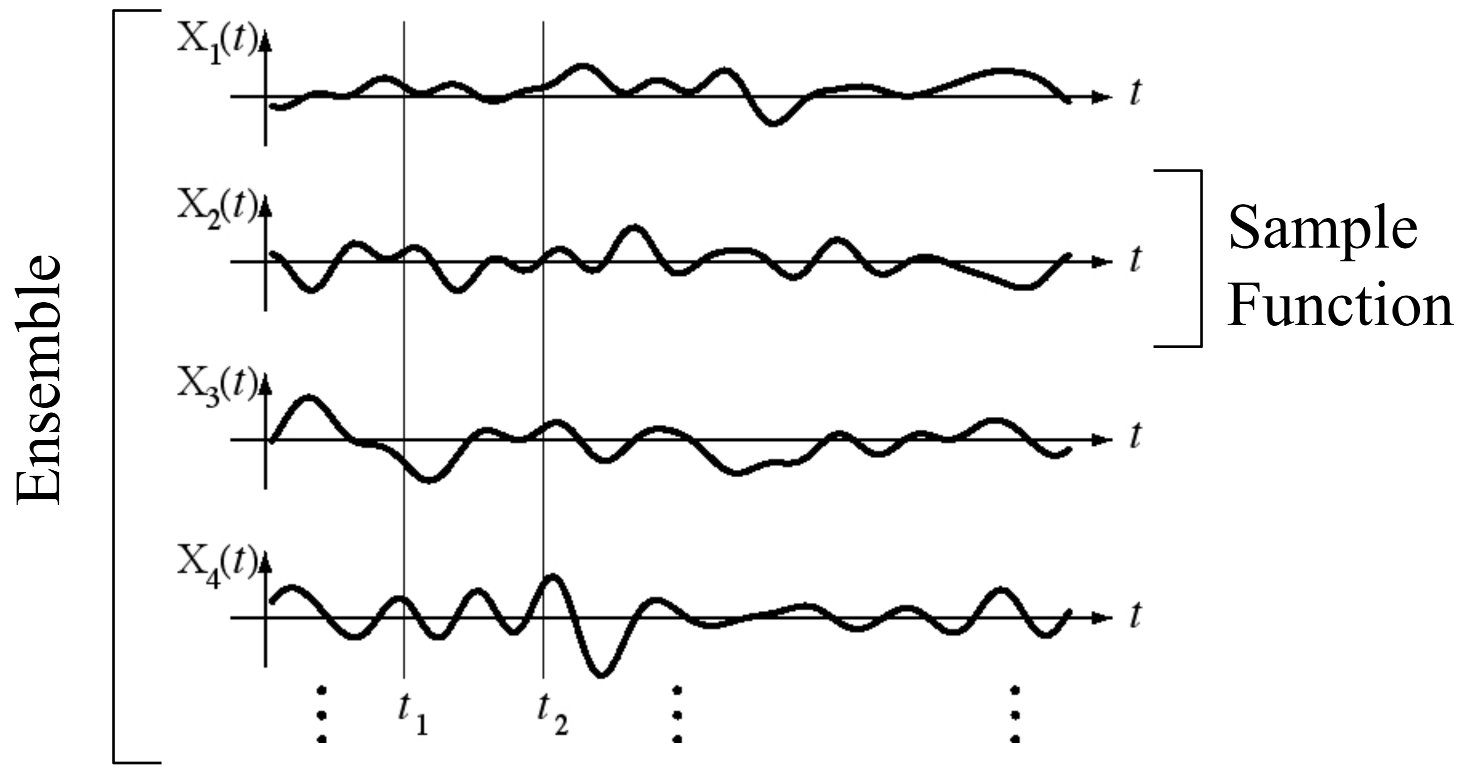
Stochastic Processes

Definition

A random variable is a number $\mathrm{X}(\zeta)$ assigned to every outcome of an experiment. A **stochastic process** is the assignment of a function of t $\mathrm{X}(t,\zeta)$ to each outcome of an experiment. The set of functions $\left\{\mathrm{X}(t,\zeta_1),\mathrm{X}(t,\zeta_2),\cdots,\mathrm{X}(t,\zeta_N)\right\}$ corresponding to the N outcomes of an experiment is called an **ensemble** and each member $\mathrm{X}(t,\zeta_i)$ is called a **sample function** of the stochastic process.

