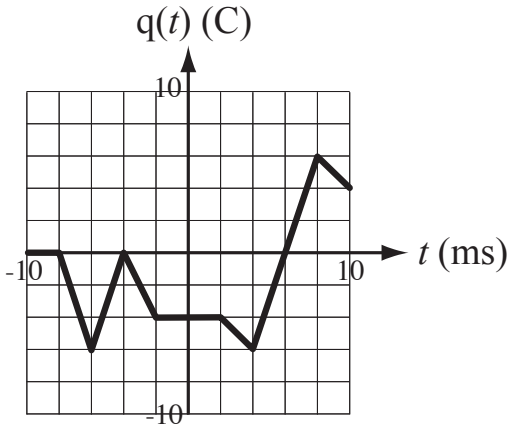
Solution of ECE 300 Test 1 S11

1. Below is a graph of the amount of charge $q(t)$ that has passed a point in a circuit versus time. Find the numerical current in amperes at that same point in the circuit at the following times.

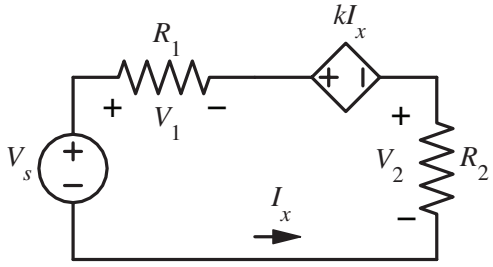


$i(1\text{ ms}) = 0$

$i(6\text{ ms}) = 12\text{C} / 4\text{ms} = 3000\text{ A}$

2. Find the numerical current I_x, the numerical voltage across and absorbed power in watts for each circuit element.

$V_s = 33\text{V}$, $R_1 = 14\Omega$, $R_2 = 9\Omega$, $k = 12$



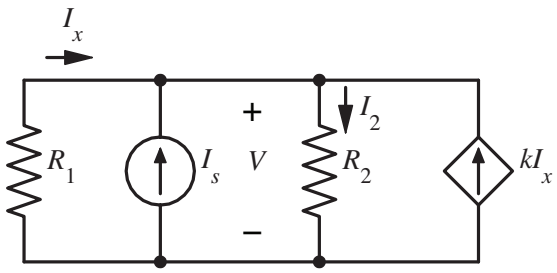
KVL: $-V_s + V_1 + kI_\text{x} + V_2 = 0 \Rightarrow -V_s - R_1 I_\text{x} + kI_\text{x} - R_2 I_\text{x} = 0 \Rightarrow I_\text{x} = \dfrac{V_s}{k - R_1 - R_2}$

$$I_\text{x} = \frac{33}{12 - 14 - 9} = -3\text{A} \Rightarrow V_1 = 42\text{V} \ , \ V_2 = 27\text{V} \ , kI_\text{x} = -36\text{V}$$

$$P_{V_s,abs} = V_s I_\text{x} = 33 \times (-3) = -99\text{W} \ , P_{R_1,abs} = I_\text{x}^2 R_1 = (-3)^2 \times 14 = 126\text{W} \ , P_{R_2,abs} = I_\text{x}^2 R_2 = (-3)^2 \times 9 = 81\text{W}$$

$$P_{kI_\text{x},abs} = -kI_\text{x} \times I_\text{x} = -12(-3)^2 = -108\text{W}$$

3. Find the numerical current through and absorbed power in watts for each circuit element.



KCL: $I_x + I_s - I_2 + kI_x = 0 \Rightarrow -\dfrac{V}{R_1} + I_s - \dfrac{V}{R_2} + 2\left(-\dfrac{V}{R_1}\right) = 0 \Rightarrow V = \dfrac{I_s}{3G_1 + G_2}$

$$V = \frac{0.05}{3\times10^{-4} + 7.6923\times10^{-5}} = 132.6531 \Rightarrow I_x = -0.01327\,\text{A}\ ,\ I_2 = 0.0102\,\text{A}$$

