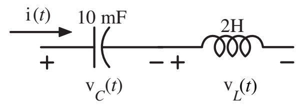
Solution of EECS 300 Test 6 F08

1. A current $\mathrm{i}(t) = (3t+4)\ \mathrm{A}$ is flowing through a $10\ \mathrm{mF}$ capacitor and a 2H inductor in series. The voltage $\mathrm{v}_C(t)$ across the capacitor at time $t=0$ is 1200 V. Find the numerical values of the capacitor voltage and the inductor voltage at time $t = 5\ \mathrm{s}$.



$$\mathrm{v}_C(t) = 1200\mathrm{V} + \frac{1}{C}\int_0^t \mathrm{i}_C(\tau)\,d\tau = \left(1200 + 100\int_0^t (3\tau+4)\,d\tau\right)\mathrm{V}$$

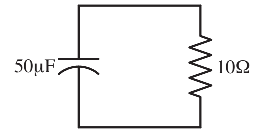
$$\mathrm{v}_C(t) = \left(1200 + 100\left[3\tau^2/2 + 4\tau\right]_0^t\right)\mathrm{V}$$

$$\mathrm{v}_C(t) = \left(1200 + 300t^2/2 + 400t\right)\mathrm{V}$$

$$\mathrm{v}_C(5) = \left(1200 + 300(25)/2 + 400(5)\right)\mathrm{V} = 6950\mathrm{V}$$

$$\mathrm{v}_L(t) = L\frac{di}{dt} = (2\mathrm{H})(3\ \mathrm{A/s}) = 6\mathrm{V} \Rightarrow \mathrm{v}_L(5) = 6\mathrm{V}$$

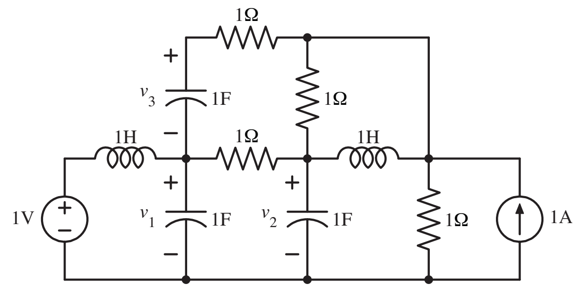
2. In the circuit below, the initial capacitor voltage at time $t = 0$ is 24V. The energy decays toward zero as time passes. How much energy will be dissipated in the resistor over the time range $0 < t < \infty$?



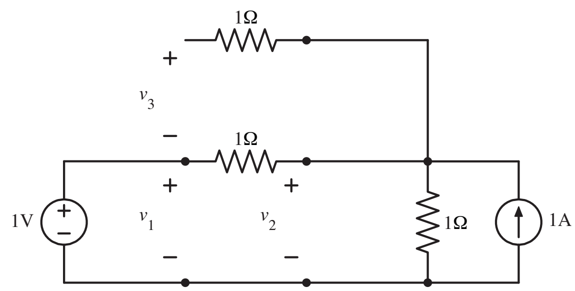
All the initial energy in the capacitor must be dissipated in the resistor. The initial energy stored in the capacitor is

$$E_C = (1/2)Cv^2 = (1/2)50\mu F(24V)^2 = 14.4mJ$$

3. The voltage source in the circuit below is a constant 1V and the current source is a constant 1A. Find the numerical values of the voltages v_1, v_2 and v_3.



We can simplify the circuit for this problem by replacing all capacitors by open circuits and all inductors by short circuits. Also the upper-right hand resistor is shorted out and can be removed without changing any voltages or currents.



Due to the 1V source acting alone:

Current source is open-circuited. Current flows through the middle resistor path and down through the right hand resistor. Half of the 1V source voltage appears across each of the two resistors in that path. $v_1 = 1V$, $v_2 = 0.5\text{ V}$ and $v_3 = -0.5V$

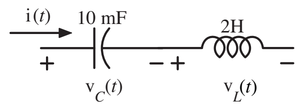
Due to the 1A source acting alone:

Voltage source is short-circuited. The source current splits between the right-hand resistor and the middle resistor, 1/2A going each way. v_1 is zero because it is across a short circuit. v_2 is 0.5 V and v_3 is 0.5 V.

