Solution of EECS 300 Test 7 F08

1. The circuit shown below has been connected this way for a very long time and, before time $t = 0$, all currents and all voltages are not changing with time.

   (a) Find the numerical value of $v_C(0^-)$.

   (b) Find the numerical value of $v_8(0^-)$.

   (c) Find the numerical value of $i_L(0^-)$.

   (d) Find the numerical value of $v_6(0^-)$.

   (e) Find the numerical value of $v_s(0^-)$.

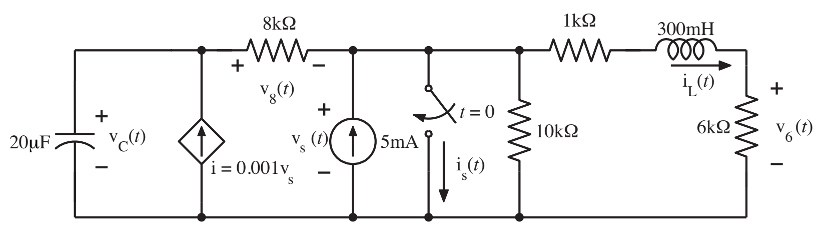
   (f) Find the numerical value of $v_C(0^+)$.

   (g) Find the numerical value of $v_8(0^+)$.

   (h) Find the numerical value of $i_L(0^+)$.

   (i) Find the numerical value of $v_6(0^+)$.

   (j) Find the numerical value of $i_s(0^+)$.



At $t = 0^-$,

$$\frac{v_c - v_s}{8000} = 0.001 v_s \quad , \quad \frac{v_s}{7000} + \frac{v_s}{10000} + \frac{v_s - v_c}{8000} = 0.005 .$$ Solving for v_s

$$\frac{v_s}{7000} + \frac{v_s}{10000} - 0.001 v_s = 0.005$$

$$v_s(0.00014286 + 0.0001 - 0.001) = 0.005$$

$$v_s = \frac{0.005}{-0.0007572} = -6.603\text{V}$$

$$\frac{v_c + 6.603}{8000} = 0.001(-6.603) \Rightarrow v_c = -59.43\text{V}$$

$$v_8 = v_c - v_s = -59.43 - (-6.603) = -52.827\text{V}$$

$$i_L = \frac{v_s}{7000} = \frac{-6.603}{7000} = -0.943\text{mA}$$

$$v_6 = 6000\, i_L = -5.66\text{V}$$

At $t = 0^+$, the switch has closed and

$$i_L = -0.943\text{mA} \quad \text{and} \quad v_c = -59.43\text{V}$$

All the inductor current now flows through the closed switch and the 1k and 6k resistors. So v_6 does not change. It is still –5.66V. The independent-current-source voltage v_s is now zero. So all the independent-current-source current flows through the closed switch. The 8k resistor and the capacitor are now in parallel so their voltages must be equal, -59.43V. The dependent-current-source current is now zero because v_s is zero. The current through the switch is the algebraic sum of the currents through the inductor, through the independent current source and through the 8k resistor. That is

$$i_s = 5\text{mA} - i_L + \frac{v_c}{8000} = 5\text{mA} - \left(-0.943\text{mA}\right) + \frac{-59.43\text{V}}{8000\Omega}$$

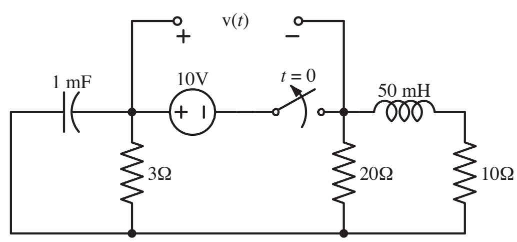
$$i_s = 5\text{mA} + 0.943\text{mA} - 7.43\text{mA} = -1.487\text{mA}$$

2. The circuit shown below has been connected this way for a very long time and, before time $t = 0$, all currents and all voltages are not changing with time. Find the numerical value of $\text{v}(t)$ at

(a) $t = -1$ ms

(b) $t = 0^+$

(c) $t = 2$ ms



(a) Before $t = 0$, $\text{v}(t)$ is the voltage across the voltage source which is 10V.