A common convention in the notation describing stochastic processes is to write the sample functions as functions of t only and to indicate the stochastic process by $\mathrm{X}(t)$ instead of $\mathrm{X}(t,\zeta)$ and any particular sample function by $\mathrm{X}_i(t)$ instead of $\mathrm{X}(t,\zeta_i)$.

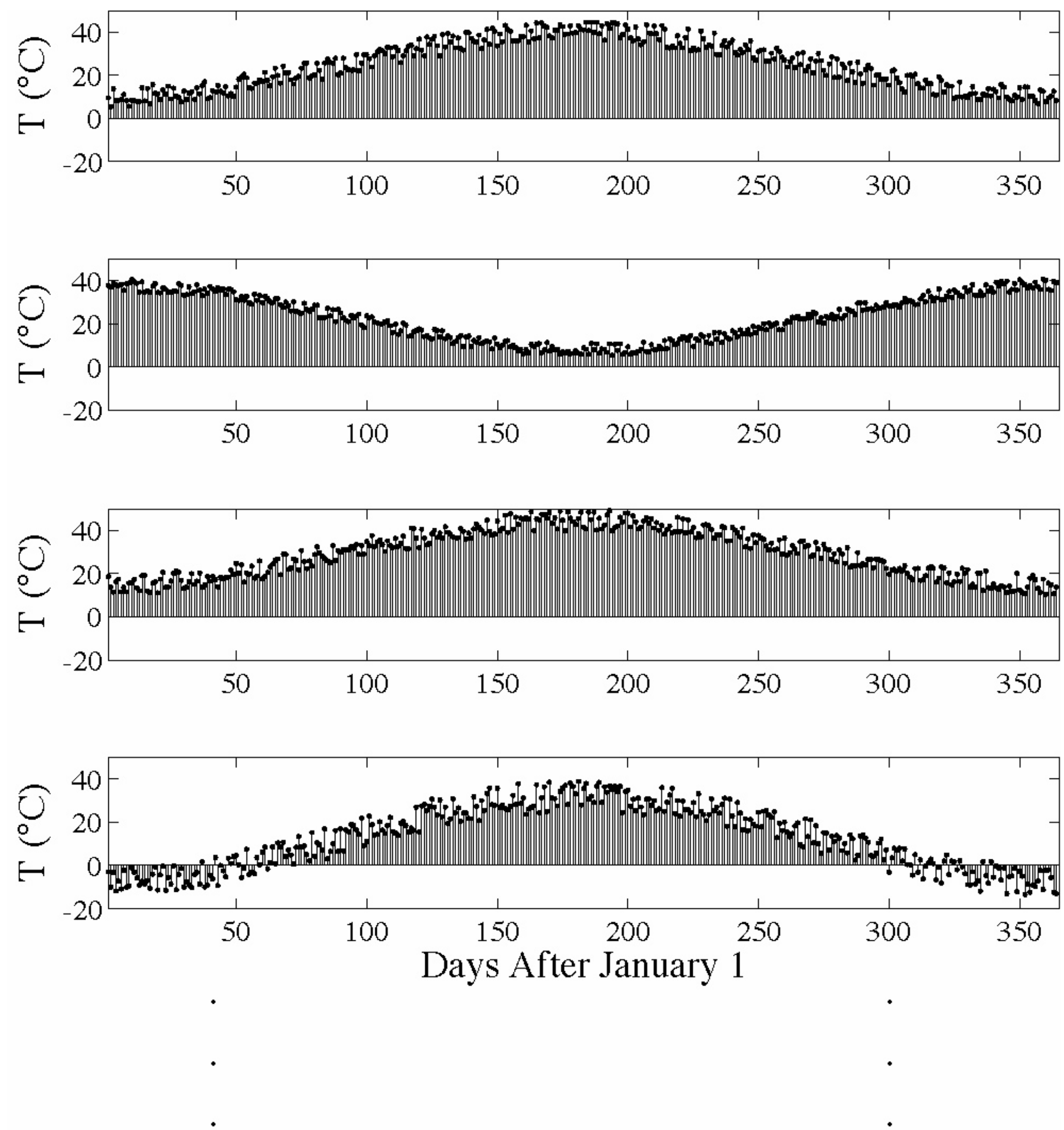
Definition



The values of $X(t)$ at a particular time t_1 define a random variable $X(t_1)$ or just X_1.