Including both sources: $v_1 = 1V$, $v_2 = 1\text{ V}$ and $v_3 = 0$

Solution of EECS 300 Test 6 F08

1. A current $\mathrm{i}(t) = (5t+2)\ \mathrm{A}$ is flowing through a $10\ \mathrm{mF}$ capacitor and a 2H inductor in series. The voltage $\mathrm{v}_C(t)$ across the capacitor at time $t=0$ is 1000 V. Find the numerical values of the capacitor voltage and the inductor voltage at time $t = 5\ \mathrm{s}$.



$$\mathrm{v}_C(t) = 1000\mathrm{V} + \frac{1}{C}\int_0^t \mathrm{i}_C(\tau)\,d\tau = \left(1000 + 100\int_0^t (5\tau + 2)\,d\tau\right)\mathrm{V}$$

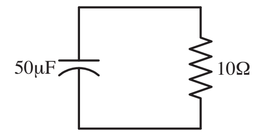
$$\mathrm{v}_C(t) = \left(1000 + 100\left[5\tau^2/2 + 2\tau\right]_0^t\right)\mathrm{V}$$

$$\mathrm{v}_C(t) = \left(1000 + 500t^2/2 + 200t\right)\mathrm{V}$$

$$\mathrm{v}_C(5) = \left(1000 + 500(25)/2 + 200(5)\right)\mathrm{V} = 8250\mathrm{V}$$

$$\mathrm{v}_L(t) = L\frac{di}{dt} = (2\mathrm{H})(5\ \mathrm{A/s}) = 10\mathrm{V} \Rightarrow \mathrm{v}_L(5) = 10\mathrm{V}$$

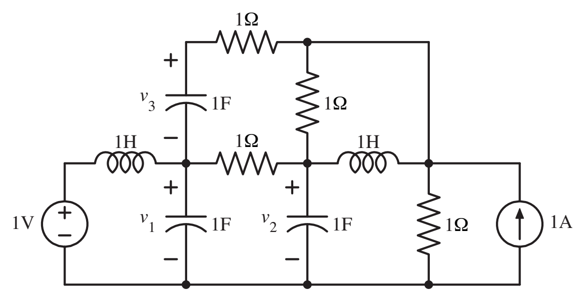
2. In the circuit below, the initial capacitor voltage at time $t = 0$ is 16V. The energy decays toward zero as time passes. How much energy will be dissipated in the resistor over the time range $0 < t < \infty$?



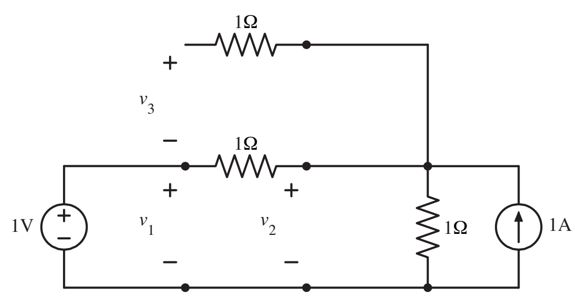
All the initial energy in the capacitor must be dissipated in the resistor. The initial energy stored in the capacitor is

$$E_C = (1/2)Cv^2 = (1/2)50\mu\text{F}(16\text{V})^2 = 6.4\text{mJ}$$

3. The voltage source in the circuit below is a constant 1V and the current source is a constant 1A. Find the numerical values of the voltages v_1, v_2 and v_3.



We can simplify the circuit for this problem by replacing all capacitors by open circuits and all inductors by short circuits. Also the upper-right hand resistor is shorted out and can be removed without changing any voltages or currents.



Due to the 1V source acting alone:

Current source is open-circuited. Current flows through the middle resistor path and down through the right hand resistor. Half of the 1V source voltage appears across each of the two resistors in that path.
$v_1 = 1\text{V}$, $v_2 = 0.5\text{ V}$ and $v_3 = -0.5\text{V}$

Due to the 1A source acting alone:

Voltage source is short-circuited. The source current splits between the right-hand resistor and the middle resistor, 1/2A going each way. v_1 is zero because it is across a short circuit. v_2 is 0.5 V and v_3 is 0.5 V.

Including both sources: $v_1 = 1\text{V}$, $v_2 = 1\text{ V}$ and $v_3 = 0$