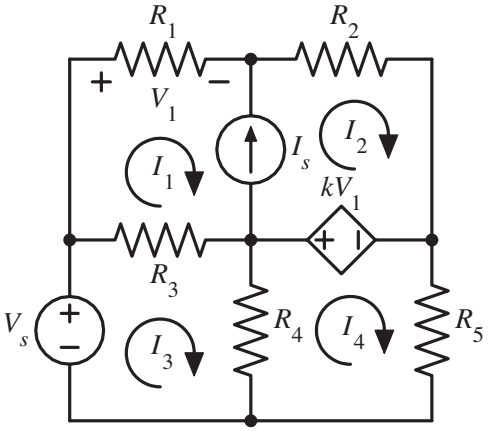
$$P_{I_s,abs} = -VI_s = -132.6531\times0.05 = -6.6327\,\text{W} \qquad P_{R_1,abs} = I_x^2 R_1 = (-0.01327)^2\times10^4 = 1.7609\,\text{W}$$

$$P_{R_2,abs} = I_2^2 R_2 = (0.0102)^2\times1.3\times10^4 = 1.3525\,\text{W}$$

$$P_{kI_x,abs} = -VkI_x = -132.6531\times2\times(-0.01327) = 3.5206\,\text{W}$$

4. Fill in the blanks in these mesh-current equations with numbers.

$V_s = 18\text{V}$, $I_s = 30\text{mA}$, $R_1 = 90\Omega$, $R_2 = 140\Omega$
$R_3 = 120\Omega$, $R_4 = 200\Omega$, $R_5 = 80\Omega$, $k = 2$



$$\begin{bmatrix} R_1+R_3 & R_2 & -R_3 & 0 \\ -R_3 & 0 & R_3+R_4 & -R_4 \\ 0 & 0 & -R_4 & R_4+R_5 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} kV_1 \\ V_s \\ -kV_1 \\ I_s \end{bmatrix}$$

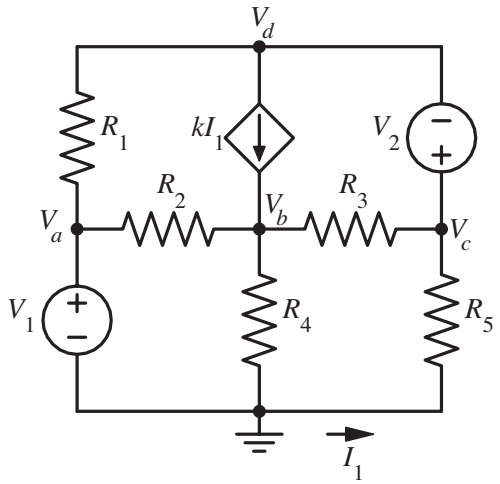
$$V_1 = I_1 R_1$$

$$\begin{bmatrix} R_1+R_3-kR_1 & R_2 & -R_3 & 0 \\ -R_3 & 0 & R_3+R_4 & -R_4 \\ kR_1 & 0 & -R_4 & R_4+R_5 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \\ I_s \end{bmatrix}$$

$$\begin{bmatrix} 30 & 140 & -120 & 0 \\ -120 & 0 & 320 & -200 \\ 180 & 0 & -200 & 280 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \\ 0.03 \end{bmatrix}$$

5. Fill in the blanks in the node-voltage equations with numbers.

$V_1 = 8\text{V}$, $V_2 = 13\text{V}$, $R_1 = 9\Omega$, $R_2 = 16\Omega$
$R_3 = 11\Omega$, $R_4 = 5\Omega$, $R_5 = 20\Omega$, $k = -1.5$



$$\begin{bmatrix} -G_2 & G_2+G_3+G_4 & -G_3 & 0 \\ -G_1 & -G_3 & G_3+G_5 & G_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} kI_1 \\ -kI_1 \\ V_1 \\ V_2 \end{bmatrix}$$

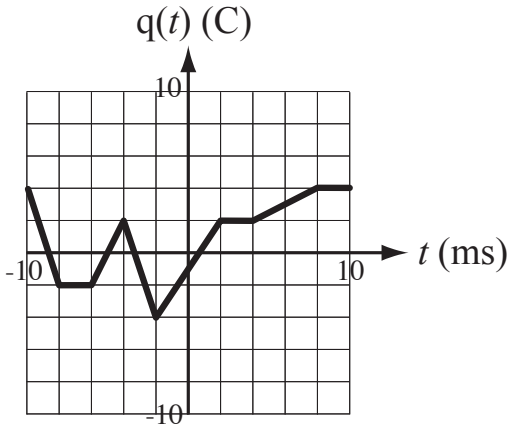
$$I_1 = -V_c G_5$$

$G_1 = 0.1111$, $G_2 = 0.0625$, $G_3 = 0.0909$, $G_4 = 0.2$, $G_5 = 0.05$

$$\begin{bmatrix} -0.0625 & 0.3534 & -0.1659 & 0 \\ -0.1111 & -0.0909 & 0.2159 & 0.1111 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 8 \\ 13 \end{bmatrix}$$