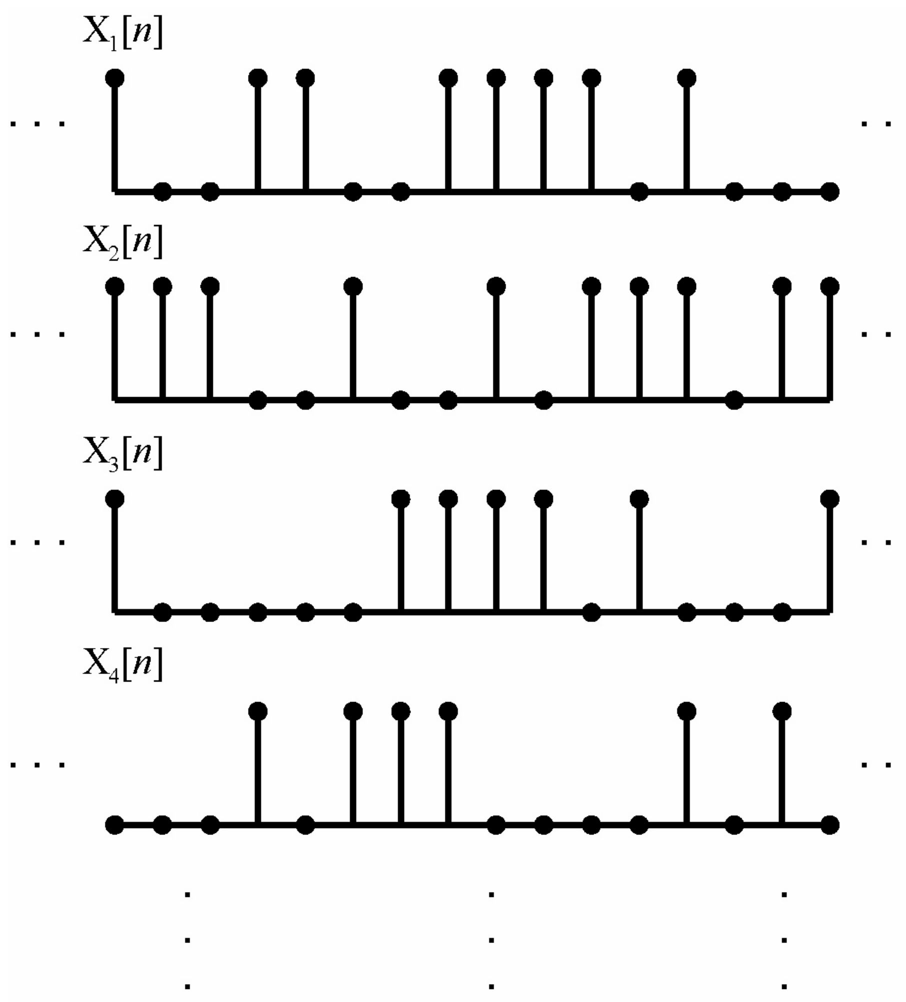
Example of a Stochastic Process

Suppose we place a temperature sensor at every airport control tower in the world and record the temperature at noon every day for a year. Then we have a discrete-time, continuous-value (DTCV) stochastic process.



