## Solution of ECE 300 Test 7 S09

1. The switch in the circuit below has been closed a very long time. Find the numerical values of  $v_c(0^+)$ ,  $i_c(0^-)$ ,  $i_c(0^+)$ ,  $v_c(200\text{ms})$  and  $i_c(200\text{ms})$ .

Just before the switch opens the capacitor has no current through it  $\left[i_{c}(0^{-})=0\right]$  and  $v_{c}(0^{-})$  is the same as the voltage across  $R_{2}$  which is 100 V  $\frac{40 \text{ k}\Omega}{40 \text{ k}\Omega + 20 \text{ k}\Omega} = 66.67 \text{ V}$ . The capacitor voltage cannot change instantaneously so the voltage is the same at  $t = 0^{+}$ , 66.67 V. At  $t = 0^{+}$  the switch is open and the voltage across  $R_{2}$  is 66.67 V. Therefore its current (downward) is 1.667 mA and that must be the negative of the capacitor current. So  $i_{c}(0^{+}) = -1.667 \text{ mA}$ . The time constant is  $\tau = 400 \text{ ms}$ .

- $v_{c}(t) = v_{c}(0^{+})e^{-2.5t}$ ,  $t > 0 \implies v_{c}(200 \text{ ms}) = 66.67e^{-0.5} = 40.44 \text{ V}$
- $i_{C}(t) = i_{C}(0^{+})e^{-2.5t}$ ,  $t > 0 \implies i_{C}(200 \text{ ms}) = -1.67e^{-0.5} = -1.011 \text{ mA}$



2. The switch in the circuit below has been open a very long time. Find the numerical values of  $v_c(0^+)$ ,

$$\mathbf{i}_{C}(0^{-}), \, \mathbf{i}_{C}(0^{+}), \, \mathbf{i}_{L}(0^{+}), \, \mathbf{v}_{L}(0^{-}), \, \mathbf{v}_{L}(0^{+}), \, \frac{d}{dt}(\mathbf{v}_{C}(t))_{t=0^{+}} \text{ and } \frac{d}{dt}(\mathbf{i}_{L}(t))_{t=0^{+}}.$$

Before the switch closes all currents and voltages are constant so we know that  $i_c(0^-) = 0$  and  $v_L(0^-) = 0$ . At  $t = 0^{-}$ the  $R_1$ current downward through is  $3A \frac{4\Omega \| 6\Omega}{15\Omega + 10\Omega + 4\Omega \| 6\Omega} = 3A \frac{2.4\Omega}{27.4\Omega} = 0.263 \text{ A}.$  So the voltage across it and  $v_c(0^-)$  are both 3.94 V. That means that  $v_c(0^+)$  is also 3.94 V because the capacitor voltage cannot change instantaneously. Before the switch closes  $i_L(0^-)$  is  $3A \times \frac{15\Omega + 10\Omega}{15\Omega + 10\Omega + 4\Omega \| 6\Omega} \times \frac{6\Omega}{6\Omega + 4\Omega} = 1.642 \text{ A}$ . Therefore  $i_L(0^+)$  is also 1.642 A because the current in an inductor cannot change instantaneously. At  $t = 0^+$  the switch is closed and the current-source current all flows through the switch. The capacitor is in parallel with  $R_1$  and  $R_3$  and its current is the negative of its voltage divided by  $R_1 \parallel R_3$  or -0.656 A. The inductor is now in parallel (and in series) with  $R_7$  and its voltage must be the negative of its current times  $R_7$  or -6.57 V. equation current and voltage for Using the defining for а capacitor,  $i(0^{+})$ -0.656 A Л

$$\frac{d}{dt} \left( v_C(t) \right)_{t=0^+} = \frac{I_C(0^-)}{C} = \frac{-0.656 \text{ A}}{5 \text{ mF}} = -131.2 \text{ V/s}.$$
 Using the defining equation for current and voltage for  $d_L(t, c_L) = \frac{V_L(0^+)}{C} = -6.57 \text{ V}$ 

an inductor, 
$$\frac{d}{dt} (i_L(t))_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{-6.57 \text{ V}}{2 \text{ H}} = -3.285 \text{ A/s}.$$

## Solution of ECE 300 Test 7 S09

1. The switch in the circuit below has been closed a very long time. Find the numerical values of  $v_c(0^+)$ ,  $i_c(0^-)$ ,  $i_c(0^+)$ ,  $v_c(200 \text{ms})$  and  $i_c(200 \text{ms})$ .