Solution of ECE 300 Test 1 S11

1. Below is a graph of the amount of charge $q(t)$ that has passed a point in a circuit versus time. Find the numerical current in amperes at that same point in the circuit at the following times.

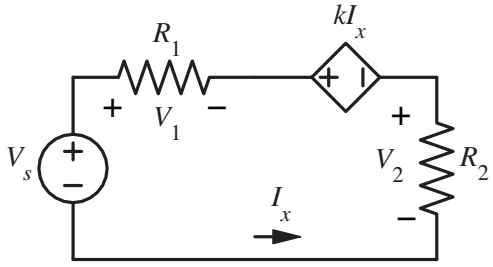


$i(-3\text{ ms}) = -6\text{C} / 2\text{ms} = -3000\text{ A}$

$i(3\text{ ms}) = 0$

2. Find the numerical current I_x, the numerical voltage across and absorbed power in watts for each circuit element.

$V_s = 22\text{V}$, $R_1 = 14\Omega$, $R_2 = 9\Omega$, $k = 12$



KVL: $-V_s + V_1 + kI_\text{x} + V_2 = 0 \Rightarrow -V_s - R_1 I_\text{x} + kI_\text{x} - R_2 I_\text{x} = 0 \Rightarrow I_\text{x} = \dfrac{V_s}{k - R_1 - R_2}$

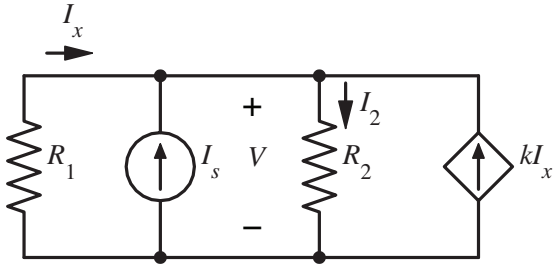
$$I_\text{x} = \frac{22}{12 - 14 - 9} = -2\text{A} \Rightarrow V_1 = 28\text{V} \ , \ V_2 = 18\text{V} \ , kI_\text{x} = -24\text{V}$$

$$P_{V_s,abs} = V_s I_\text{x} = 22 \times (-2) = -44\text{W} \ , P_{R_1,abs} = I_\text{x}^2 R_1 = (-2)^2 \times 14 = 56\text{W} \ , P_{R_2,abs} = I_\text{x}^2 R_2 = (-2)^2 \times 9 = 36\text{W}$$

$$P_{kI_\text{x},abs} = -kI_\text{x} \times I_\text{x} = -12(-2)^2 = -48\text{W}$$

3. Find the numerical current through and absorbed power in watts for each circuit element.

$I_s = 25\text{mA}$, $R_1 = 10k\Omega$, $R_2 = 13k\Omega$, $k = 2$



KCL: $I_x + I_s - I_2 + kI_x = 0 \Rightarrow -\dfrac{V}{R_1} + I_s - \dfrac{V}{R_2} + 2\left(-\dfrac{V}{R_1}\right) = 0 \Rightarrow V = \dfrac{I_s}{3G_1 + G_2}$

$$V = \frac{0.025}{3\times10^{-4} + 7.6923\times10^{-5}} = 66.3265 \Rightarrow I_x = -0.0066\text{A} \ , \ I_2 = 0.0051\text{A}$$

