Solution of ECE 300 Test 3 S09

1. Fill in the blanks with numbers in the following sets of mesh and nodal equations for the circuit below.

$$\begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}$$

$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

$$2i_1 + 20 + 10(i_2 - i_3) = 0$$
$$10(i_3 - i_2) + 40(i_3 - i_4) + 3i_x = 0 \text{ and } i_x = i_4$$
$$8i_4 + 40(i_4 - i_3) - 20 = 0$$
$$i_2 - i_1 = -4$$

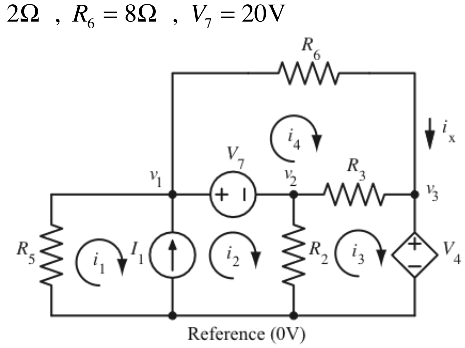
$$\begin{bmatrix} 2 & 10 & -10 & 0 \\ 0 & -10 & 50 & -37 \\ 0 & 0 & -40 & 48 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -20 \\ 0 \\ 20 \\ -4 \end{bmatrix}$$

$$\frac{v_1}{2} + \frac{v_1 - v_3}{8} + \frac{v_2 - v_3}{40} + \frac{v_2}{10} = -4$$
$$v_1 = v_2 + 20$$
$$\left.\begin{aligned} v_3 &= 3i_x \\ i_x &= \frac{v_1 - v_3}{8} \end{aligned}\right\} \Rightarrow v_3 = 3\frac{v_1 - v_3}{8} \Rightarrow -11v_3 + 3v_1 = 0$$

$$\begin{bmatrix} 5/8 & 1/8 & -3/20 \\ 1 & -1 & 0 \\ 3 & 0 & -11 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 20 \\ 0 \end{bmatrix}$$

$I_1 = -4\,\text{A}$, $R_2 = 10\Omega$, $R_3 = 40\Omega$, $V_4 = \left(3\frac{\text{V}}{\text{A}}\right) i_x$

$R_5 = 2\Omega$, $R_6 = 8\Omega$, $V_7 = 20\text{V}$



2. In the circuit below find the numerical power in watts absorbed by each resistor.

Mesh current analysis, i_1 in the left mesh, i_2 in the right mesh and i_3 in the top mesh.

$$\left\{\begin{array}{l} i_1 = I_1 = 2 \\ 20(i_2-2)+10(i_2-i_3)+5i_2=0 \\ 40+10(i_3-i_2)+5(i_3-2)=0 \end{array}\right\} \Rightarrow \begin{bmatrix} 35 & -10 \\ -10 & 15 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 40 \\ -30 \end{bmatrix} \Rightarrow \Delta = 425$$

$$i_2 = (1/425)\begin{vmatrix} 40 & -10 \\ -30 & 15 \end{vmatrix} = \frac{300}{425} = 0.7059\text{A}$$

$$i_3 = (1/425)\begin{vmatrix} 35 & 40 \\ -10 & -30 \end{vmatrix} = \frac{-650}{425} = -1.529\text{A}$$

$$P_2 = (2+1.529)^2 \times 5 = 62.27\text{W} \ , \ P_3 = (-1.529-0.7059)^2 \times 10 = 49.95\text{W}$$
$$P_4 = (2-0.7059)^2 \times 20 = 33.49\text{W} \ , \ P_5 = (0.7059)^2 \times 5 = 2.491\text{W}$$
$$P_6 = 2^2 \times 15 = 60\text{W}$$