Just before the switch opens the capacitor has no current through it  $[i_c(0^-)=0]$  and  $v_c(0^-)$  is the same as the voltage across  $R_2$  which is 60 V  $\frac{40 \text{ k}\Omega}{40 \text{ k}\Omega + 20 \text{ k}\Omega} = 40 \text{ V}$ . The capacitor voltage cannot change instantaneously so the voltage is the same at  $t = 0^+$ , 40 V. At  $t = 0^+$  the switch is open and the voltage across  $R_2$  is 40 V. Therefore its current (downward) is 1 mA and that must be the negative of the capacitor current. So  $i_c(0^+) = -1 \text{ mA}$ . The time constant is  $\tau = 400 \text{ ms}$ .

 $\mathbf{v}_{c}(t) = \mathbf{v}_{c}(0^{+})e^{-2.5t}$ ,  $t > 0 \implies \mathbf{v}_{c}(200 \text{ ms}) = 40e^{-0.5} = 24.264 \text{ V}$  $\mathbf{i}_{c}(t) = \mathbf{i}_{c}(0^{+})e^{-2.5t}$ ,  $t > 0 \implies \mathbf{i}_{c}(200 \text{ ms}) = -1e^{-0.5} = -0.607 \text{ mA}$ 



2. The switch in the circuit below has been open a very long time. Find the numerical values of  $v_c(0^+)$ ,

$$\mathbf{i}_{C}(0^{-}), \, \mathbf{i}_{C}(0^{+}), \, \mathbf{i}_{L}(0^{+}), \, \mathbf{v}_{L}(0^{-}), \, \mathbf{v}_{L}(0^{+}), \, \frac{d}{dt}(\mathbf{v}_{C}(t))_{t=0^{+}} \text{ and } \frac{d}{dt}(\mathbf{i}_{L}(t))_{t=0^{+}}.$$

Before the switch closes all currents and voltages are constant so we know that  $i_c(0^-)=0$  and  $v_L(0^-)=0$ . At  $t=0^-$  the current downward through  $R_1$  is  $5A\frac{4\Omega||6\Omega}{15\Omega+10\Omega+4\Omega||6\Omega} = 5A\frac{2.4\Omega}{27.4\Omega} = 0.438 \text{ A}$ . So the voltage across it and  $v_c(0^-)$  are both 6.57 V. That means that  $v_c(0^+)$  is also 6.57 V because the capacitor voltage cannot change instantaneously. Before the switch closes  $i_L(0^-)$  is  $5A \times \frac{15\Omega+10\Omega}{15\Omega+10\Omega+4\Omega||6\Omega} \times \frac{6\Omega}{6\Omega+4\Omega} = 2.74 \text{ A}$ . Therefore  $i_L(0^+)$  is

also 2.74 A because the current in an inductor cannot change instantaneously. At  $t = 0^+$  the switch is closed and the current-source current all flows through the switch. The capacitor is in parallel with  $R_1$  and  $R_3$  and its current is the negative of its voltage divided by  $R_1 \parallel R_3$  or -1.095 A. The inductor is now in parallel (and in series) with  $R_7$  and its voltage must be the negative of its current times  $R_7$  or -10.95 V. Using the defining equation for current and voltage for a capacitor,  $\frac{d}{dt} (v_c(t))_{t=0^+} = \frac{i_c(0^+)}{C} = \frac{-1.095 \text{ A}}{5 \text{ mF}} = -219 \text{ V/s}$ . Using the defining equation for current and voltage  $\frac{d}{dt} (v_c(t))_{t=0^+} = \frac{v_t(0^+)}{C} = -10.95 \text{ V}$ .

for an inductor,  $\frac{d}{dt}(i_L(t))_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{-10.95 \text{ V}}{2 \text{ H}} = -5.475 \text{ A/s}.$ 

## Solution of ECE 300 Test 7 S09

1. The switch in the circuit below has been closed a very long time. Find the numerical values of  $v_c(0^+)$ ,  $i_c(0^-)$ ,  $i_c(0^+)$ ,  $v_c(200\text{ms})$  and  $i_c(200\text{ms})$ .