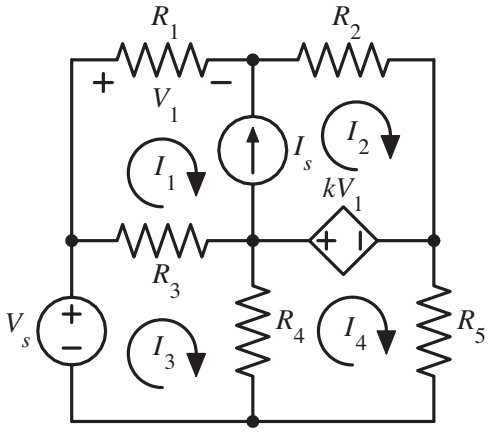
$P_{I_s,abs} = -VI_s = -66.3265\times0.025 = -1.6582\text{W}$ $\qquad$ $P_{R_1,abs} = I_x^2 R_1 = (-0.0066)^2 \times 10^4 = 0.4402\text{W}$

$$P_{R_2,abs} = I_2^2 R_2 = (0.0051)^2 \times 1.3\times10^4 = 0.3381\text{W}$$

$$P_{kI_x,abs} = -VkI_x = -66.3265\times2\times(-0.0066) = 0.8801\text{W}$$

4. Fill in the blanks in these mesh-current equations with numbers.

$V_s = 15\text{V}$, $I_s = 50\text{mA}$, $R_1 = 110\Omega$, $R_2 = 100\Omega$
$R_3 = 130\Omega$, $R_4 = 220\Omega$, $R_5 = 70\Omega$, $k = 3$



$$\begin{bmatrix} R_1+R_3 & R_2 & -R_3 & 0 \\ -R_3 & 0 & R_3+R_4 & -R_4 \\ 0 & 0 & -R_4 & R_4+R_5 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} kV_1 \\ V_s \\ -kV_1 \\ I_s \end{bmatrix}$$

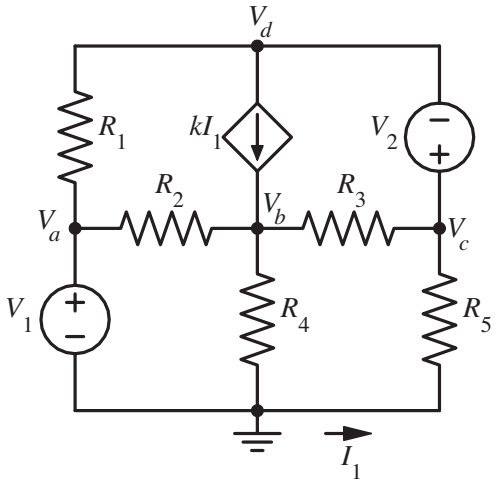
$$V_1 = I_1 R_1$$

$$\begin{bmatrix} R_1+R_3-kR_1 & R_2 & -R_3 & 0 \\ -R_3 & 0 & R_3+R_4 & -R_4 \\ kR_1 & 0 & -R_4 & R_4+R_5 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \\ I_s \end{bmatrix}$$

$$\begin{bmatrix} -90 & 100 & -130 & 0 \\ -130 & 0 & 350 & -220 \\ 330 & 0 & -220 & 290 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \\ 0 \\ 0.05 \end{bmatrix}$$

5. Fill in the blanks in the node-voltage equations with numbers.

$V_1 = 12\text{V}$, $V_2 = 18\text{V}$, $R_1 = 7\Omega$, $R_2 = 13\Omega$
$R_3 = 9\Omega$, $R_4 = 11\Omega$, $R_5 = 15\Omega$, $k = -2.5$



$$\begin{bmatrix} -G_2 & G_2+G_3+G_4 & -G_3 & 0 \\ -G_1 & -G_3 & G_3+G_5 & G_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} kI_1 \\ -kI_1 \\ V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = -V_c G_5$$

$$\begin{bmatrix} -G_2 & G_2+G_3+G_4 & -G_3-2.5G_5 & 0 \\ -G_1 & -G_3 & G_3+G_5+2.5G_5 & G_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_1 \\ V_2 \end{bmatrix}$$

$G_1 = 0.1429$, $G_2 = 0.0769$, $G_3 = 0.1111$, $G_4 = 0.0909$, $G_5 = 0.0667$

$$\begin{bmatrix} -0.0769 & 0.2789 & -0.2778 & 0 \\ -0.1429 & -0.1111 & 0.3444 & 0.1429 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \\ 18 \end{bmatrix}$$