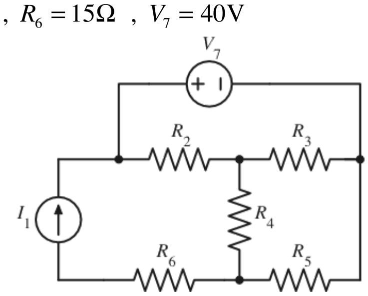
Node voltage analysis, reference node at the bottom of R_4, v_1 at the top of the current source, v_2 between R_2 and R_3, v_3 at the right end of R_3 and v_4 at the bottom of the current source.

$$\frac{v_1-v_2}{5} - 2 + \frac{v_3-v_2}{10} + \frac{v_3}{5} = 0$$
$$\frac{v_2-v_1}{5} + \frac{v_2}{20} + \frac{v_2-v_3}{10} = 0$$
$$\frac{v_4}{15} + 2 = 0 \Rightarrow v_4 = -30$$
$$v_1 = v_3 + 40$$

$$\left\{\begin{array}{l} \dfrac{v_1-v_2}{5} - 2 + \dfrac{v_1-40-v_2}{10} + \dfrac{v_1-40}{5} = 0 \\ \dfrac{v_2-v_1}{5} + \dfrac{v_2}{20} + \dfrac{v_2-v_1+40}{10} = 0 \end{array}\right\} \Rightarrow \begin{bmatrix} 1/2 & -3/10 \\ -3/10 & 7/20 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 14 \\ -4 \end{bmatrix} \Rightarrow \Delta = 0.085$$

$$\left\{\begin{array}{l} v_1 = (1/0.085)\begin{vmatrix} 14 & -3/10 \\ -4 & 7/20 \end{vmatrix} = \dfrac{3.7}{0.085} = 43.53\text{V} \\ v_2 = (1/0.085)\begin{vmatrix} 1/2 & 14 \\ -3/10 & -4 \end{vmatrix} = \dfrac{2.2}{0.085} = 25.88\text{V} \end{array}\right\} \Rightarrow v_3 = 3.53\text{V}$$

$I_1 = 2\text{A}$, $R_2 = 5\Omega$, $R_3 = 10\Omega$, $R_4 = 20\Omega$
$R_5 = 5\Omega$, $R_6 = 15\Omega$, $V_7 = 40\text{V}$



Solution of ECE 300 Test 3 S09

1. Fill in the blanks with numbers in the following sets of mesh and nodal equations for the circuit below.

$$\begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}$$

$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

$$2i_1 + 20 + 12\left(i_2 - i_3\right) = 0$$

$$12\left(i_3 - i_2\right) + 30\left(i_3 - i_4\right) + 2i_x = 0 \text{ and } i_x = i_4$$

$$8i_4 + 30\left(i_4 - i_3\right) - 20 = 0$$

$$i_2 - i_1 = -4$$

$$\begin{bmatrix} 2 & 12 & -12 & 0 \\ 0 & -12 & 42 & -28 \\ 0 & 0 & -30 & 38 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -20 \\ 0 \\ 20 \\ -4 \end{bmatrix}$$

$$\frac{v_1}{2} + \frac{v_1 - v_3}{8} + \frac{v_2 - v_3}{30} + \frac{v_2}{12} = -4$$

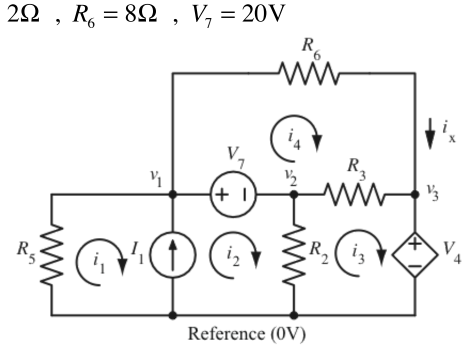
$$v_1 = v_2 + 20$$

$$\left.\begin{aligned} v_3 &= 2i_x \\ i_x &= \frac{v_1 - v_3}{8} \end{aligned}\right\} \Rightarrow v_3 = 2\frac{v_1 - v_3}{8} \Rightarrow -10v_3 + 2v_1 = 0$$

$$\begin{bmatrix} 5/8 & 21/180 & -19/120 \\ 1 & -1 & 0 \\ 2 & 0 & -10 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 20 \\ 0 \end{bmatrix}$$