Just before the switch opens the capacitor has no current through it  $\left[i_c(0^-)=0\right]$  and  $v_c(0^-)$  is the same as the voltage across  $R_2$  which is 40 V  $\frac{40 \text{ k}\Omega}{40 \text{ k}\Omega + 20 \text{ k}\Omega} = 26.67 \text{ V}$ . The capacitor voltage cannot change instantaneously so the voltage is the same at  $t = 0^+$ , 26.67 V. At  $t = 0^+$  the switch is open and the voltage across  $R_2$  is 26.67 V. Therefore its current (downward) is 0.667 mA and that must be the negative of the capacitor current. So  $i_c(0^+) = -0.667 \text{ mA}$ . The time constant is  $\tau = 400 \text{ ms}$ .

- $\mathbf{v}_{C}(t) = \mathbf{v}_{C}(0^{+})e^{-2.5t}$ ,  $t > 0 \implies \mathbf{v}_{C}(200 \text{ ms}) = 26.67e^{-0.5} = 16.18 \text{ V}$
- $i_C(t) = i_C(0^+)e^{-2.5t}$ ,  $t > 0 \implies i_C(200 \text{ ms}) = -0.667e^{-0.5} = -0.4044 \text{ mA}$



2. The switch in the circuit below has been open a very long time. Find the numerical values of  $v_c(0^+)$ ,

$$\mathbf{i}_{C}(0^{-}), \, \mathbf{i}_{C}(0^{+}), \, \mathbf{i}_{L}(0^{+}), \, \mathbf{v}_{L}(0^{-}), \, \mathbf{v}_{L}(0^{+}), \, \frac{d}{dt}(\mathbf{v}_{C}(t))_{t=0^{+}} \text{ and } \frac{d}{dt}(\mathbf{i}_{L}(t))_{t=0^{+}}.$$

Before the switch closes all currents and voltages are constant so we know that  $i_{C}(0^{-}) = 0$  and  $v_L(0^-) = 0$ . At  $t = 0^{-}$ the downward current through *R*, is  $2A \frac{4\Omega || 6\Omega}{15\Omega + 10\Omega + 4\Omega || 6\Omega} = 2A \frac{2.4\Omega}{27.4\Omega} = 0.175 \text{ A}$ . So the voltage across it and  $v_c(0^-)$  are both 2.63 V. That means that  $v_c(0^+)$  is also 2.63 V because the capacitor voltage cannot change instantaneously. Before the switch closes  $i_L(0^-)$  is  $2A \times \frac{15\Omega + 10\Omega}{15\Omega + 10\Omega + 4\Omega ||6\Omega} \times \frac{6\Omega}{6\Omega + 4\Omega} = 1.095 \text{ A}$ . Therefore  $i_L(0^+)$  is also 1.095 A because the current in an inductor cannot change instantaneously. At  $t = 0^+$  the switch is closed and the current-source current all flows through the switch. The capacitor is in parallel with  $R_1$  and  $R_3$  and its current is the negative of its voltage divided by  $R_1 \parallel R_3$  or -0.437 A. The inductor is now in parallel (and in series) with  $R_{7}$  and its voltage must be the negative of its current times  $R_{7}$  or -4.38 V. Using the defining equation for current and voltage for а capacitor,  $\frac{d}{dt} \left( v_c(t) \right)_{t=0^+} = \frac{i_c(0^+)}{C} = \frac{-0.437 \text{ A}}{5 \text{ mF}} = -87.4 \text{ V/s}.$  Using the defining equation for current and voltage for

an inductor,  $\frac{d}{dt} (i_L(t))_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{-4.38 \text{ V}}{2 \text{ H}} = -2.19 \text{ A/s}.$ 