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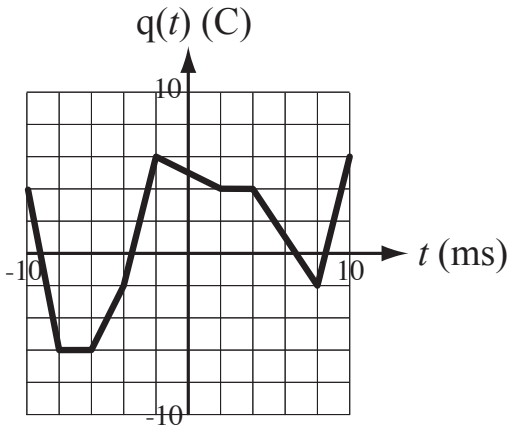
1. Below is a graph of the amount of charge $q(t)$ that has passed a point in a circuit versus time. Find the numerical current in amperes at that same point in the circuit at the following times.

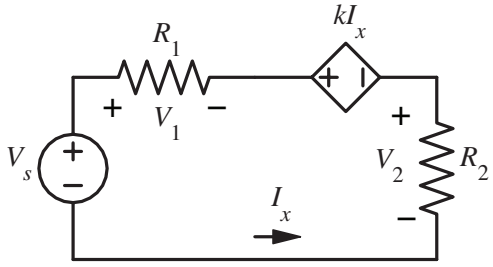


$i(-3\text{ ms}) = 8\text{C} / 2\text{ms} = 4000\text{ A}$

$i(3\text{ ms}) = 0$

2. Find the numerical current I_x, the numerical voltage across and absorbed power in watts for each circuit element.

$V_s = 11\text{V}$, $R_1 = 14\Omega$, $R_2 = 9\Omega$, $k = 12$



KVL: $-V_s + V_1 + kI_\text{x} + V_2 = 0 \Rightarrow -V_s - R_1 I_\text{x} + kI_\text{x} - R_2 I_\text{x} = 0 \Rightarrow I_\text{x} = \dfrac{V_s}{k - R_1 - R_2}$

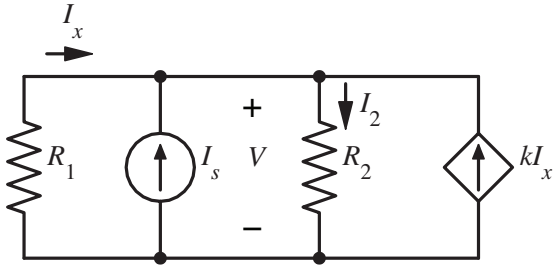
$$I_\text{x} = \frac{11}{12 - 14 - 9} = -1\text{A} \Rightarrow V_1 = 14\text{V} \ , \ V_2 = 9\text{V} \ , kI_\text{x} = -12\text{V}$$

$$P_{V_s,abs} = V_s I_\text{x} = 11 \times (-1) = -11\text{W} \ , P_{R_1,abs} = I_\text{x}^2 R_1 = (-1)^2 \times 14 = 14\text{W} \ , P_{R_2,abs} = I_\text{x}^2 R_2 = (-1)^2 \times 9 = 9\text{W}$$

$$P_{kI_\text{x},abs} = -kI_\text{x} \times I_\text{x} = -12(-1)^2 = -12\text{W}$$

3. Find the numerical current through and absorbed power in watts for each circuit element.

$I_s = 100\text{mA}$, $R_1 = 10k\Omega$, $R_2 = 13k\Omega$, $k = 2$



KCL: $I_x + I_s - I_2 + kI_x = 0 \Rightarrow -\dfrac{V}{R_1} + I_s - \dfrac{V}{R_2} + 2\left(-\dfrac{V}{R_1}\right) = 0 \Rightarrow V = \dfrac{I_s}{3G_1 + G_2}$

$$V = \frac{0.1}{3\times10^{-4} + 7.6923\times10^{-5}} = 265.3062 \Rightarrow I_x = -0.0265\text{A} \ , \ I_2 = 0.0204\text{A}$$