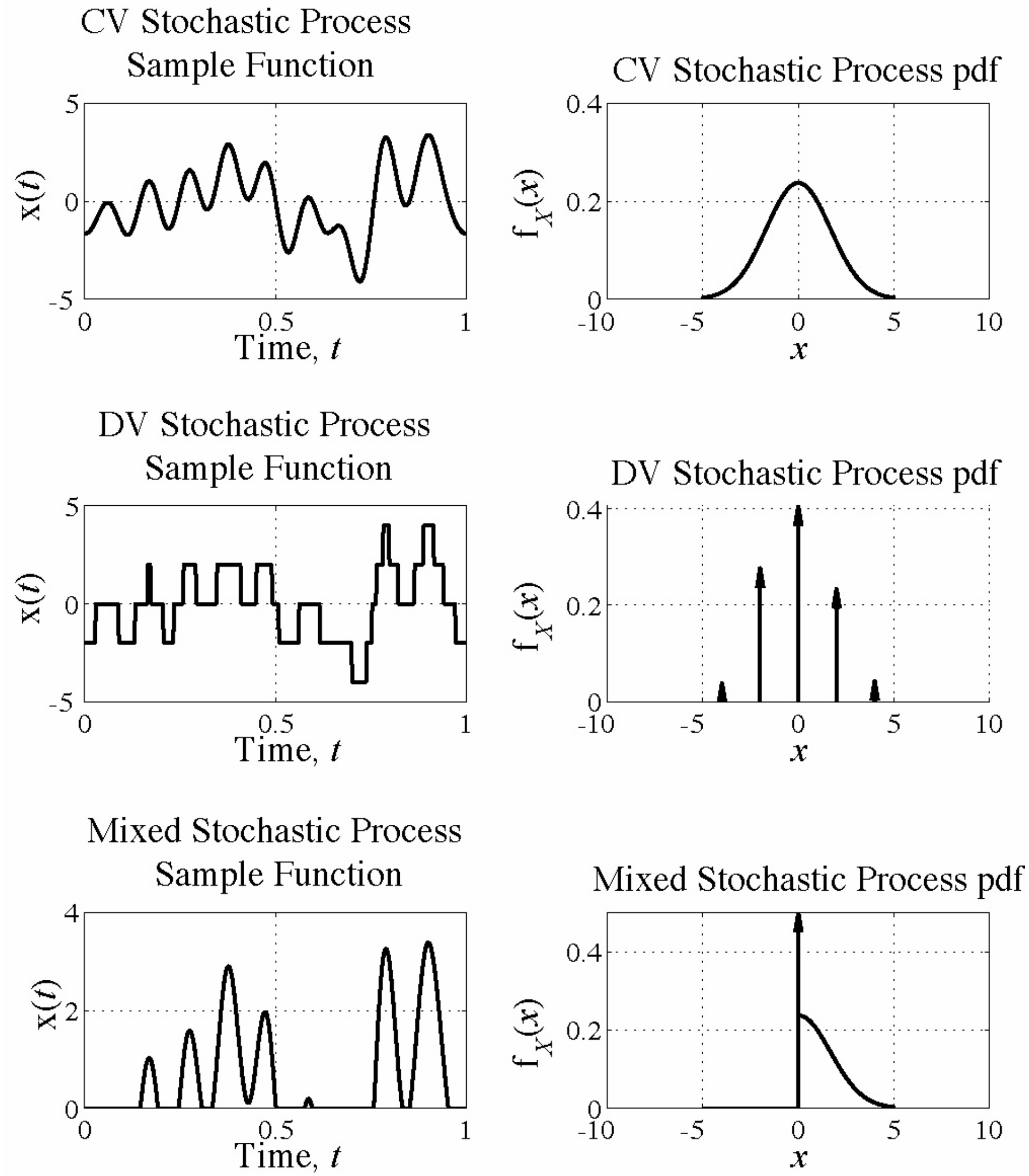
Example of a Stochastic Process

Suppose there is a large number of people, each flipping a fair coin every minute. If we assign the value 1 to a head and the value 0 to a tail we have a discrete-time, discrete-value (DTDV) stochastic process



Continuous-Value vs. Discrete-Value

A continuous-value (CV) random process has a pdf with no impulses. A discrete-value (DV) random process has a pdf consisting only of impulses. A mixed random process has a pdf with impulses, but not just impulses.



Deterministic vs. Non-Deterministic

A stochastic process is deterministic if a sample function can be described by a mathematical function such that its future values can be computed. The randomness is in the ensemble, not in the time functions. For example, let the sample functions be of the form, $\mathrm{X}(t) = A\cos(2\pi f_0 t + \theta)$ and let the parameter θ be random over the ensemble but constant for any particular sample function. All other stochastic processes are non-deterministic.

Stationarity

If all the mltivariate statistical descriptors of a stochastic process are not functions of time, the stochastic process is said to be **strict - sense stationary** (**SSS**).

A random process is **wide - sense stationary** (**WSS**) if $\mathrm{E}\left(\mathrm{X}\left(t_1\right)\right)$ is independent of the choice of t_1 and $\mathrm{E}\left(\mathrm{X}\left(t_1\right)\mathrm{X}\left(t_2\right)\right)$ depends only on the difference between t_1 and t_2.

Ergodicity

If all of the sample functions of a random process have the same statistical properties the random process is said to be **ergodic**. The most important consequenceof ergodicity is that ensemble moments can be replaced by time moments.

$$\mathrm{E}\left(X^n\right) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} X^n\left(t\right) dt$$

Every ergodic random process is also stationary.

Measurement of Process Parameters

The mean value of an ergodic stochastic process can be estimated by $\bar{X} = \frac{1}{T}\int_0^T \mathrm{X}(t)\,dt$ where $\mathrm{X}(t)$ is a sample function of that stochastic process. In practical situations, this function is usually not known. Instead samples from it are known. Then the estimate of the mean value would be $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ where X_i is a sample from $\mathrm{X}(t)$.

Measurement of Process Parameters

To make a good estimate, the samples from the random process should be independent.