(b) At $t = 0^+$, the current through the inductor and the voltage across the capacitor are the same as they were at $t = 0^-$. At $t = 0^-$ the inductor current is

$$\frac{10\text{V}}{3\Omega + 20\Omega \| 10\Omega} \times \frac{20\Omega}{20\Omega + 10\Omega} = \frac{10\text{V}}{9.667\Omega} \times \frac{2}{3} = 0.6897\text{A}$$

flowing from right to left in the diagram.

At $t = 0^-$ the capacitor voltage is

$$\frac{10\text{V}}{3\Omega + 20\Omega \| 10\Omega} \times 3\Omega = 3.103\text{V}$$

positive polarity on the right side in the diagram.

At $t = 0^+$:
The switch has just opened so the inductor current must now flow through the $20\Omega - 10\Omega - 50\text{mH}$ loop and the voltage across the 20Ω resistor is $0.6897\text{A} \times 20\Omega = 13.794\text{V}$, positive polarity on top in the diagram. The voltage across the 3Ω resistor is 3.103V positive polarity on top in the

diagram because it is in parallel with the capacitor. Therefore, summing voltages around the $3\Omega - 20\Omega - \mathrm{v}(t)$ loop yields $\mathrm{v}(0^+) = 3.103\mathrm{V} - 13.794\mathrm{V} = -10.691\mathrm{V}$.

(c) After $t = 0$ the voltages across the 3Ω resistor and the 20Ω resistor both decay toward zero but with different time constants. The time constant for the 3Ω resistor is $1\mathrm{mF} \times 3\Omega = 3\mathrm{ms}$ and the time constant for the 20Ω resistor is $50\mathrm{mH} / 30\Omega = 1.667\mathrm{ms}$. So the 3Ω resistor voltage is

$$\mathrm{v}_3(t) = 3.103e^{-t/0.003}\mathrm{V} = 3.103e^{-333.33t}\mathrm{V}$$

positive polarity on top in the diagram and the 20Ω resistor voltage is

$$\mathrm{v}_{20}(t) = 13.794e^{-t/0.001667}\mathrm{V} = 13.794e^{-600t}\mathrm{V}$$

positive polarity on top in the diagram. Therefore, at $t = 2\mathrm{ms}$,

$$\mathrm{v}(0.002) = 3.103e^{-333.33(0.002)} - 13.794e^{-600(0.002)} = 1.593\mathrm{V} - 4.155\mathrm{V} = -2.562\mathrm{V}.$$

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   (a) Find the numerical value of $v_C(0^-)$.

   (b) Find the numerical value of $v_8(0^-)$.

   (c) Find the numerical value of $i_L(0^-)$.

   (d) Find the numerical value of $v_6(0^-)$.

   (e) Find the numerical value of $v_s(0^-)$.

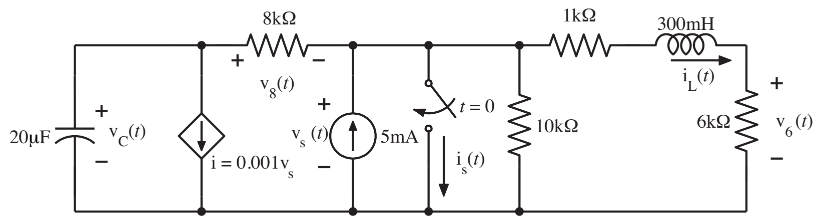
   (f) Find the numerical value of $v_C(0^+)$.

   (g) Find the numerical value of $v_8(0^+)$.

   (h) Find the numerical value of $i_L(0^+)$.

   (i) Find the numerical value of $v_6(0^+)$.

   (j) Find the numerical value of $i_s(0^+)$.



At $t = 0^-$,

$$\frac{v_c - v_s}{8000} + 0.001v_s = 0 \quad , \quad \frac{v_s}{7000} + \frac{v_s}{10000} + \frac{v_s - v_c}{8000} = 0.005.$$ Solving for v_s