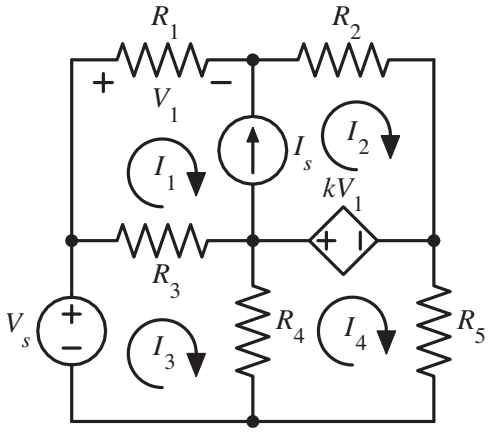
$P_{I_s,abs} = -VI_s = -265.3062\times0.1 = -26.5306\text{W}$ $\qquad$ $P_{R_1,abs} = I_x^2 R_1 = (-0.0265)^2 \times 10^4 = 7.0436\text{W}$

$$P_{R_2,abs} = I_2^2 R_2 = (0.0204)^2 \times 1.3\times10^4 = 5.41\text{W}$$

$$P_{kI_x,abs} = -VkI_x = -265.3062\times2\times(-0.0265) = 14.0824\text{W}$$

4. Fill in the blanks in these mesh-current equations with numbers.

$V_s = 9\text{V}$, $I_s = 80\text{mA}$, $R_1 = 50\Omega$, $R_2 = 110\Omega$
$R_3 = 170\Omega$, $R_4 = 140\Omega$, $R_5 = 60\Omega$, $k = 1.5$



$$\begin{bmatrix} R_1+R_3 & R_2 & -R_3 & 0 \\ -R_3 & 0 & R_3+R_4 & -R_4 \\ 0 & 0 & -R_4 & R_4+R_5 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} kV_1 \\ V_s \\ -kV_1 \\ I_s \end{bmatrix}$$

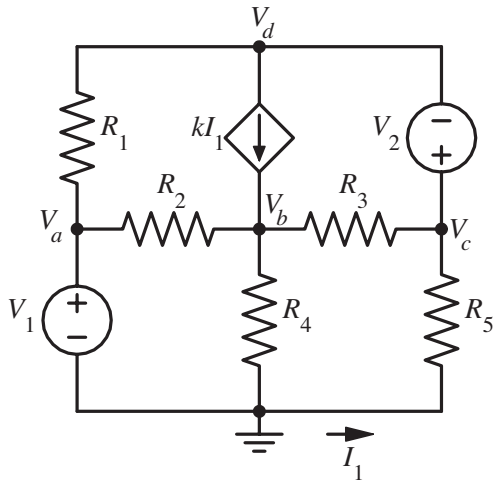
$$V_1 = I_1 R_1$$

$$\begin{bmatrix} R_1+R_3-kR_1 & R_2 & -R_3 & 0 \\ -R_3 & 0 & R_3+R_4 & -R_4 \\ kR_1 & 0 & -R_4 & R_4+R_5 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \\ I_s \end{bmatrix}$$

$$\begin{bmatrix} 145 & 110 & -170 & 0 \\ -170 & 0 & 310 & -140 \\ 75 & 0 & -140 & 200 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 0 \\ 0.08 \end{bmatrix}$$

5. Fill in the blanks in the node-voltage equations with numbers.

$V_1 = 6\text{V}$, $V_2 = 4\text{V}$, $R_1 = 15\Omega$, $R_2 = 8\Omega$
$R_3 = 7\Omega$, $R_4 = 16\Omega$, $R_5 = 18\Omega$, $k = -0.5$



$$\begin{bmatrix} -G_2 & G_2+G_3+G_4 & -G_3 & 0 \\ -G_1 & -G_3 & G_3+G_5 & G_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} kI_1 \\ -kI_1 \\ V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = -V_c G_5$$

$$\begin{bmatrix} -G_2 & G_2+G_3+G_4 & -G_3-0.5G_5 & 0 \\ -G_1 & -G_3 & G_3+G_5+0.5G_5 & G_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_1 \\ V_2 \end{bmatrix}$$

$G_1 = 0.0667$, $G_2 = 0.125$, $G_3 = 0.1429$, $G_4 = 0.0625$, $G_5 = 0.0556$

$$\begin{bmatrix} -0.125 & 0.3304 & -0.1706 & 0 \\ -0.0667 & -0.1429 & 0.2262 & 0.0667 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \\ 4 \end{bmatrix}$$