$I_1 = -4\text{A}$, $R_2 = 12\Omega$, $R_3 = 30\Omega$, $V_4 = \left(2\frac{\text{V}}{\text{A}}\right)i_x$

$R_5 = 2\Omega$, $R_6 = 8\Omega$, $V_7 = 20\text{V}$



2. In the circuit below find the numerical power in watts absorbed by each resistor.

Mesh current analysis, i_1 in the left mesh, i_2 in the right mesh and i_3 in the top mesh.

$$\left\{\begin{array}{l} i_1 = I_1 = 2 \\ 20(i_2-2)+10(i_2-i_3)+5i_2=0 \\ 40+10(i_3-i_2)+10(i_3-2)=0 \end{array}\right\} \Rightarrow \begin{bmatrix} 35 & -10 \\ -10 & 20 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 40 \\ -20 \end{bmatrix} \Rightarrow \Delta = 600$$

$$i_2 = (1/600)\begin{vmatrix} 40 & -10 \\ -20 & 20 \end{vmatrix} = \frac{600}{600} = 1\text{A}$$

$$i_3 = (1/600)\begin{vmatrix} 35 & 40 \\ -10 & -20 \end{vmatrix} = \frac{-300}{600} = -0.5\text{A}$$

$$P_2 = (2+0.5)^2 \times 10 = 62.5\text{W} \ , \ P_3 = (1+0.5)^2 \times 10 = 22.5\text{W}$$
$$P_4 = (2-1)^2 \times 20 = 20\text{W} \ , \ P_5 = (1)^2 \times 5 = 5\text{W}$$
$$P_6 = 2^2 \times 15 = 60\text{W}$$

Node voltage analysis, reference node at the bottom of R_4, v_1 at the top of the current source, v_2 between R_2 and R_3, v_3 at the right end of R_3 and v_4 at the bottom of the current source.

$$\frac{v_1-v_2}{10} - 2 + \frac{v_3-v_2}{10} + \frac{v_3}{5} = 0$$
$$\frac{v_2-v_1}{10} + \frac{v_2}{20} + \frac{v_2-v_3}{10} = 0$$
$$\frac{v_4}{15} + 2 = 0 \Rightarrow v_4 = -30$$
$$v_1 = v_3 + 40$$

$$\left\{\begin{array}{l} \dfrac{v_1-v_2}{10} - 2 + \dfrac{v_1-40-v_2}{10} + \dfrac{v_1-40}{5} = 0 \\ \dfrac{v_2-v_1}{10} + \dfrac{v_2}{20} + \dfrac{v_2-v_1+40}{10} = 0 \end{array}\right\} \Rightarrow \begin{bmatrix} 2/5 & -2/10 \\ -2/10 & 1/4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 14 \\ -4 \end{bmatrix} \Rightarrow \Delta = 0.06$$

$$\left\{\begin{array}{l} v_1 = (1/0.06)\begin{vmatrix} 14 & -2/10 \\ -4 & 1/4 \end{vmatrix} = \dfrac{2.7}{0.06} = 45\text{V} \\ v_2 = (1/0.06)\begin{vmatrix} 2/5 & 14 \\ -2/10 & -4 \end{vmatrix} = \dfrac{1.2}{0.06} = 20\text{V} \end{array}\right\} \Rightarrow v_3 = 5\text{V}$$

$I_1 = 2\text{A}$, $R_2 = 10\Omega$, $R_3 = 10\Omega$, $R_4 = 20\Omega$
$R_5 = 5\Omega$, $R_6 = 15\Omega$, $V_7 = 40\text{V}$