$$\frac{v_s}{7000} + \frac{v_s}{10000} + 0.001v_s = 0.005$$

$$v_s(0.00014286 + 0.0001 + 0.001) = 0.005$$

$$v_s = \frac{0.005}{0.0012428} = 4.023\text{V}$$

$$\frac{v_c - 4.023}{8000} = -0.001(4.023) \Rightarrow v_c = -28.161\text{V}$$

$$v_8 = v_c - v_s = -28.161 - 4.023 = -32.184\text{V}$$

$$\text{i}_L = \frac{v_s}{7000} = \frac{4.023}{7000} = 0.575\text{mA}$$

$$v_6 = 6000\,\text{i}_L = 3.448\text{V}$$

At $t = 0^+$, the switch has closed and

$$i_L = 0.575\text{mA} \quad \text{and} \quad v_c = -28.161\text{V}$$

All the inductor current now flows through the closed switch and the 1k and 6k resistors. So v_6 does not change. It is still 3.448V. The independent-current-source voltage v_s is now zero. So all the independent-current-source current flows through the closed switch. The 8k resistor and the capacitor are now in parallel so their voltages must be equal, -28.161V. The dependent-current-source current is now zero because v_s is zero. The current through the switch is the algebraic sum of the currents through the inductor, through the independent current source and through the 8k resistor. That is

$$i_s = 5\text{mA} - i_L + \frac{v_c}{8000} = 5\text{mA} - 0.575\text{mA} + \frac{-28.161\text{V}}{8000\Omega}$$

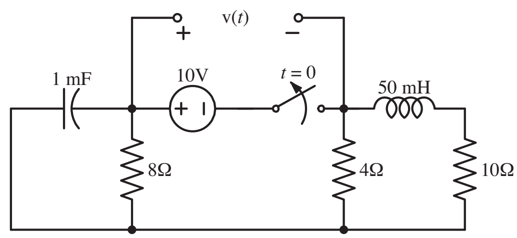
$$i_s = 5\text{mA} - 0.575\text{mA} - 3.52\text{mA} = 0.905\text{mA}$$

2. The circuit shown below has been connected this way for a very long time and, before time $t = 0$, all currents and all voltages are not changing with time. Find the numerical value of $\text{v}(t)$ at

(a) $t = -1\text{ ms}$

(b) $t = 0^+$

(c) $t = 2\text{ ms}$



(a) Before $t = 0$, $\text{v}(t)$ is the voltage across the voltage source which is 10V.

(b) At $t = 0^+$, the current through the inductor and the voltage across the capacitor are the same as they were at $t = 0^-$. At $t = 0^-$ the inductor current is

$$\frac{10\text{V}}{8\Omega + 4\Omega \| 10\Omega} \times \frac{4\Omega}{4\Omega + 10\Omega} = \frac{10\text{V}}{10.857\Omega} \times \frac{4}{14} = 0.263\text{A}$$

flowing from right to left in the diagram.

At $t = 0^-$ the capacitor voltage is

$$\frac{10\text{V}}{8\Omega + 4\Omega \| 10\Omega} \times 8\Omega = 7.369\text{V}$$

positive polarity on the right side in the diagram.

At $t = 0^+$:

The switch has just opened so the inductor current must now flow through the $4\Omega - 10\Omega - 50\text{mH}$ loop and the voltage across the 4Ω resistor is $0.263\text{A} \times 4\Omega = 1.052\text{V}$, positive polarity on top in the diagram. The voltage across the 8Ω resistor is 7.369V positive polarity on top in the diagram because it is in parallel with the capacitor. Therefore, summing voltages around the $8\Omega - 4\Omega - \mathrm{v}(t)$ loop yields $\mathrm{v}(0^+) = 7.369\mathrm{V} - 1.052\mathrm{V} = 6.317\mathrm{V}$.

(c) After $t = 0$ the voltages across the 8Ω resistor and the 4Ω resistor both decay toward zero but with different time constants. The time constant for the 8Ω resistor is $1\mathrm{mF} \times 8\Omega = 8\mathrm{ms}$ and the time constant for the 4Ω resistor is $50\mathrm{mH} / 14\Omega = 3.57\mathrm{ms}$. So the 8Ω resistor voltage is

$$\mathrm{v}_8(t) = 7.369e^{-t/0.008}\mathrm{V} = 7.369e^{-125t}\mathrm{V}$$

positive polarity on top in the diagram and the 4Ω resistor voltage is

$$\mathrm{v}_4(t) = 1.052e^{-t/0.00357}\mathrm{V} = 1.052e^{-280.1t}\mathrm{V}$$

positive polarity on top in the diagram. Therefore, at $t = 2\mathrm{ms}$,

$$\mathrm{v}(0.002) = 7.369e^{-125(0.002)} - 1.052e^{-281.1(0.002)} = 5.739 - 0.6 = 5.139\mathrm{V}.$$