between $-\pi$ and π.

(a) Compute the mean and standard deviation of X. 3, 5.385

(b) Is X stationary? Yes

6-4. Redo Exercise 6-3 with the distribution of θ changed to uniform between $-\pi/2$ and $\pi/2$.

(a) Compute the mean and standard deviation of X.

$$E(X) = 3 + (10/\pi)\cos(\omega t)$$

$$\sigma_X = \sqrt{29 + \frac{40\cos(\omega t)}{\pi} - \cos^2(\omega t)}$$

(b) Is X stationary? No.

6-5. Redo Exercise 6-3 with $\theta = 0$.

(a) Compute the mean and standard deviation of X.

$$E(X) = 3 + 5\cos(\omega t)$$

$$\sigma_X = \sqrt{4 + 25\cos^2(\omega t)}$$

(b) Is X stationary? No.